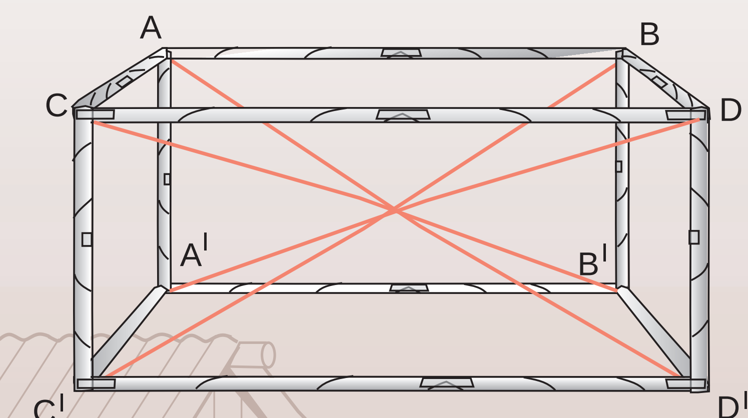
EDUCATION MATHEMATICS ELEMENTARY

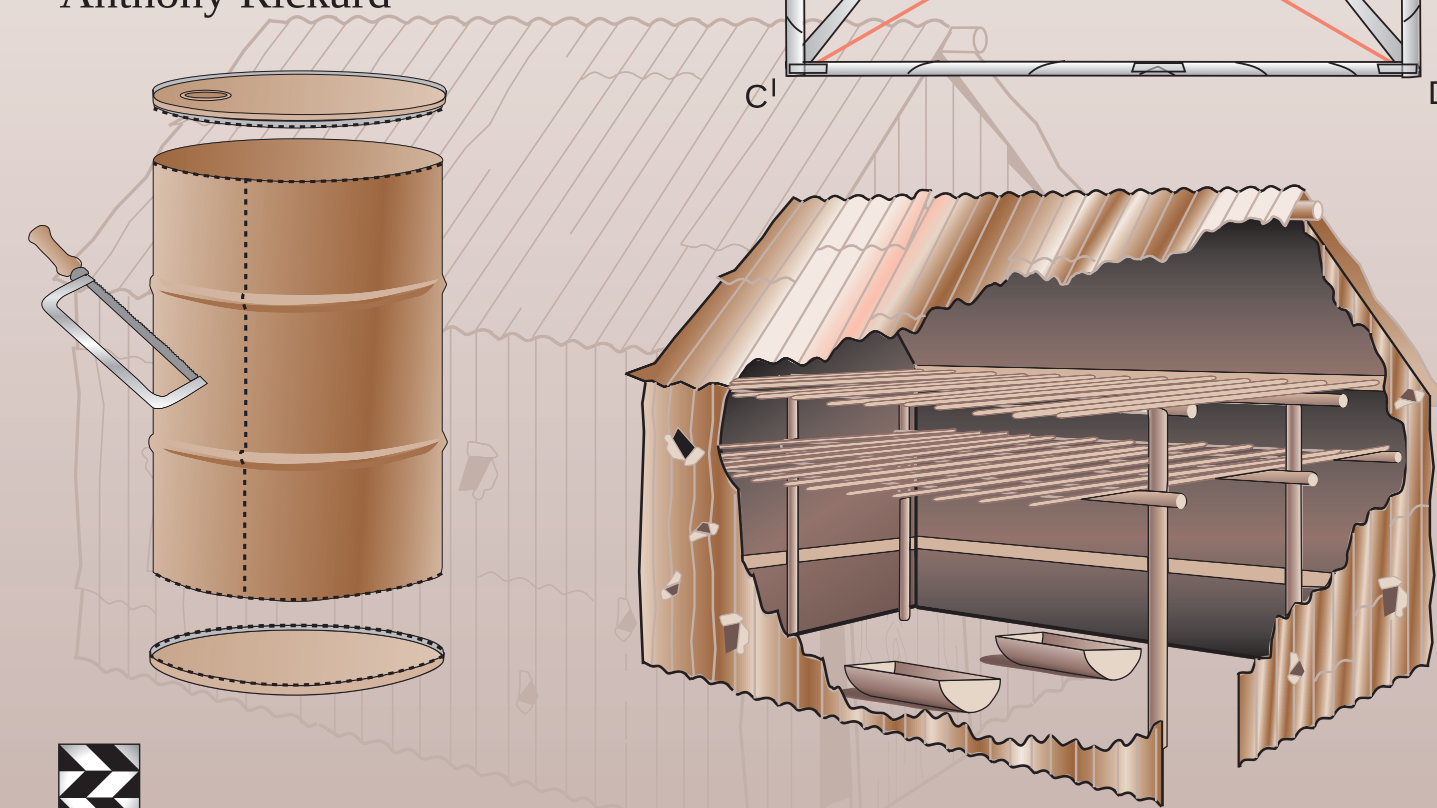
Building a Smokehouse

GRADE LEVEL
6/7
K 1 2 3 4 5 6 7

The Geometry of Prisms

Melissa Kagle
Valerie Barber
Jerry Lipka
Ferdinand Sharp
Anthony Rickard





MCC

Part of the series *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders©*

Building a Smokehouse The Geometry of Prisms

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders is the result of a long-term collaboration. These supplemental math modules for grades K-7 bridge the unique knowledge of Yup'ik elders with school-based mathematics. This series challenges students to communicate and think mathematically as they solve inquiry-oriented problems. Problems are constructed with constraints so that students can explore and understand mathematical relationships, properties of geometrical shapes, place value understanding, conjecturing, and proofs. The modules tap into students' creative, practical, and analytical thinking. Our classroom-based research strongly suggests that students engaged in this curriculum can develop deeper mathematical understandings than students who engage only with a procedure-oriented, paper and pencil curriculum.

Also in this series for Grade 2

Going to Egg Island: Adventures in Grouping and Place Values Students learn to group objects, compose and decompose numbers using the Yup'ik counting system (base 20 and sub-base 5), and use place value charts in multiple bases. Package includes a storybook, *Egg Island*, five posters, two CD-Roms, and coloring book.

Picking Berries: Connections Between Data Collection, Graphing, and Measuring Students engage in a series of hands-on activities that help them explore data, graphic representation, and linear measuring. Package includes a CD-Rom, one poster, and two storybooks, *Big John and Little Henry* and *Berry Picking*.

Patterns and Parkas: Investigating Geometric Principles, Shapes, Patterns and Measurement Students learn about shapes including squares, rectangles, triangles, and parallelograms, and their properties. They learn a variety of ways to make these shapes and how Yup'ik elders use these shapes to create patterns. As the students make shapes, they learn about symmetry and congruence and learn how to prove that a shape is a square or rectangle. They connect learning in the community to learning in school. The package includes a DVD documenting elders making patterns and *Iluvaktuq*, a storybook about a famous Yup'ik warrior.

Also in this series for Grades 3-5

Designing Patterns: Exploring Shapes and Area Students learn how to create a rhombus from a rectangle with only one cut. They explore properties of the resulting shapes, lines of symmetry, and part-to-part and part-to-whole relationships while designing linear patterns. Package includes a DVD demonstration by elders, a CD-Rom, and a storybook of the great Yup'ik warrior, *Iluvaktuq*.

Also in this series for Grade 6-7

Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area Following an elder demonstration, students must lay out a rectangular base and prove it is a rectangle. In so doing, they explore various properties of quadrilaterals, including measurements of perimeter and area. They further investigate the key relationship between area and dimensions of a rectangle when perimeter is held constant. Package includes two posters and a CD-Rom.

Salmon Fishing: Investigations into Probability Students use activities based on subsistence and commercial fishing in Southwest Alaska to investigate various topics within probability, such as experimental and theoretical probability, the law of large numbers, sample space, and equally and unequally likely events. Package includes two posters, a CD-Rom, and an interactive Excel spreadsheet.

Star Navigation: Explorations into Angles and Measurement Students become apprentices to Frederick George, an expert star navigator, by learning his methods of using hand measures to approximate angles. They learn ways of observing, measuring, and navigating during the daytime and at night, including specific details of the location and orientation of the Big Dipper and Cassiopeia. Through these experiences students refine their understanding of angle measurements and how they differ from linear measures. Package includes two posters, a CD-Rom, *The Star Navigation Reader* with traditional stories and nonfiction accounts related to navigating, and a DVD with the Morning Star song and dance.

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The *Supplemental Math Modules* curriculum was developed at the University of Alaska Fairbanks

Part of the series *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders ©*

Building a Smokehouse:

The Geometry of Prisms

Part of the Series

Math in a Cultural Context:

Lessons Learned from Yup'ik Eskimo Elders

Grade 6

Melissa Kagle

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Developed at University of Alaska Fairbanks, Fairbanks, Alaska

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders © was developed at the University of Alaska Fairbanks. This material is based upon work supported by the National Science Foundation under grant #9618099, *Adapting Yup'ik Elders' Knowledge: Pre-K-to-6th Math and Instructional Materials Development,* and U.S. Department of Education grant #5356A030033, *Developing and Implementing Culturally Based Curriculum and Teacher Preparation.*

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It is the collaboration between Yup'ik elders and educational specialists that makes the MCC series a dynamic curriculum. The Smokehouse module has benefitted from the input of many people, all of whom deserve my thanks.

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Last but not least, thank you to my loving family, who supported me in pursuing this dream. Although this has been a long-term collaborative endeavor, I hope that we have taken a small step to meet the desires of the elders for the next generation so that they will be flexible thinkers able to effectively function in the Yup'ik and Western worlds.

Introduction

Math in a Cultural Context:

Lessons Learned from Yup'ik Eskimo Elders

Introduction to the Series

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders is a supplemental math curriculum based on the traditional wisdom and practices of the Yup'ik Eskimo people of southwest Alaska. The kindergarten to seventh-grade math modules that you are about to teach are the result of more than a decade of collaboration between math educators, teachers, Yup'ik Eskimo elders, and educators to connect cultural knowledge to school mathematics. To understand the rich environment from which this curriculum came, imagine traveling on a snowmachine over the frozen tundra and finding your way based on the position of the stars in the night sky. Consider paddling a sleek kayak across open waters shrouded in fog, yet knowing which way to travel toward land by the pattern of the waves, or imagine building a kayak or making clothing and accurately sizing them by visualizing or using body measures. This is a small sample of the activities that modern Yup'ik people engage in. The mathematics embedded in these activities formed the basis for this series of supplemental math modules. Each module is independent and lasts from three to eight weeks.

From 2001 through fall 2005, with the exception of one urban trial, students who used these modules consistently outperformed at statistically significant levels over students who only used their regular math textbooks. This was true for urban as well as rural students, both Caucasian and Alaska Native. We believe that this supplemental curriculum will motivate your students and strengthen their mathematical understanding because of the engaging content, hands-on approach to problem solving, and the emphasis on mathematical communication. Further, these modules build on students' everyday experiences and intuitive understandings, particularly in geometry, which is underrepresented in school.

A design principle used in the development of these modules is that the activities allow students to explore mathematical concepts semi-autonomously. Though use of hands-on materials, students can "physically" prove conjectures, solve problems, and find patterns, properties, short cuts, or generalize. The activities incorporate multiple modalities and can challenge students with diverse intellectual needs. Hence, the curriculum is designed for heterogeneous groups with the realization that different students will tap into different cognitive strengths. According to Sternberg and his colleagues (1997, 1998), by engaging students creatively, analytically, and practically, students will have a more robust understanding of the concept. This allows for shifting roles and expertise among students rather than privileging those students with analytic knowledge.

The modules explore the everyday application of mathematical skills such as grouping, approximating, measuring, proportional thinking, informal geometry, and counting in base twenty and then the modules present these in terms of formal mathematics. Students move from the concrete and applied to more formal and abstract math. The activities are designed for students to be able to:

- learn to solve mathematical problems that support an in-depth understanding of mathematical concepts;
- derive mathematical formulas and rules from concrete and practical applications;
- become flexible thinkers by learning that there is more than one method of solving a mathematical problem;
- learn to communicate and think mathematically while they demonstrate understanding to peers; and
- learn content across the curriculum, since the lessons comprise Yup'ik Eskimo culture, literacy, geography, and science.

Beyond meeting some of the content (mathematics) and process standards of the National Council of Teachers of Mathematics (2000), the curriculum design and its activities respond to the needs of diverse learners. Many activities are designed for group work. One of the strategies for using group work is to provide leadership opportunities to students who may not typically be placed in that role. Also, the modules tap into a wide array of intellectual abilities—practical, creative, and analytic. We assessed modules tested in rural and urban Alaska and found that students who were only peripherally involved in math became more active participants when using this curriculum.

Our goal is for students to learn to reason mathematically by constructing models and analyzing practical tasks for their embedded mathematics. This enables them to generate and discover mathematical rules and formulas. In this way, we offer students a variety of ways to engage the math material through practical activity, spatial/visual learning, analytic thinking, and creative thinking. Students are constantly encouraged to communicate mathematically by presenting their understandings while other students are encouraged to provide alternate solutions, strategies, and counter arguments. This process also strengthens their deductive reasoning.

Pedagogical Approach Used in the Modules

The concept of third space is embedded within each module. Third space relates to a dynamic and creative place between school-based knowledge, everyday knowledge, and knowledge related to other nonmainstream cultural groups. Third space also includes local knowledge such as ways of measuring and counting that are distinct from school-based notions, and it is about bringing these elements together in a creative, respectful, and artful manner. Within this creative and evolving space, pedagogical content knowledge can develop creatively from both Western schooling and local ways. In particular, this module pays close attention to expert-apprentice modeling because of its prevalent use among Yup'ik elders and other Alaska Native groups.

Design

The curriculum design includes strategies that engage students:

- cognitively, so that students use a variety of thinking strategies (analytic, creative, and practical);
- socially, so that students with different social, cognitive, and mathematical skills use those strengths to lead and help solve mathematical problems;
- pedagogically, so that students explore mathematical concepts and communicate and learn to reason mathematically by demonstrating their understanding of the concepts; and
- practically, as students apply or investigate mathematics to solve problems from their daily lives.

The organization of the modules follows five distinct approaches to teaching and learning that converge into one system.

Expert-Apprentice Modeling

The first approach, expert-apprentice modeling, comes from Yup'ik elders and teachers and is supported by research in anthropology and education. Many lessons begin with the teacher (the expert) demonstrating a concept to the students (the apprentices). Following the theoretical position of the Russian psychologist Vygotsky (cited in Moll, 1990), expert Yup'ik teachers (Lipka and Yanez, 1998), and elders, students begin to appropriate the knowledge of the teacher (who functions in the role of expert), as the teacher and the more adept apprentices help other students learn. This establishes a collaborative classroom setting in which student-to-student and student-to-teacher dialogues are part of the classroom fabric.

More recently, we have observed experienced teachers using joint productive activity—the teacher works in parallel with students modeling an activity, a concept, or a skill. When effectively implemented, joint productive activity appears to increase student ownership of the task and increases their responsibility and motivation.

The typical authority structure surrounding classrooms changes as students take on more of the responsibility for learning. Social relations in the classroom become more level. In the case of this module the connections between out-of-school learning and in-school learning are strengthened through pedagogical approaches such as expert-apprentice modeling and joint productive activity when those are approaches of the community.

Reform-oriented Approach

The second pedagogical approach emphasizes student collaboration in solving "deeper" problems (Ma, 1999). This approach is supported by research in math classrooms and particularly by recent international studies (Stevenson et al., 1990; Stigler and Hiebert, 1998) strongly suggesting that math problems should be more in-depth and challenging and that students should understand the underlying principles, not merely use procedures competently. The modules present complex problems (two-step open-ended problems) that require students to think more deeply about mathematics.

Multiple Intelligences

Further, the modules tap into students' multiple intelligences. While some students may learn best from hands-on, real-world related problems, others may learn best when abstracting and deducing. This module provides opportunities to guide both modalities. Robert Sternberg's work (1997, 1998) influenced the development of these modules. He has consistently found that students who are taught so that they use their analytic, creative, and practical intelligences will outperform students who are taught using one modality, most often analytic. Thus, we have shaped our activities to engage students in this manner.

Mathematical Argumentation and Deriving Rules

The purpose of math communication, argumentation, and conceptual understanding is to foster students' natural abilities. These modules support a math classroom environment in which students explore the underlying mathematical rules as they solve problems. Through structured classroom communication, students will learn to work collaboratively in a problem-solving environment in which they learn both to appreciate alternative solutions and strategies and to evaluate these strategies and solutions. They will present their mathematical solutions to their peers. Through discrepancies in strategies and solutions, students will communicate with and help each other to understand their reasoning and mathematical decisions. Mathematical discussions are encouraged to strengthen students' mathematical and logical thinking as they share their findings. This requires classroom norms that support student communication and learning from errors rather than to criticizing them. Activities in the module are organized by connecting tasks and materials so that the possible explorations are limited, increasing the likelihood that students will understand the concept or deduce principles. Students are given the opportunity to support their conceptual understanding by practicing it in the context of a particular problem.

Familiar and Unfamiliar Contexts Challenge Students' Thinking

By working in unfamiliar settings and facing new and challenging problems, students learn to think creatively. They gain confidence in their abilities to solve both everyday problems and abstract mathematical questions, and their entire realm of knowledge and experience expands. Further, by making the familiar unfamiliar and by working on novel problems, students are encouraged to connect what they learn from one setting (everyday problems) with mathematics in another setting. For example, most sixth-grade students know about rectangles and how to calculate the area of a rectangle, but if you ask students to go outside and find the four corners of an eight-foot-by twelve-foot-rectangle without using rulers or similar instruments, they are faced with a challenging problem. As they work through this everyday application (which is needed to build any rectangular structure) and as they "prove" to their classmates that they do, in fact, have a rectangular base, they expand their knowledge of rectangles. In effect they must shift their thinking from considering rectangles as physical entities or as prototypical examples to understanding the salient properties of a rectangle. Similarly, everyday language, conceptions, and intuition may, in fact, be in the way of mathematical understanding and the precise meaning of mathematical terms. By treating familiar knowledge in unfamiliar ways, students explore and confront their own mathematical understandings and begin to understand the world of mathematics. These major principles guide the overall pedagogical approach to the modules.

Literacy Counts!: Developing Language and Literacy in MCC

As MCC has developed over the years, the importance of the role of literacy has also grown. The inclusion of culturally based stories has proved to contribute to students' engagement with the math modules as well as provide cultural grounding for the module activities. MCC modules have also made use of literacy-based activities, such as journaling, to further students' understanding of math concepts and vocabulary. Building on these trends and practices, *Literacy Counts!: A Teacher's Guide to Developing Literacies across the Curriculum* was developed by our literacy team of Joan Parker Webster, Evelyn Yanez, and Dora Andrew-Ihrke.

There are two strands within the Literacy Counts! approach: (a) Strand 1 is designed to develop literacy in the traditional sense of linguistic modes (speaking, writing, reading, listening, and presenting) as well as other non-linguistic modes, such as two-dimensional drawings or constructing three-dimensional models to communicate mathematically, and practice mathematics; and (b) Strand 2 is designed to develop multiple literacies (linguistic, visual, kinesthetic, dramatic, etc.), through the use of culturally relevant stories and nonfiction literature that accompany MCC modules.

The Organization of the Modules

The curriculum includes modules for kindergarten through seventh grade. Modules are divided into sections: activities, explorations, and exercises, with some variation between each module. Supplementary information is included in Cultural Notes, Teacher Notes, and Math Notes. Each module follows a particular cultural story line, and the mathematics connect directly to it. Some modules are designed around a children's story, and an illustrated text is included for the teacher to read to the class.

The module is a teacher's manual. It begins with a general overview of the activities ahead, an explanation of the math and pedagogy of the module, teaching suggestions, and a historical and cultural overview of the curriculum in general and of the specific module. Each activity includes a brief introductory statement, an estimated duration, goals, materials, any pre-class preparatory instructions for the teacher, and the procedures for the class to carry out the activity. Assessments are placed at various stages, both intermittently and at the end of activities.

Illustrations help to enliven the text. Yup'ik stories and games are interspersed and enrich the mathematics. Transparency masters, worksheet masters, and suggestions for additional materials are attached at the end of each activity. An overhead projector is necessary. Blackline masters that can be made into overhead transparencies are an important visual enhancement of the activities, stories, and games. Such visual aids also help to further classroom discussion and understanding.

Resources and Materials Required to Teach the Modules

Materials

The materials and tools limit the range of mathematical possibilities, guiding students' explorations so that they focus upon the intended purpose of the lesson. For example, in one module, latex sheets are used to explore concepts of topology. Students can manipulate the latex to the degree necessary to discover the mathematics of the various activities and apply the rules of topology.

For materials and learning tools that are more difficult to find or that are directly related to unique aspects of this curriculum, we provide detailed instructions for the teacher and students on how to make those tools. For example, in *Going to Egg Island: Adventures in Grouping and Place Values,* students use a base twenty abacus. Although the project has produced and makes available a few varieties of wooden abaci, detailed instructions are provided for the teacher and students on how to make simple, inexpensive, and usable abaci with beads and pipe cleaners.

Each module and each activity lists all of the materials and learning tools necessary to carry it out. Some of the tools are expressly mathematical, such as interlocking centimeter cubes, abaci, and compasses. Others are particular to the given context of the problem, such as latex and black and white geometric pattern pieces. Many of the materials are items a teacher will probably have on hand, such as paper, markers, scissors, and rulers. Students learn to apply and manipulate the materials. The value of caring for the materials is underscored by the precepts of subsistence, which is based on processing raw materials and foods with maximum use and minimum waste. Periodically, we use food as part of an activity. In these instances, we encourage minimal waste.

DVDs

To convey the knowledge of the elders underlying the entire curriculum more vividly, we have produced a few DVDs to accompany some of the modules. For example, the *Going to Egg Island: Adventures in Grouping and Place Values* module includes DVDs of Yup'ik elders demonstrating some traditional Yup'ik games. We also have footage and recordings of the ancient chants that accompanied these games. This module includes the DVD *Tumartat: Putting the Pieces Together to Make a Whole*.

Yup'ik Language Glossary and Math Terms Glossary

To help teachers and students get a better feel for the Yup'ik language, its sounds, and the Yup'ik words used to describe mathematical concepts in this curriculum, we have developed a Yup'ik glossary on CD-ROM. Each word is recorded in digital form and can be played back in Yup'ik. The context of the word is provided, giving teachers and students a better sense of the Yup'ik concept, not just its Western "equivalent." Pictures and illustrations often accompany the word for additional clarification.

Yup'ik Values

There are many important Yup'ik values associated with each module. The elders counsel against waste. They value listening, learning, working hard, being cooperative, and passing knowledge on to others. These values are expressed in the content of the Yup'ik stories that accompany the modules, in the cultural notes, and in various activities. Similarly, Yup'ik people as well as other traditional people continue to produce, build, and make crafts from raw materials. Students who engage in these modules also learn how to make simple mathematical tools fashioned around such themes as Yup'ik border patterns and building model kayaks, fish racks, and smokehouses. Students learn to appreciate and value other cultures.

Cultural Notes

Most of the mathematics used in the curriculum comes from our direct association and long-term collaboration with Yup'ik Eskimo elders and teachers. We have included many cultural notes to describe and explain more fully the purposes, origins, and variations associated with a particular traditional activity. Each module is based on a cultural activity and follows a Yup'ik cultural story line along which the activities and lessons unfold.

Math Notes

We want to ensure that teachers who may want to teach these modules but feel unsure of some of the mathematical concepts will feel supported by the Math Notes. These provide background material to help teachers better understand the mathematical concepts presented in the activities and exercises of each module. For example, in the *Perimeter and Area* module, the Math Notes give a detailed description of a rectangle and describe the geometric proofs one would apply to ascertain whether or not a shape is a rectangle. One module explores rectangular prisms and the geometry of three-dimensional objects; the Math Notes include information on the geometry of rectangular prisms, including proofs, to facilitate the instructional process. In every module, connections are made between the "formal math," its practical application, and the classroom strategies for teaching the math.

Teacher Notes

The main function of the Teacher Notes is to focus on the key pedagogical aspects of the lesson. For example, they provide suggestions on how to facilitate students' mathematical understanding through classroom organization strategies, classroom communication, and ways of structuring lessons. Teacher Notes also make suggestions for ways of connecting out-of-school knowledge with schooling.

Assessment

Assessment and instruction are interrelated throughout the modules. Assessments are embedded within instructional activities, and teachers are encouraged to carefully observe, listen, and challenge their students' thinking. We call this active assessment, which allows teachers to assess how well students have learned to solve the mathematical and cultural problems introduced in a module.

Careful attention has been given to developing assessment techniques and tools that evaluate both the conceptual and procedural knowledge of students. We agree with Ma (1999) that having one type of knowledge without the other, or not understanding the link between the two, will produce only partial understanding. The goal here is to produce relational understanding in mathematics. Instruction and assessment have been developed and aligned to ensure that both types of knowledge are acquired; this has been accomplished using both traditional and alternative techniques. The specific details and techniques for assessment (when applicable) are included within activities. The three main tools for collecting and using assessment data follow.

Math Notebooks

In recent years, NCTM has promoted standards that incorporate math notebooks as part of math instruction. Journaling has most often occurred as a tool for reflecting on what was learned. In contrast, math notebooks, which are incorporated in strand one of Literacy Counts, are used by students to record what they are thinking and learning about math concepts before, during, and after the activities in the modules. Through the use of math notebooks, students build their content knowledge while at the same time developing their literacy skills through reading, writing, drawing, and graphic representations. Math notebooks also play an important role in helping students develop math vocabulary.

Observation

Observing and listening to students lets teachers learn about the strategies that they use to analyze and solve various problems. Listening to informal conversations between students as they work cooperatively on problems provides further insight into their strategies. Through observation, teachers also learn about their students' attitudes toward mathematics and their skills in cooperating with others. Observation is an excellent way to link assessment with instruction.

Adaptive Instruction

The goal of the summary assessment in this curriculum is to adapt instruction to the skills and knowledge needed by a group of students. From reviewing notebook notes to simply observing, teachers learn which mathematical processes their students are able to effectively use and which ones they need to practice more. Adaptive assessment and instruction complete the link between assessment and instruction.

An Introduction to the Land and Its People, Geography, and Climate

Flying over the largely uninhabited expanse of southwest Alaska on a dark winter morning, one looks down at a white landscape interspersed with trees, winding rivers, rolling hills, and mountains. One sees a handful of lights sprinkled here, a handful there. Half of Alaska's 600,000-plus population lives in Anchorage. The other half is dispersed among smaller cities such as Fairbanks and Juneau and among the over 200 rural villages that are scattered across the state. Landing on the village airstrip, which is usually gravel and, in the winter, covered with smooth, hard-packed snow, one is taken to the village by either car or snowmachine. Few villages or regional centers are connected to a road system. The major means of transportation between these communities is by small plane, boat, and snowmachine, depending on the season.

It is common for the school to be centrally located. Village roads are usually unpaved, and people drive cars, four-wheelers, and snowmachines. Houses are typically made from modern materials and have electricity and running water. Over the past 20 years, Alaska villages have undergone major changes, both technologically and culturally. Most now have television, a full phone system, modern water and sewage treatment facilities, an airport, and a small store. Some also have a restaurant, and a few even have a small hotel and taxicab service. Access to medical care and public safety are still sporadic, with the former usually provided by a local health care worker and a community health clinic, or by health care workers from larger cities or regional centers who visit on a regular basis. Serious medical emergencies require air evacuation to either Anchorage or Fairbanks.

The Schools

Years of work have gone into making education as accessible as possible to rural communities. Almost every village has an elementary school, and most have a high school. Some also have a higher education satellite facility, computer access to higher education courses, or options that enable students to earn college credits while in their respective home communities. Vocational education is taught in some of the high schools, and there are also special vocational education facilities in some villages. While English has become the dominant language throughout Alaska, many Yup'ik children in the villages still learn Yup'ik at home.

Yup'ik Village Life Today

Most villagers continue to participate in the seasonal rounds of hunting, fishing, and gathering. Although many modern conveniences are located within the village, when one steps outside of its narrow bounds, one is immediately aware of one's vulnerability in this immense and unforgiving land, where one misstep can lead to disaster. Depending upon their location (coastal community, riverine, or interior), villagers hunt and gather the surrounding resources. These include sea mammals, fish, caribou, and many types of berries. The seasonal subsistence calendar illustrates which activities take place during the year (see Figure i). Knowledgeable elders know how to cross rivers and find their way through ice fields, navigating the seemingly featureless tundra by using directional indicators such as frozen grass and the constellations in the night sky. All of this can mean the difference between life and death. In the summer, when this largely treeless, moss- and grass-covered plain thaws into a large swamp dotted with small lakes, the consequences of ignorance, carelessness, and inexperience can be just as devastating. Underwater hazards in the river, such as submerged logs, can capsize a boat, dumping the occupants into the cold, swift current. Overland travel is much more difficult during the warm months due to the marshy ground and many waterways, and one can easily become disoriented and get lost. The sea is also integral to life in this region and requires its own set of skills and specialized knowledge to be safely navigated.

The Importance of the Land: Hunting and Gathering

Basic subsistence skills include knowing how to read the sky to determine the weather and make appropriate travel plans, being able to read the land to find one's way, knowing how to build an emergency shelter and, in the greater scheme, how to hunt and gather food and properly process and store it. In addition, the byproducts of subsistence activities, such as carved walrus tusks, pelts, and skins are made into clothing or decorative items and a variety of other utilitarian arts and crafts products and provide an important source of cash for many rural residents.

Hunting and gathering are still of great importance in modern Yup'ik society. A young man's first seal hunt is celebrated; family members who normally live and work in one of the larger cities will often fly home to help when the salmon are running, and whole families still gather to go berry picking. The importance of hunting and gathering in daily life is further reflected in the legislative priorities expressed by rural residents in Alaska. These focus on such things as subsistence hunting regulations, fishing quotas, resource development, and environmental issues that affect the well-being of game animals and subsistence vegetation.

Conclusion

We developed this curriculum in a Yup'ik context. The traditional subsistence and other skills of the Yup'ik people incorporate spatial, geometrical, and proportional reasoning and other mathematical reasoning. We have attempted to offer you and your students a new way to approach and apply mathematics while also learning about Yup'ik culture. Our goal has been to present math as practical information that is inherent in everything we do. We hope your students will adopt and incorporate some of this knowledge and add it to the learning base.

We hope you and your students will benefit from the mathematics, culture, geography, and literature embedded in the *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders* series. The elders who guided this work emphasized that the next generation of children should be flexible thinkers and leaders. In a small way, we hope that this curriculum guides you and your students along this path.

Tua-i-ngunrituq [This is not the end].

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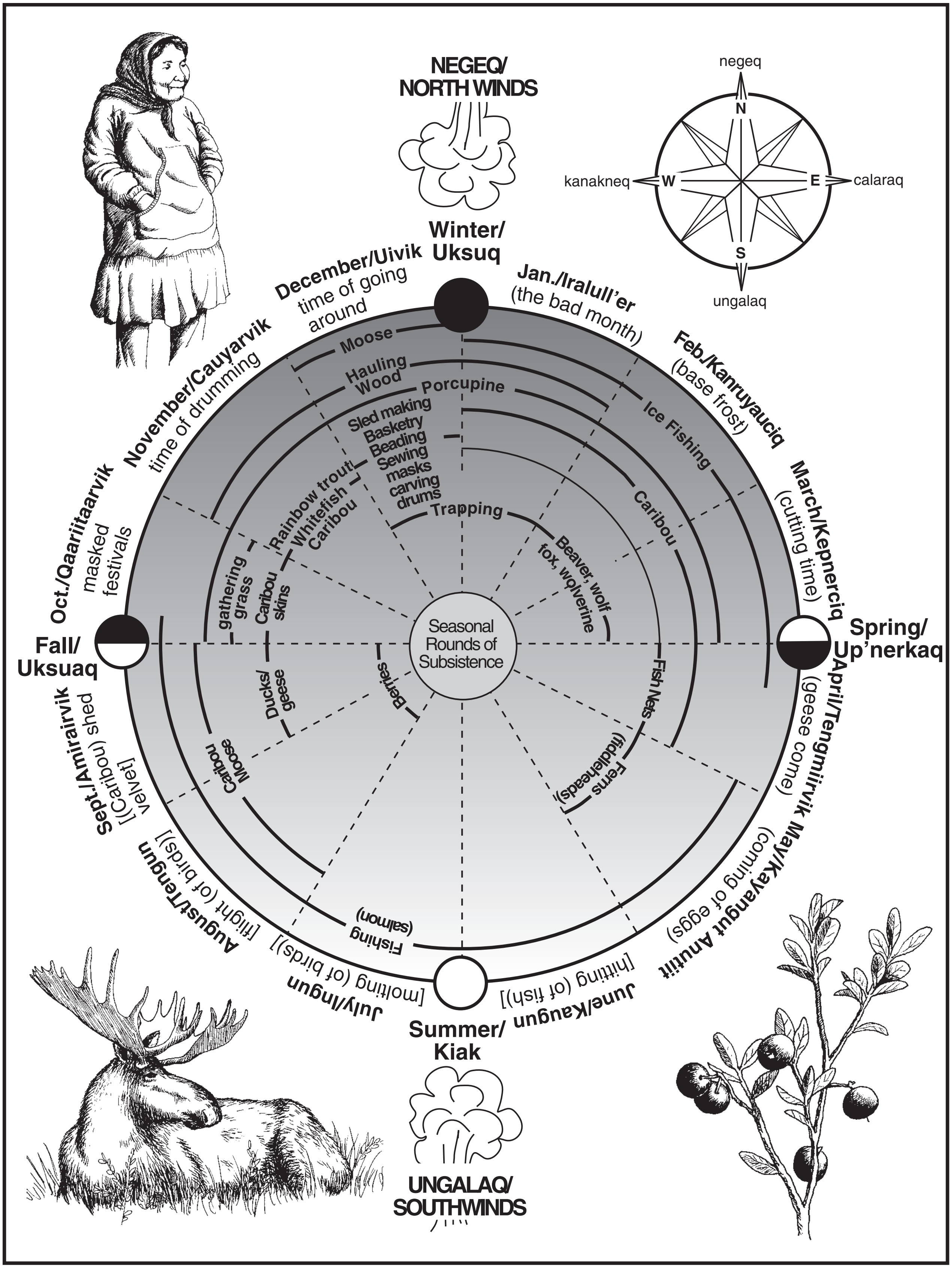


Fig. i: Yearly subsistence calendar

Introduction to the Module

The mathematics in this module is set in the context of the Yup'ik people's main summer subsistence fishing season, July through September. During this time, families and communities up and down the Alaska coast catch, clean, and begin the process of drying their annual catch of salmon. In this module, students will learn how to build a smokehouse modeled after ones currently used by the Yup'ik people. In the process, they will be acquiring and utilizing math skills necessary for the problems they encounter. This module incorporates geometry, which tends to be underrepresented in the primary and middle school grades. The NCTM Standards emphasize geometry for grade levels 6-7. Geometry is an intuitive discipline and is important because it is a way of describing our synthetic as well as natural universe and is used daily by people from all walks of life. This module draws upon students' intuitive grasp of geometry, turning it into a more formal understanding of geometry and geometric vocabulary. Geometric explorations help students develop problem-solving skills through spatial reasoning and manipulation. As students gain experience through model building and exploration, they go from an intuitive understanding of properties of two- and three-dimensional figures to a more formal understanding by applying the concepts they learn. These experiences will build a solid foundation for future study of geometry.

Mathematics of the Module

In this module, students learn properties of three-dimensional shapes by building smokehouse models. As students construct their models they develop an understanding of mathematical concepts related to three-dimensional geometry (See Figure 1). For example, by framing the smokehouse walls, students learn about the edges and faces of rectangular prisms. They identify congruent faces, describe the angles between faces, and determine the number of faces in the three-dimensional shape. By framing their model smokehouse roofs, they explore the properties of triangular prisms. As they cover their smokehouse models with "siding," students learn to calculate surface area.

Students generalize and make conjectures using knowledge of properties of rectangles and triangles and then prove or disprove their ideas through hands-on activities. They generalize by making conjectures about properties of three-dimensional shapes and explore ways of proving or disproving their conjectures.

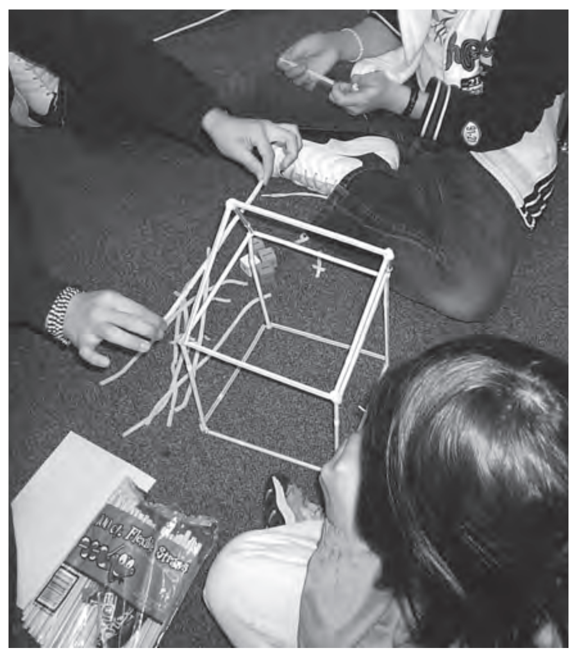


Fig. 1: Smokehouse model.

Generalizing is a theme of this module and is a key component of mathematical thinking that is often overlooked. It can be argued that multiplication is a way to generalize adding, just as adding is a generalization of counting. Algebra is a way to generalize the principles and methods of arithmetic. Algebraic expressions provide ways to describe the relationships among quantities that can vary. When expressed as equations, these relationships can be used to find unknown values of the variables. Identifying patterns, creating expressions to describe those patterns, and then applying the patterns in other situations is the most basic form of generalization. In *Building a Smokehouse*, students look at how properties of shapes can be generalized so that an understanding of one shape can lead to insights about related ones. Generalization can be mistaken, so the process becomes another form of conjecturing and proving that requires encouragement to rework over and over if necessary. The more we can encourage students to think in terms of generalizing, the more we can help students to think mathematically. In the module, places where students are asked to generalize are marked with "sticky notes" such as on page 51.

The smokehouse module builds on the students' prior knowledge of area and perimeter and the properties of a rectangle that were developed in the *Math in a Cultural Context* (MCC) series in *Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area*. In *Building a Smokehouse*, students design and build a model structure, from the rectangular foundation to the triangular prism roof, using nonstandard measurements. Through manipulative activities, students develop concepts necessary to generalize and solve problems.

Learning Mathematics in a Cultural Context

Building a Smokehouse: The Geometry of Prisms engages students with the mathematics of three-dimensional figures within the cultural context of constructing a smokehouse. You may teach about the Yup'ik culture by interspersing information about the geography of southwest Alaska, the importance of smoking salmon, and subsistence living. You should help students make connections between fishing, food preservation, and river ecosystems. You may also take some time at the beginning of the module to establish the cultural material included in this introduction.

Building smokehouses at fish camp is an important part of traditional Yup'ik subsistence in rural Alaska. Since smokehouses can be modeled using rectangular and triangular prisms, students learn about and apply the mathematical properties of prisms as they design a smokehouse and study how a smokehouse functions. For example, students learn that the sides of a smokehouse can be modeled by individual rectangles connecting to form a rectangular prism and that the roof can be modeled by a triangular prism. Decomposing the smokehouse into rectangular and triangular prisms also motivates study of surface area, slope, and an introduction to cylinders. All of these concepts are connected to smokehouses so that mathematical learning is anchored to important cultural practices. This connection provides students with a real world, cultural context in which to explore mathematics, and mathematics with which to study and understand cultural practices.

Students' prior knowledge of rectangles and triangles serves as a foundation on which to build understanding of prisms. For example, students learn that the faces of rectangular prisms are rectangles and that the sides the rectangles share form the edges of the prism. Mathematical findings such as these are then used to identify, explore, and formally define key properties of rectangular prisms. The same process is generalized to triangles to learn about triangular prisms. Finding the total area of all the faces of a prism in order to determine the amount of material required to build a smokehouse naturally brings out the concept of surface area. Triangular prisms, used to model the roof of a smokehouse, motivate the concept of slope, which is used to measure the angle, or pitch, of the roof. Finally, students are introduced to cylinders and surface area as they learn how oil drums are dismantled to build the exterior surface of a smokehouse. In the last section of the module, students learn about the measurement of surface area and develop strategies for determining the surface area of a cylinder and of rectangular and triangular prisms.

All of the mathematics learned throughout the module in geometry, algebra, and measurement are directly connected to the Alaska context and traditional Yup'ik practices and knowledge of building smokehouses at fish camp. All of these investigations take a constructivist approach to teaching and learning where students, under the engaged guidance of their teacher, build on, modify, and refine their own knowledge of mathematics. This process of teaching and learning is consistent with Yup'ik cultural practices of expert-apprentice modeling and the application of knowledge to solve problems.

In fact, during the summer of 2002 at the Math in a Cultural Context Summer Math Institute elders, teachers, and mathematicians built model smokehouses. The following pictures illustrate the process of using cultural, practical, creative, and analytic knowledge. Henry Alakayak, an elder from Manokotak, took a leadership role at this institute. Figures 2 and 3 show Henry Alakayak measuring pieces for the model smokehouse. After Henry Alakayak and the others at the Summer Math Institute completed their models (practical thinking), they explored some of the properties (analytic thinking) of a rectangular prism (smokehouse model). This approach of building models and exploring geometric properties is a central feature of this module. (See Figure 4)

Fig. 2: Henry Alakayak works on his model smokehouse. Photo courtesy of Nina Knapp.

Fig. 3: Detail of Henry Alakayak measuring. Photo courtesy of Nina Knapp.

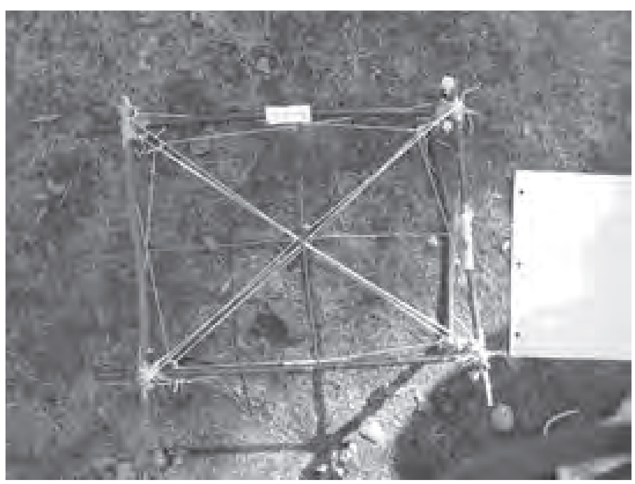


Fig. 4: Model used by elders and teachers for exploring the properties of a prism. Photo courtesy of Nina Knapp.

A note from the Series Editor:

We are please to state that the *Math in a Cultural Context* series has been extensively tested and the results are very positive. When this module was tested, students who used it outperformed students who used their regular curriculum at statistically significant levels and with a large effect size (.84). We are confident that your students will learn.

Sections of the module

In the first section of the module, students review what they know about quadrilaterals. Students will determine key properties of rectangles (e.g., opposite sides are parallel and of equal length) and use them to derive additional properties of rectangles. For example, students learn that the diagonals of a rectangle are always of equal length and that this is not true for other kinds of parallelograms. This is the mathematical reason that the rectangular base of a smokehouse is determined by measuring the lengths of the diagonals. Applying this knowledge will support students' development of reasoning and proof as they learn to show whether a particular parallelogram is or is not a rectangle (e.g., if the diagonals of a parallelogram are equal, then it must be a rectangle).

In the second section of the module, students build a model of a smokehouse with a flat roof. This helps them generalize their knowledge about rectangles into three dimensions as they learn about rectangular prisms. Students learn that just as rectangles have opposite sides that are parallel and equal, rectangular prisms have opposite faces made up of congruent parallel rectangles.

In the third section of the module, students learn about the concept of slope: a ratio that is a measure of the steepness of a line or plane and is determined by dividing the vertical and horizontal distances between two points on the line or plane. For example, a tilted plane that rises 8 units and runs 10 units has a slope of 8/10 = 4/5 = 0.80. Students learn to think of this measure as the familiar rise/run or "change in y" divided by "change in x" mathematical definition of slope. Students then connect the concept of slope to smokehouses by using slope to measure the pitch, or steepness, of the roof of the smokehouse. Students use their mathematical knowledge about slope to reason that the greater the slope, or pitch, the steeper the line or plane. This is connected to the smokehouse by exploring how the steeper roof more easily sheds snow or rain. This is important because too much snow accumulating on the roof could cause the smokehouse to collapse, and standing water on the roof increases the chances that the roof may leak.

Knowledge of slope and rectangular prisms are then linked to help students learn about triangular prisms. Students learn definition and properties of triangular prisms as a generalization of the definitions and properties of triangles and rectangular prisms. Eventually, students analyze their smokehouse models as a composite of two prisms—the floor and sides of the smokehouse form a rectangular prism, and the roof sections form a triangular prism.

In the fourth and final section of the module, the concept of surface area is developed as a way for students to determine how much material is required to cover the outside of a smokehouse. Surface area is defined as the total area of all the sides and roof of the smokehouse. The properties of rectangular and triangular prisms are used to determine the surface areas of those shapes and thus the total surface area for a smokehouse. Old oil drums are commonly used to cover the outside of smokehouses. To deal with this, students explore the properties and surface area of a cylinder. Students decompose the surface area of a cylinder into two circles (the top and bottom of the oil drum) and a rectangle (the side of the cylinder cut vertically and unwrapped to form a rectangle).

Throughout the module, students are asked to formulate ideas about relationships and properties, and then test these ideas by building models, trying to find counterexamples, or reasoning from known properties and relationships to prove or verify their ideas. The module also supports students in connecting their ideas about mathematical concepts, strategies, and relationships back to smokehouses to provide a concrete application and interesting context for the mathematics.

Unpacking the Mathematics

Mathematical Reasoning and Proof

Identifying properties of shapes, and then using them to develop conjectures and proofs about shapes, is central to the module. For example, students learn that rectangles are one kind of quadrilateral, that they are also parallelograms, and that they have diagonals of equal length. Other quadrilaterals, such as isosceles trapezoids, also have diagonals of equal length. By the end of the first section of the module, students should be able to demonstrate that an isosceles trapezoid is not a parallelogram and that, therefore, while it has diagonals of the same length, a rectangle is still the only *parallelogram* that has diagonals of equal length. Making such distinctions is key to developing skill and understanding with mathematical reasoning by using mathematical properties to develop mathematical generalizations. In sections two and three of the module, students further develop their skills in mathematical reasoning and proof by generalizing properties of quadrilaterals and rectangles into three dimensions as they investigate rectangular and triangular prisms.

Algebraic Concept of Slope and Proportional Reasoning

While this module focuses primarily on the NCTM content standards of geometry and measurement, the algebraic concept of slope is introduced in section three because it arises naturally within the context of building smokehouses. The steepness of a smokehouse roof is explicitly connected to formal mathematics by having students make inclined planes and record their data to investigate slope as a constant ratio of the rise to the run. This also helps students develop understanding of proportional reasoning and using mathematics to model real situations. When they study algebra more formally, students can apply their knowledge of slope to lines on the coordinate plane, using the y- and x-axes to measure rise and run. They will also learn that the slope of a line is a common way of modeling proportional relationships. In this way, the activities of this module help prepare students for more formal algebraic studies.

Measuring Surface Area

The measurement of surface area is developed in Section 4 of the module as a generalization of the two-dimensional concept of perimeter to three dimensions. The mathematical processes of generalizing and developing strategies with reasoning and proof are connected to surface area so students can articulate formal mathematical approaches to determining the surface areas of rectangular and triangular prisms and cylinders. The smokehouse context for these investigations serves as a critical bridge from cultural practices and knowledge to formal mathematical concepts and procedures.

Teaching the Module

This smokehouse module may be taught within a math class or can be integrated with language arts, science, or social studies lessons. Allow room for the students' smokehouse models to be displayed and remain up throughout the module. Student-generated lists will also need to be displayed throughout the module.

Building a Smokehouse Model

The main activity in this module is the construction of three-dimensional models of smokehouses. Model sizes should be determined by the amount of space you are able to give up for the duration of the unit and the type of materials you are able to use. Models can be constructed using different types of materials as listed in Activity 3.

The mathematics of the module are taught through the construction of the model. The activities give students a focused way to explore three-dimensional geometry. After each step in the model building, there are group and whole class discussions as well as notebook entries that are designed to help students understand the math that is embedded in making the model. Guiding questions are provided as well as background information on three-dimensional geometry to help generate rich discussions in the class about the math topics and help students formalize their emerging understandings.

Students work in construction groups starting in Activity 2 and remain in these groups throughout the module. We feel that three students is the ideal number for a construction group, but two to four students per group will also work. We have given some suggestions for group roles in Activity 2. These roles can be switched for the different activities.

Pedagogical Strategies Emphasized in this Module

Joint Activity

A key pedagogical strategy used in this module is one we call "joint activity." This joint activity stems from a Yup'ik teaching method where the expert (in this case, the teacher) works on the same task as the students. In this module, the teacher constructs a model smokehouse along with the students. This strategy allows students needing further guidance to observe and continue learning before working independently. Also, this demonstrates teacher engagement with the content. It also provides you with your own data to work on to help guide students. Other activities are designed to facilitate students' investigations by asking them guiding rather than leading questions, posing counterexamples, modeling how to be inquisitive, and making conjectures about generalizing and constructing proofs. Activities are developed to use a hands-on and constructivist approach, tapping into students' analytical, practical, and creative skills. As students work, you will need to determine how much guidance is needed to direct them toward the end goals without short-circuiting the discovery process. We have flagged activities that lend themselves particularly well to this strategy.

Fig. 5: Joint activity: Teacher Bernardo Untalasco works on his smokehouse model while students work on their models.

Questioning Strategies

Questioning strategies are an important part of increasing understanding. The lessons make suggestions for questions to pose, but the most effective use of questioning requires taking student-generated ideas and exploring them as they emerge. Leave plenty of time for thinking about responses. Allowing students time to write in their journals before and after discussion can make for more thoughtful discussions.

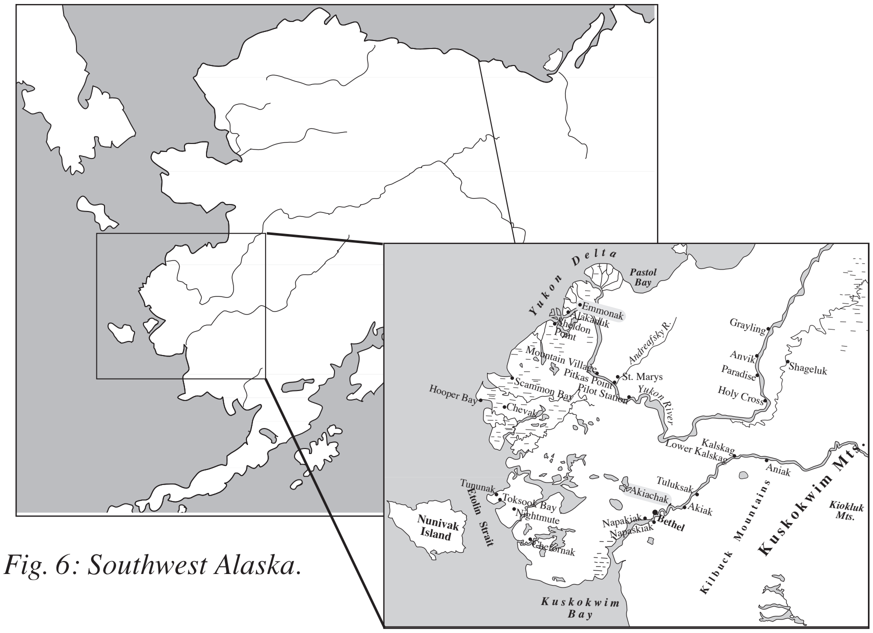


Fig. 6: Southwest Alaska.

Constrained Exploration

The *Building a Smokehouse* module relies on the strategy of constrained exploration. Students have the opportunity to explore the concepts being introduced—in this case the geometry of three dimensional prisms—on their own, but the activities are designed to help focus these explorations so that students are guided toward pertinent understandings. These hands-on, concrete experiences allow students to understand sophisticated math concepts. We have flagged ways in which further explorations might be used.

Literacy Counts

Literacy Counts is a supplemental literacy curriculum designed to accompany *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders* (MCC). As MCC has developed over the years, the importance of the role of literacy has also grown. The inclusion of culturally based stories has proved to contribute to students' engagement with the math modules as well as provide cultural grounding for the module activities. MCC modules have also made use of literacy-based activities, such as writing in journals, to further students' understanding of math concepts and vocabulary.

We provide activities that build students' understanding of math vocabulary and concepts. Often these activities use culturally mediated ways including kinesthetic (symbolic representation through dance or pantomime), drama, math notebooks and journaling, illustrations, and models.

Assessment

The two main forms of assessment in this module are students' math notebooks, in which they record their findings about the geometry topics, and posters constructed by each group that show how they used mathematical properties to construct their model smokehouses. The poster is begun in Activity 2 and completed in Activity 11.

The poster activity and math notebooks will help you monitor students' understanding of concepts. It will also allow students an opportunity to justify, explain, and clarify their work in writing. Checking math notebooks can be done in several ways. If you have time, collecting, reading, and responding daily is a wonderful way to guide your pace of instruction and get insight into students' understanding of math concepts. Otherwise, random checks for completion or classroom discussion are good alternatives. At the end of the unit, the math notebooks will be a record of the unit to take home and share with parents. It may be advantageous to designate an area in the room to store math notebooks daily since they will be an essential part of the assessment and students will need them throughout the module. Guide your students as to the format that you prefer in the math notebooks, such as dates at the top of entries, restating the question before writing, using whole sentences, or using straight edges for geometric drawings.

Cultural Background

Smoking Salmon

The smoking process is a critical step in preserving salmon for the long Alaska winter. Properly cutting, drying, and storing extends the freshness of the fish throughout the winter. Improperly curing the fish results in early spoilage and molding, so that it has to be thrown away or used for dog food.

Preparing to smoke the year's catch means repairing the existing smokehouse or building a new one, collecting the wood needed for smoking, building or repairing the wooden racks from which to hang the salmon for smoking, and having containers, such as buckets, for storing or distributing smoked salmon.

Background Information on Smokehouses

Smokehouse designs, the type of wood used to smoke the fish, and the taste of the final product vary in Alaska between families and communities due to personal and cultural preferences, and because of environmental factors. For those teaching this module in rural Alaska, you may want to use the smokehouse design, method of construction, and way of smoking that corresponds to your community.

Smokehouse Description and Design

Two styles of smokehouses are presented in the module, those of the Yukon and Kuskokwim river regions.

Yukon River Delta Style

Figure 7 shows a sketch of the Charles' smokehouse, which is on the Yukon River delta. Note the two fire pits located two-thirds of the way toward the back of the smokehouse. Placement of these pits sets up airflow conducive to the smoking process. The Charles' smokehouse has two levels that are used at different stages in the smoking process. In the first stage, Walkie Charles hangs fresh salmon on the lower level near the fire pits for a number of days to dry and cure. They are then moved forward on the lower level as new fish are added to the back. As they dry further, they are eventually moved to the upper level and finally back to the lower level along the periphery of the smokehouse for storage and slightly more smoke. Rotating the salmon

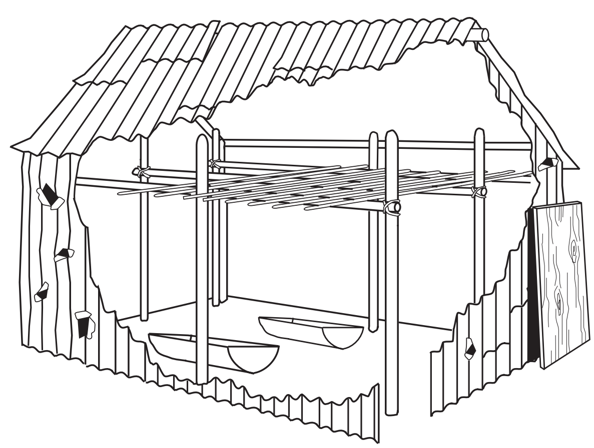


Fig. 7: A sketch of the Charles' smokehouse.

to different levels and locations allows it to dry and smoke properly without cooking it.

Figure 9 shows that the smokehouse is well ventilated through open spaces in the four walls. It is important not to make the smokehouse airtight as it impedes the airflow, resulting in fish that tastes too smoky or bitter. For this reason, most smokehouses are built out of scrap material such as tin sheets which already have gaping nail holes in them. Because of the need for airflow and ventilation, a gap is left at the top of the walls. The smokehouse roof is constructed to overhang the walls to keep out the rain.

Fig. 8: Maggie Charles, Walkie's mom, in front of drying fish that will be placed in her smokehouse. Photo courtesy of Maggie and Walkie Charles.

Kuskokwim River Delta Style

The George family's smokehouse, shown in Figure 10, is in the Kuskokwim River region. The George family has a single-level smokehouse, which follows the custom of their region. The first salmon to be smoked are the kings, which are the first salmon of the summer fishing season to enter the Kuskokwim River and migrate past his village. Frederick loads four paired kings (*inglukegtat taryaqviit*) on a rack and the king salmon are dried for a while on the outside fish drying rack (*qer'aq*). The drying pole (*initaq*) used outside the fish racks will fit in the smokehouse. This allows Frederick to easily transfer the properly dried fish from outside to inside the smokehouse. Frederick follows his father's sequential method of hanging fish in the smokehouse. When the fish are first brought into the smokehouse, they are placed in the right front section. As they dry, they are relocated to the rear of the smokehouse.

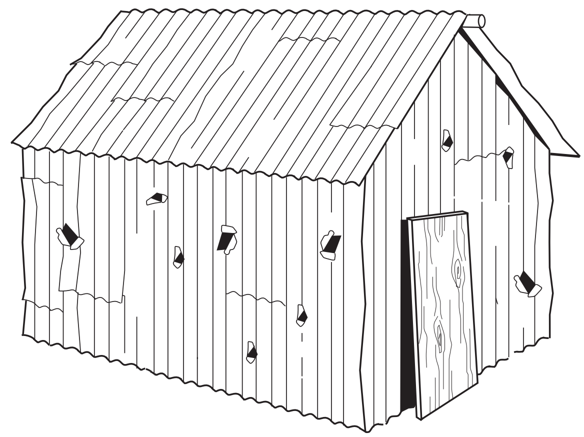


Fig. 9: Smokehouses should be well ventilated.

Smoking and Curing

Both Frederick and Walkie emphasize that it is extremely important to carefully monitor the salmon during the smoking and drying process. Too much smoke can make the fish bitter and too little smoke may not cure them properly. On cloudy days when the flies are active, it is important to have constant smoke, not only to cure the salmon but also to keep out the flies so they do not lay eggs on the salmon. During rainy days, the constant smoke will keep the dampness out to prevent the fish from molding and spoiling. On sunny days, Frederick waits until nightfall to smoke the salmon to keep the salmon from drying out.

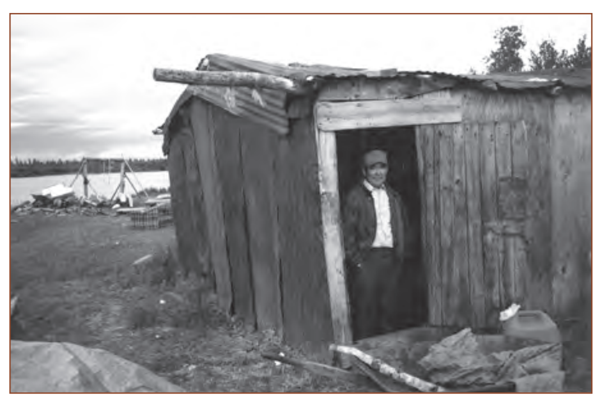


Fig. 10: Frederick George standing in the doorway of his smokehouse. Photo courtesy of Susan Addington.

The spacing between the hung salmon, their proximity to the fire pit and the smoke, and the style of the cut all affect the smoking process. For example, the king salmon (Figure 12) are placed close together with the backbone in between them (the chums are often hung to dry without the backbone). The backbones are paired and tied with twine before hanging the fish to dry. In this way both sides dry approximately at the same time. If the backbones were attached to the skin side of the fish, the skinned section would dry quicker than the backbone. For the chum, the women cut the blanket side (by a quarter inch) and cut the meat facing the inner side, then hang the fish by overlapping and alternating the blankets. Because of the overlapping, the smoke is trapped under the blankets and the fish can receive a better curing process (see Figure 13).

Smokehouse designs reflect the environmental conditions of different regions. For example, in Southeast Alaska, where the summer climate is cool and wet, smokehouses have less ventilation, the salmon is cut thinner to hasten the drying process, and fish racks are not used.

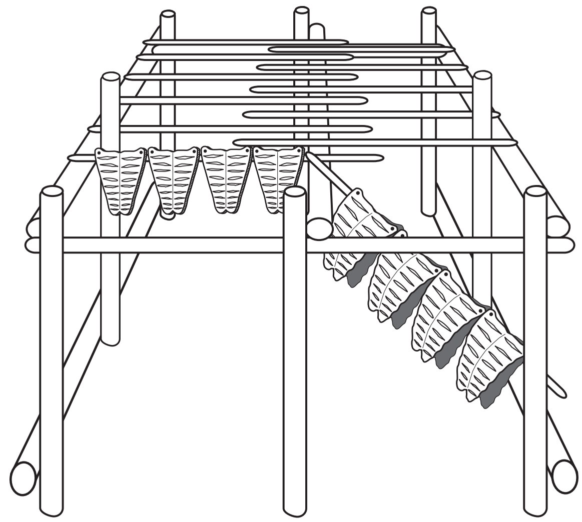


Fig. 11: The drying racks can be moved to the smokehouse.

Fig. 12: Hanging up kings to smoke. Photo courtesy of Susan Addington.

Fig. 13: Mary George checks the kings to see if they're done. Photo courtesy of Susan Addington.

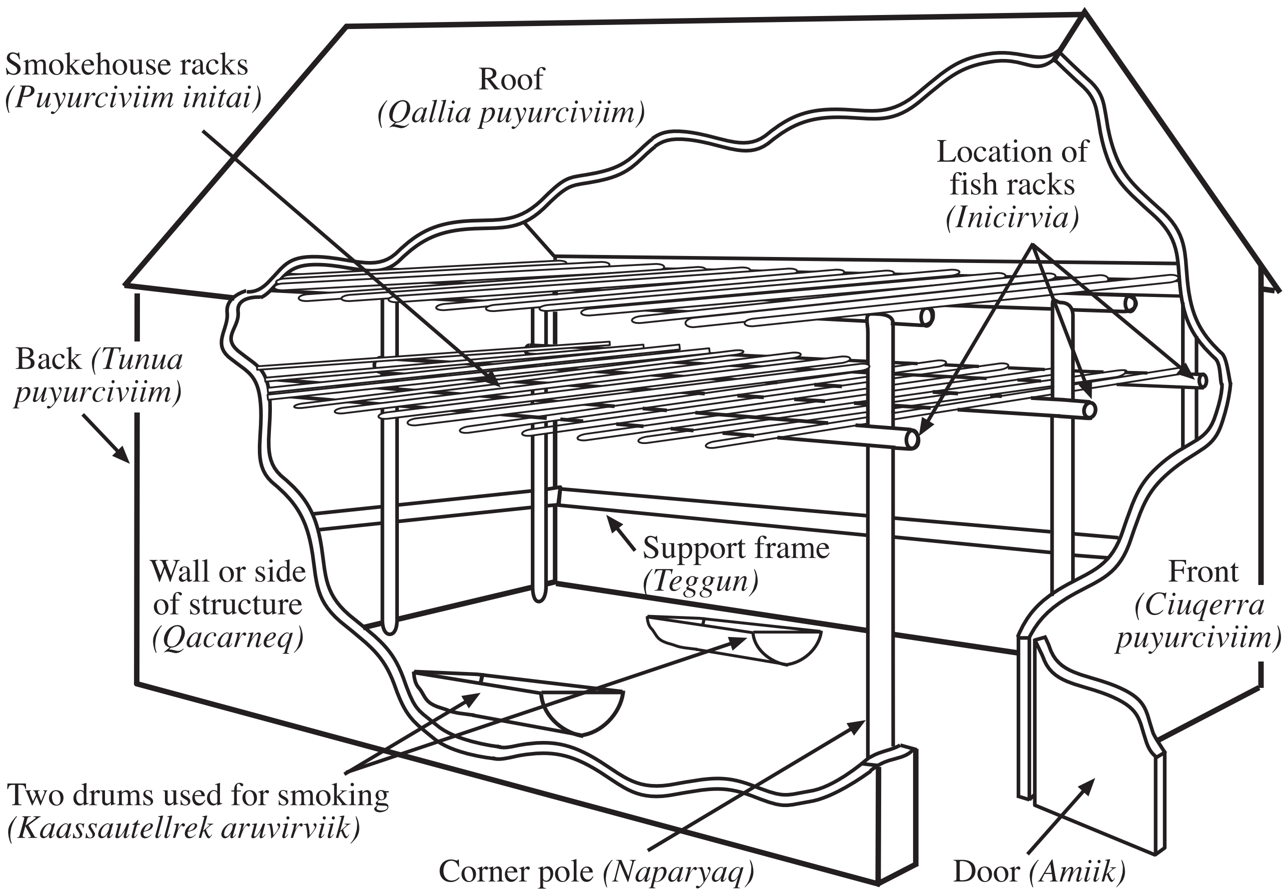


Fig. 14. Yup'ik names for parts of a smokehouse. This image contains abbreviated vocabulary. For full definitions, see page 17.

Smokehouse Terms

The Yup'ik names for each of the parts of the smokehouse are shown in Figure 14. The Yup'ik terms given come from three different Yup'ik regions reflecting the Lower Yukon, Kuskokwim, and Bristol Bay dialects.

Additional Smokehouse Terms

Aciqliit—the bottom tier of the racks.
Agagliiyaq—the side beam.
Agluq—the large center beam.
Caniqaq—the side wall.
Canirun—the cross beam of the structure.
Canirutii egkum—the smokehouse's rear cross beam.
Egkuq—the side wall of the smokehouse.
Puyurciviim initai—smokehouse racks.
Qacarneq—the wall or side of the structure.
Qiliin tegg'utai—the trusses (the roof's support beams).
Quleqliit—the top racks.

Fig. 15: Smelts strung on willow poles.

Types of Trees Used

Depending on the types of materials available, people in different areas have different ways of constructing their smokehouses and smoking their fish. For the smokehouse frame, spruce (*kevraartuq* in Kuskokwim, *nekevaartuq* in Bristol Bay) is most often used. For smoking the salmon, Frederick (in the Kuskokwim region) uses young willow trees (*enrilnguat*) that have sweet buds. Dead fallen cottonwood (*equgniilnguut*) are also used for smoking because they are moist and burn slowly. The people who live on the Yukon Delta, where trees are sparse, use driftwood for constructing the smokehouse and smoking the fish. Branches from willows with small leaves are used for stringing up smelts. A smelt is a small fish that the Yup'ik catch and preserve in addition to salmon.

Fig. 16: Hundreds of smelts drying on racks. Photo courtesy of the Anchorage Museum of Art and History.

Fig. 17: Smokehouse in winter snow. Photo courtesy of Susan Addington.

NCTM Standards and this Module

The skills and knowledge emphasized in these activities relate directly to the NCTM (National Council of Teachers of Mathematics) standards. The specific NCTM standards addressed by this module focus on algebra, geometry, problem-solving, reasoning and proof, and communication and connections.

Standard 2: Algebra
- Use mathematical models to represent and understand quantitative relationships
- Represent and analyze mathematical situations and structures using algebraic symbols

Standard 3: Geometry
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Use visualization, spatial reasoning and geometric modeling to solve problems

Standard 4: Measurement
- Understand measurable attributes of objects and units, systems and processes of measurement
- Apply appropriate techniques, tools and formulas to determine measurements

Standard 6: Problem Solving
- Build new mathematical knowledge through problem-solving
- Solve problems that arise in mathematics and other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem-solving

Standard 7: Reasoning and Proof
- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjecture
- Develop and evaluate mathematical arguments and proof
- Select and use various types of reasoning and methods of proof

Standard 8: Communications
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

Standard 9: Connections
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Standard 10: Representations
- Create and use representations to organize, record and communicate mathematical ideas
- Select apply and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social and mathematical phenomena

Master Materials List

Teacher Provides

Blocks
Brads
Butcher paper
Corrugated cardboard
Cubes of any size, all equal
Grid paper
Gumdrops or marshmallows
Index cards or old manila folders
Model building materials: straws and pipecleaners; newspaper
Masking tape
Math notebooks
Pennies
Post-it® notes
Preserved foods such as canned or freeze-dried
Rulers
Siding material: aluminum foil, butcher paper, newspaper
Scissors
Simulated oil drums with a different diameter and height than the soda cans (optional)
Smoked fish, moose, or other meat (optional)
Soda cans
String
Tape
Toothpicks
Your own model with part of a shed roof constructed

Package Includes

CD-ROM, Yup'ik Glossary
Smokehouses Poster

Blackline Masters for Transparencies

Akiachak Smokehouse
Elders Using Diagonals to Construct a Rectangle
The Five Salmon Species
Flattening an Oil Drum
Flow of Air Through a Smokehouse
Grid Paper (optional)
Map of Southwest Alaska
Photos of Southwest Alaska
Slope Data Table
Smokehouse (*Puyurcivik*)
Smokehouse with Two-Tiered Racks
Types of Smokehouses
Yup'ik Preservation Methods

Blackline Masters for Handouts

Additional Practice Problems for Activities 4, 5, 7, and 10
Grid Paper
Plane Shapes
Properties of Quadrilaterals
Shape Strips
Slope Data Table
Vocabulary Map

Master Vocabulary List

Angle—a geometric figure formed when two lines, rays or line segments meet at a point. The meeting point is called the vertex of the angle. An angle is measured in degrees of rotation between the two lines, rays or line segments. An angle can measure between 0 and 360 degrees.

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth.

Base—the bottom of a geometric shape, usually parallel to the ground or the bottom of a page.

Bisect—to cut or divide into two equal parts.

Bisector—something that cuts or divides onto two equal parts.

Circumference—the length of the boundary of a circle.

Center—a point that is equidistant from all points in a circle; a point that is the intersection of the diagonals of a square, rectangle, rhombus or parallelogram.

Centroid—the center of mass of a figure. The centroid of a triangle is the intersection of the medians. The centroid of a rectangle is the intersection of the diagonals.

Congruence—the property of two shapes that are exactly equal in size and shape.

Congruent—geometric figures (or parts of figures) that are the same shape and size. Two shapes are congruent if one shape can be slid, flipped and/or rotated so that that shape fits exactly on top of the other one. Parts of a shape, for example, sides or angles may also be considered congruent.

Conjecture—a statement about a mathematical fact, relationship or generalization that is based on careful observation or experimentation, but which has not been proven. (See Proof.)

Cube—a solid figure in which all six faces are equal-sized squares.

Cylinder—a solid figure with two parallel circular bases.

Diagonal—a line joining two non-consecutive vertices of a plane or solid figure. For a quadrilateral, a line joining opposite vertices.

Diameter—a line segment that goes through the center of a circle and whose end points are on the circle.

Edge—a line segment formed by the intersection of two faces of a solid figure.

Face—a flat side of a solid figure.

Generalize—to take one idea and apply it to a related topic in a different domain.

Height—the distance from the base of an object to its highest point.

Heptagon—a polygon with seven sides.

Heptagonal prism—a prism with heptagonal (7-sided) bases.

Hexagon—a polygon with six sides.

Hexagonal prism—a prism with hexagonal (6-sided) bases.

Horizontal—parallel to the horizon or to a baseline.

Hypotenuse—the side opposite the right angle in a right triangle.

Isosceles triangle—a triangle with at least two sides the same length. The base angles of an isosceles triangle are also equal.

Isosceles triangular prism—a prism whose bases are isosceles triangles.

Kite—a quadrilateral with exactly one line of symmetry.

Length—the measure of distance between two ends of a line segment.

Line of symmetry—a line that is a property of a geometric figure (a shape, design or pattern); it divides the figure into two equal parts, such that when the figure is reflected about that line, the result is identical to the starting figure. For example, a square has four lines of symmetry: vertical horizontal, and diagonal lines through the center of a square are all lines of symmetry. An informal test for a line of symmetry in a two-dimensional shape is to fold the shape along a line through its center. If both sides match exactly after folding the line is a line of symmetry.

Line segment—a part of a line with two endpoints and a definite length.

Median—a line connecting a vertex of a triangle with the midpoint of the opposite side.

Midpoint—a point that bisects a line segment into two equal lengths.

Midpoint connector—a line connecting the midpoints of opposite sides of a quadrilateral.

Nonstandard measures—a system of measurement using units that are derived from a specific situation and are not based on standardized units such as inches, centimeters, or kilogram. For example, measuring a length on the ground in terms of the length of an individual's foot, or the number of paces it takes to walk a distance.

Oblique prism—a prism whose faces are not perpendicular to its bases.

Octagon—a polygon with eight sides.

Octagonal prism—a prism with octagonal (8-sided) faces.

Parallel—in a plane, lines that do not intersect; In three dimensions, planes that do not intersect.

Parallelogram—a quadrilateral with two pairs of parallel and equal sides.

Pentagon—a five-sided polygon.

Pentagonal prism—a prism with pentagonal (5-sided) bases.

Perpendicular—two lines or planes intersecting at a right angles.

Plane—a flat surface that extends infinitely in all directions; mathematically, a plane is considered to have zero thickness.

Pi—a constant represented by the Greek letter π that expresses the ratio of the circumference to the diameter of a circle. It appears in many mathematical expressions and its value is approximately 3.14159.

Polygon—a closed plane figure bounded by three or more line segments.

Preservation—to prepare (food) for future use, as by canning or salting, so as to prevent it from decaying or spoiling.

Prism—a solid figure that has two congruent parallel polygons as bases and rectangles or parallelograms for all its other faces. There are two types of prisms, right prisms and oblique prisms.

Proof—a mathematical argument—based on logical reasoning—that demonstrates that a particular fact or relationship is true.

Properties—the parts of a geometric shape: sides, angles, symmetries and their relationships, that define a particular shape as a unique shape. For example, the inherent properties of a square are that it has four equal sides and four equal angles. From this we can derive other properties such as the fact that all the angles are 90 degrees, that its diagonals meet in the center, and that a square has four lines of symmetry.

Pyramid—a solid figure with one base that is a polygon and triangles for its other faces.

Quadrilateral—a four-sided polygon.

Ratio—a comparison of two quantities by division.

Rectangle—a four-sided polygon with four right angles and opposite sides congruent.

Rectangular prism—a prism with rectangular bases.

Right angle—an angle that measures 90˚, or one fourth of a full rotation. This is the angle found in squares, rectangles, and right prisms.

Right prism—a prism whose faces are perpendicular to its bases.

Right triangle—a triangle with one right angle.

Right triangular prism—a prism with right triangle bases.

Rise—the height measured between two points on a line segment (or plane surface), used to calculate its slope.

Rhombus (pl. rhombi)—a quadrilateral with four equal sides. Alternative definition: a parallelogram with four equal sides.

Run—the horizontal distance measured between two points on a line segment or plane surface, used to calculate its slope.
Slope—the ratio of the rise divided by the run of a line segment or plane surface. A measure of its steepness.
Smokehouse—a covered structure used to smoke fish and meat. It has a fire pit in the floor and racks to hang the objects. The sides are vented so the smoke can move freely through the structure.
Square—a regular quadrilateral. All sides have the same length and all the angles are right angles. Alternative definition: a rectangle with all sides the same length.
Square units—units of area, such as square inches, square centimeters or square miles.
Solid figure—a three dimensional shape.
Standard measures—a system of measurement using standardized units such as inches, meters, or kilograms.
Subsistence—means of support or livelihood; often the barest means of food, clothing, and shelter needed to sustain life.
Surface area—the sum of the areas of all the faces, or surfaces, of a solid figure.
Trapezoid—a quadrilateral with exactly two parallel sides.
Three-dimensional—a quality of space measured in three mutually perpendicular directions, such as length, width, and height.
Triangle—a polygon with three sides.
Triangular prism—a prism with triangular bases.
Two-dimensional—a quality of space measured in two mutually perpendicular directions, such as length and width.
Vertex (pl. vertices)—the point or points at which sides of a polygon, lines of an angle, or edges of a solid figure meet. Vertices of polygons are sometimes called corners.
Vertical—perpendicular to the plane of the horizon or to a primary axis.
Width—a length measuring an object from side to side.

Yup'ik Vocabulary List

Aciqliit—the bottom tier of the racks.
Agagliiyaq—the side beam.
Agluq—the large center beam.
Amiik—door.
Caniqaq—the side wall.
Canirun—the cross beam of the structure.
Canirutii egkum—the smokehouse's rear cross beam.
Ciuqerra puyurciviim—front of the smokehouse.
Egkuq—the side wall of the house or the smokehouse.
Enrilnguat—young willow trees.
Equgniilnguut—dead fallen cottonwood.
Inglukegtat taryaqviit—pairing two king salmon for smoking.
Inicirvia—the location where you place the fish rack.
Initaq—the drying pole.
Kassautelleq aruvirvik (also spelled **Kassautellrek aruvirviik** for two objects)**—**half of a container that was used for gas or fuel oil now used for smoking.
Kevraartuq (Kuskokwim) or **Nekevaartuq** (Bristol Bay)**—**spruce tree.
Naparyaq—corner pole.

Puyurciviim initai—smokehouse racks.
Qacarneq—the wall or side of the structure.
Qallia puyurciviim—roof or top of the smokehouse.
Qer'aq—fish rack.
Qiliin tegg'utai—the trusses (the roof's support beams).
Quleqliit—the top tier of the racks.
Teggun—support frame.
Tunua puyurciviim—back part of the smokehouse (outside).
Uluaq—a traditional curved knife.

Note: The Yup'ik vocabulary includes words not necessarily in the module but words associated with the process of smoking salmon and especially words associated with the smokehouse. Thus they are a general reference.

Photography Credits

Anchorage Museum of History and Art:
Smelts drying on racks; (12, left)
Maggie and Walkie Charles:
Maggie Charles (9, top; 26, center)
Nina Knapp:
Henry Alakayak (3, top left)
Detail of Henry Alakayak (3, top right)
Smokehouse model (3, bottom)
Susan Addington:
Frederick George (9, bottom; 30, bottom)
Hanging kings up to smoke (10, center; 30, center right)
Mary George (10, bottom)
Smokehouse in winter snow (12, right)
Akiachak smokehouse (98; 30, top left)

Section 1

Smokehouses, Rectangles, and Proof

Activity 1: Introduction to Smokehouses

In this module, students will build a model of a smokehouse to explore properties of three-dimensional geometric shapes, especially prisms. The smokehouse model is made of two different types of prisms, rectangular and triangular. By physically building the model, students will have a hands-on way to learn about prisms and their properties. Each activity is structured to first give students the opportunity to build two-dimensional and three-dimensional shapes and then use the background knowledge gained to understand the mathematics of what they're doing. Each of the activities builds on the previous ones, becoming more complex over the course of the module.

This activity gives students a context for understanding the smokehouse and its role in the preservation of food, which is essential to many cultures. Students gain insight into the process of preserving food—salmon in this case—as they learn a specific way in which Yup'ik people preserve salmon. The smokehouse is the place where the salmon preservation occurs For the Yup'ik people, living a subsistence lifestyle means preserving the foods obtained through hunting, fishing, and gathering. One of the ways that food is preserved is through smoking. This activity sets students up for designing and building model smokehouses in subsequent activities.

Goals

- To understand the role that the smokehouse plays in the Yup'ik culture
- To understand the importance of preserving food and the function of the smokehouse

Materials

- Butcher paper
- Index cards or Post-it® notes
- Math notebooks
- Smoked fish, moose, or other meat (optional)
- Poster, Smokehouses
- Preserved foods such as canned or freeze-dried
- Transparency, The Five Salmon Species
- Transparency, Map of Southwest Alaska
- Transparency, Photos of Southwest Alaska
- Transparency, Smokehouse (*Puyurcivik*)
- Transparency, Yup'ik Preservation Methods

Duration

One class period.

Vocabulary

Preservation—to prepare (food) for future use, as by canning or salting, so as to prevent it from decaying or spoiling.

Smokehouse—a covered structure used to smoke fish and meat. It has a fire pit in the floor and racks to hang the objects. The sides are vented so the smoke can move freely through the structure.

Subsistence—means of support or livelihood; often the barest means of food, clothing, and shelter needed to sustain life.

Preparation

Hang up the Smokehouses Poster where it will be visible throughout the unit. Bring in examples of preserved foods and, if available, smoked fish or other meat. Have students bring in examples of foods from home that have been dried or smoked, which will both give students a sense of the variety of foods that are prepared in this way and, where appropriate, help students to connect the module to their own experiences. Students will need math notebooks for making illustrations as they define properties of prisms.

Instructions

1. Tell students that they will build a model of a smokehouse and by doing this learn geometry. If you are in an area where smokehouses are used, ask students to describe smokehouses they have seen or used,

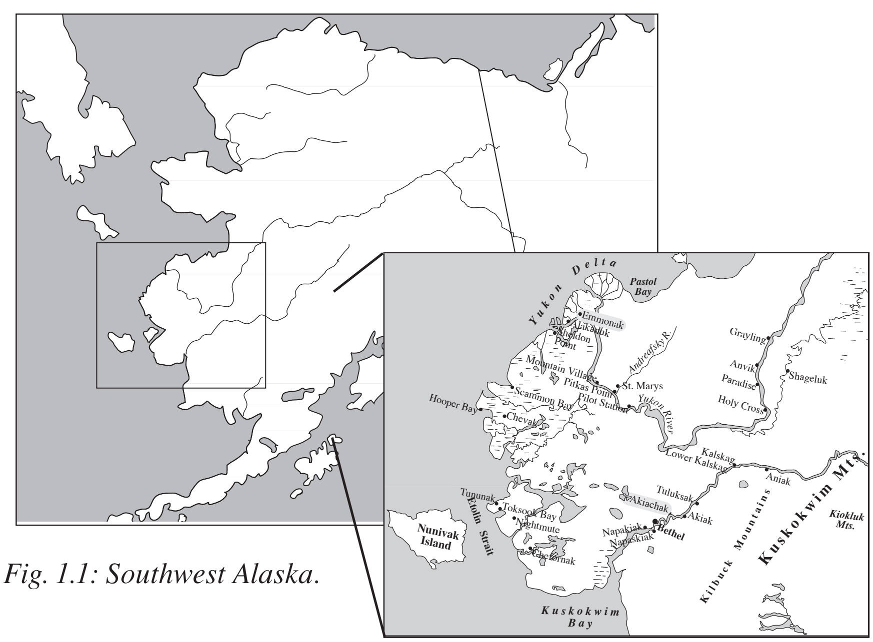


Fig. 1.1: Southwest Alaska.

especially their shape. If not, use the smokehouse examples provided on the Smokehouse Poster or transparency. Ask students to make conjectures about how smokehouses might work to dry fish. Explain that today's activity will introduce them to the function of the smokehouse in food preservation.

2. Using the transparency, Map of Southwest Alaska (Figure 1.1) as a reference, indicate Southwest Alaska, where the Yup'ik live, pointing out the lower Yukon and the Kuskokwim rivers. Also show the Photos of Southwest Alaska transparency to give students a better sense of this area of Alaska. Some of the information in this unit comes from the Yup'ik people living in this region.

3. Show the different types of preserved food you have brought in as examples so students can see a range of preservation methods. Emphasize that food preservation is a requirement in all societies, is accomplished in a wide variety of ways, and is something that is taken for granted by many.

4. **Math Notebooks.** To prepare for the next step, have students do a quick entry in their math notebooks about the foods they eat.

5. Break the class into groups of two and have each pair take one or more of the foods that were listed in the notebook entry and come up with a way to preserve each of their assigned foods so that they will not spoil in the short term. Have pairs share their preservation methods with the whole class.

6. Next, classify the types of food preservation the class has listed into categories, such as canning, freezing, smoking, drying, pickling, etc. and list each method at the top of a piece of butcher paper. Have students label index cards or Post-it® notes with different types of preserved food and place them on the piece of butcher paper labeled with the appropriate preservation method. Emphasize that food preservation is a requirement in all societies and is accomplished in a wide variety of ways.

7. Show the Yup'ik Preservation Methods transparency and ask students which foods they listed might have been preserved using each method. Have students give personal experiences with each of the types of food preservation shown. Explain that this module will focus on one method of preservation mainly used by the Yup'ik for preserving fish: smoking in a smokehouse. Show and discuss foods students have brought from their homes.

Fig. 1.2: Yup'ik preservation methods: cold storage, smoking, canning, drying.

Cultural Note

The Yup'ik people prefer cohos and red salmon for smoking. Chums are not used as frequently. If kings are to be used, they must be dried on the fish rack for a longer time due to their higher fat content.

8. Show the transparency, The Five Salmon Species. Explain that salmon is an important subsistence food for the Yup'ik. If possible, have smoked salmon or other smoked meat available for students to try.

9. Show students the transparency, Smokehouse (*Puyurcivik*) (see Figure 1.3) and discuss how the smokehouse works (see Cultural Note on Smokehouses and the Introduction to the Module for more information). **Note:** The transparency, Smokehouse, contains abbreviated vocabulary. For full vocabulary, see page 17.

Smokehouse *(Puyurcivik)*

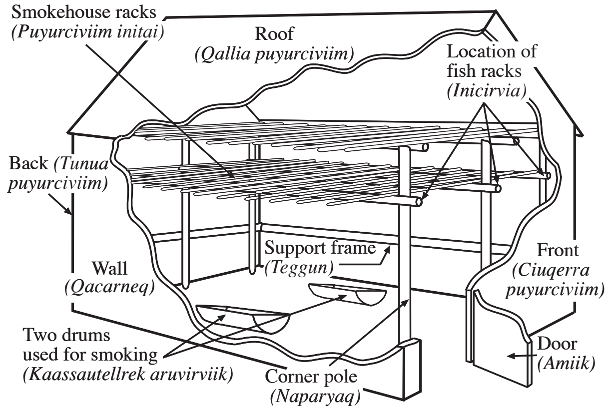


*Fig. 1.3: A smokehouse (*puyurcivik*).*

Math Notebook: In their math notebooks, students should write down one question that they have about smokehouses.

Homework: Have students write a paragraph talking about their own use of a smokehouse or other ways to preserve food. Encourage students to ask adults in their home to share their knowledge about this topic.

Cultural Note on Smokehouses

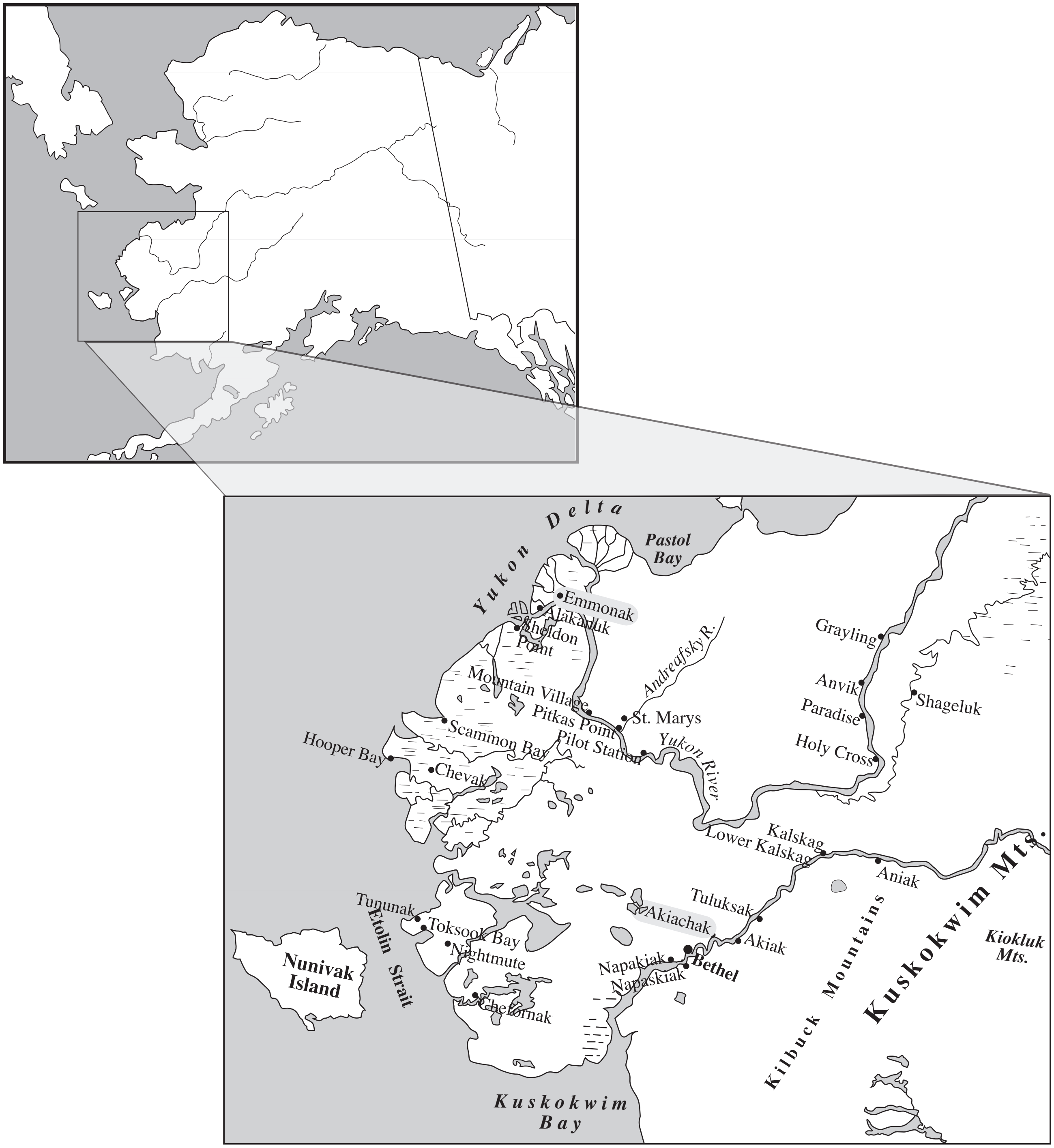
Small, windowless shed-like structures are ubiquitous in the villages and fish camps of southwest Alaska. These structures are smokehouses used to preserve the family's salmon catch each season for use throughout the year. Smokehouses range in size from quite small (4 by 6 feet) to massive, double-storied structures; the size of the smokehouse gives you an idea of how many people (and dogs) are in the family of the owner.

In southwest Alaska, the salmon are first filleted using an *uluaq*, a traditional curved knife, then put on fish racks to dry and brought into the smokehouse when the flesh has begun to harden; therefore, fish racks and smokehouses are usually in close proximity. Some families simply bring the racks laden with fish into the smokehouse.

A smoldering fire of green wood fills the smokehouse with smoke. Although different methods are used in different regions and by different families, the goal of smoking remains the same no matter the specific procedure used: the fish must be completely dried without being cooked.

Further cultural notes on smokehouses can be found in the Introduction and throughout the module.

Map of Southwest Alaska



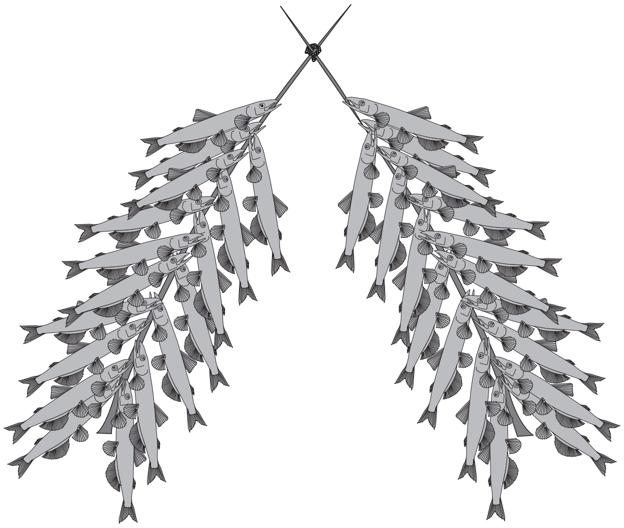
Photos of Southwest Alaska

Small smokehouse made from plywood

Maggie Charles in front of her fish racks

Frederick George's smokehouse in Akiachak

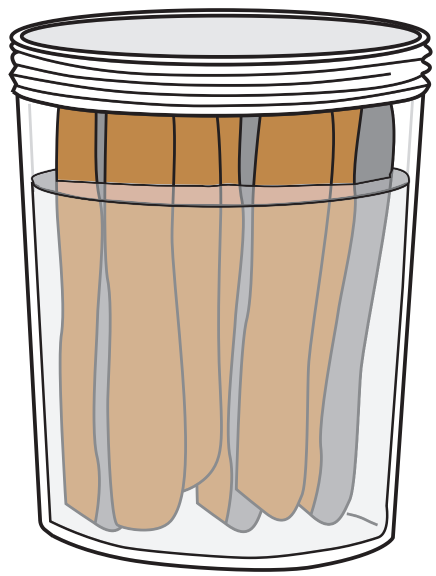
Yup'ik Preservation Methods



Drying smelts on a stick

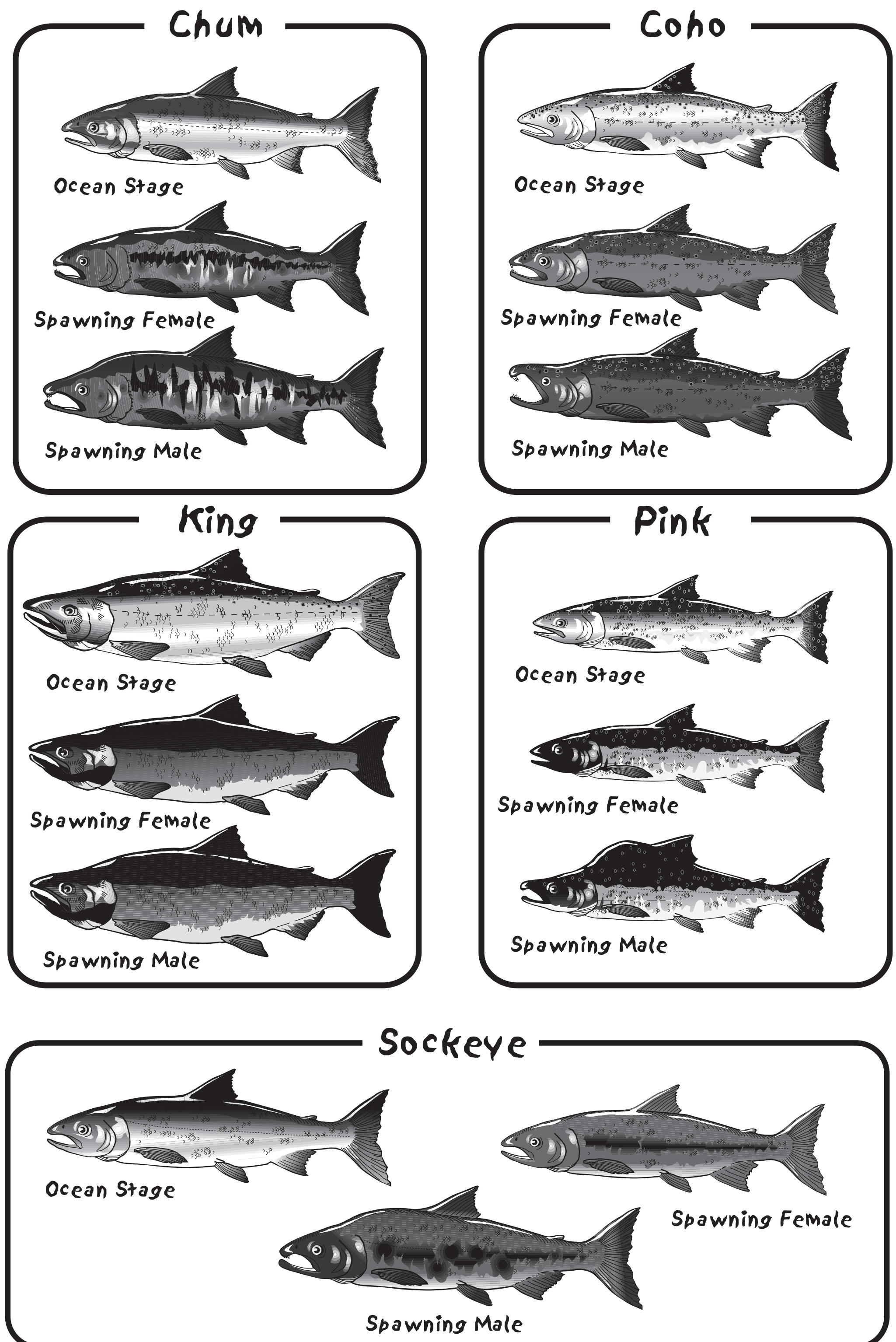
Cold storage

Smoking

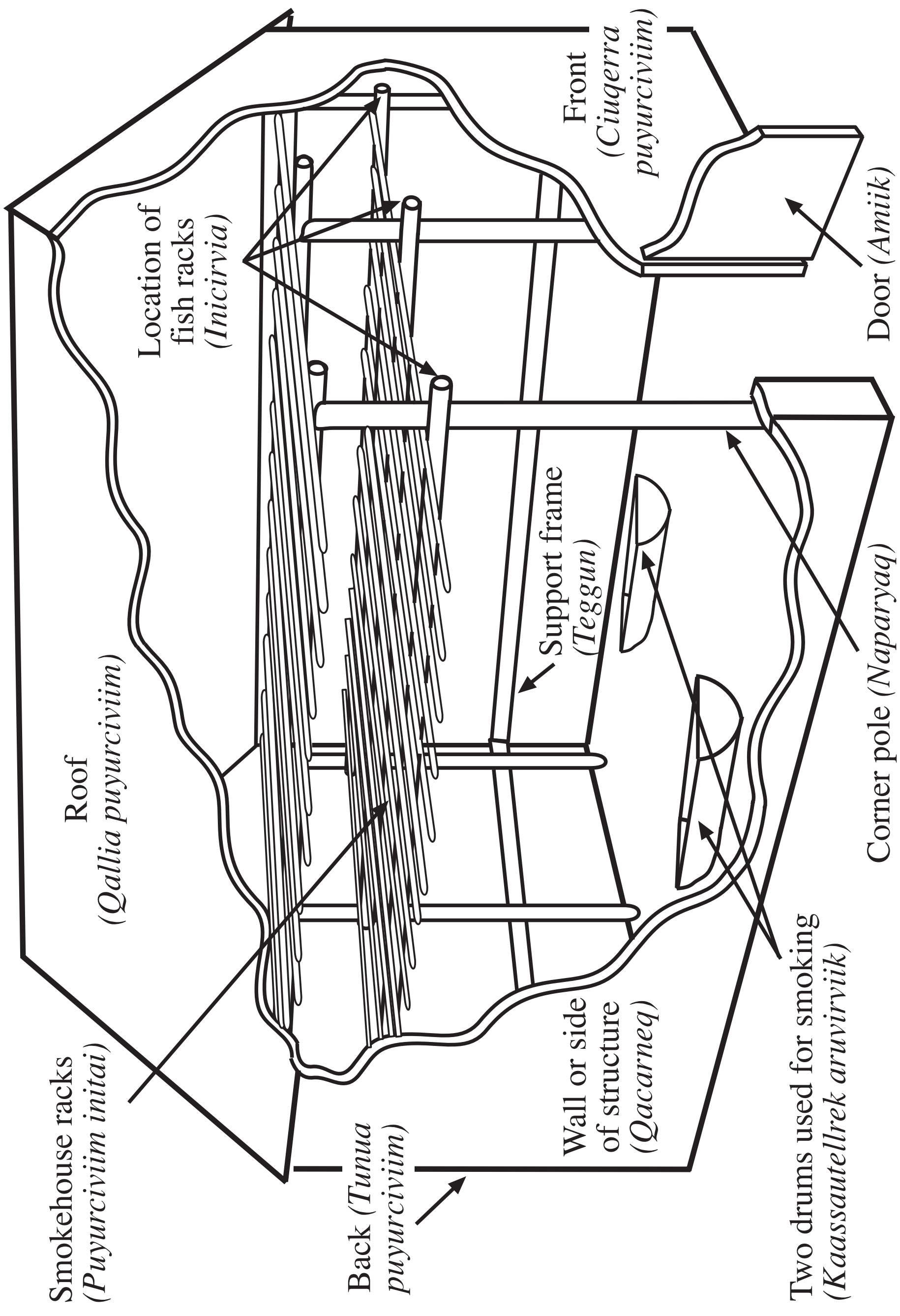


Canning

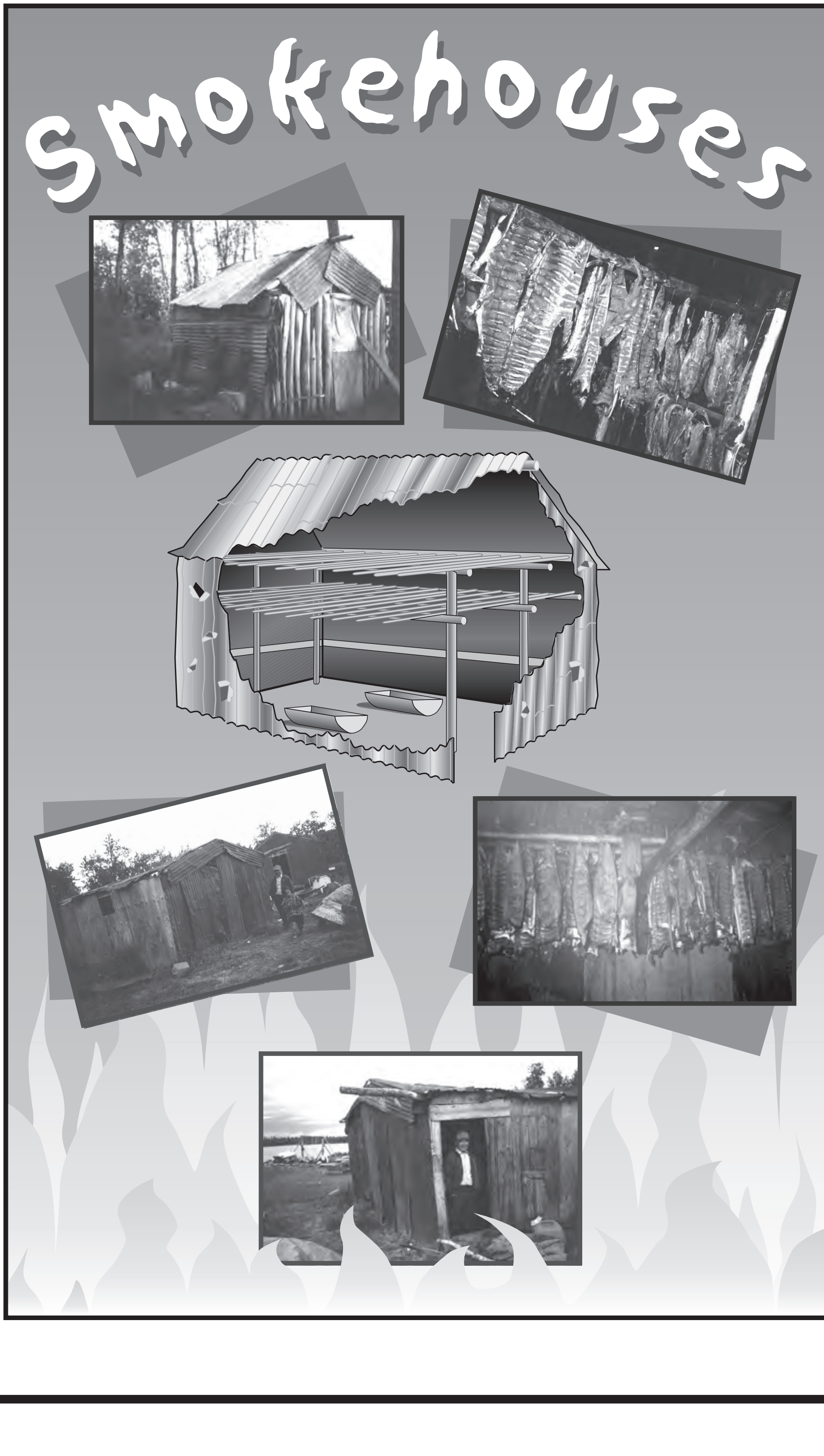
The Five Salmon Species



Smokehouse (*Puyurcivik*)



Smokehouse Poster



Activity 2: Building the Base

Students will gain experience in building a smokehouse. They will use a few materials that some elders use to determine if the shape of their building is a rectangle.

The activity starts with students constructing rectangles and other quadrilaterals using shape strips (see Figure 2.1). Students will use the shape strips to make several quadrilaterals, starting with a rectangle, by attaching the strips together with brads at the marks. Making shapes with this tool gives students a hands-on way to explore properties of quadrilaterals such as side lengths and angles. The shape strips are also used to introduce students to derived properties of quadrilaterals such as diagonals and mid-point connectors, and their properties. If the diagonals of a quadrilateral are equal and the opposite sides are equal, then it proves that the shape is a rectangle. Diagonals are used by Yup'ik elders and other builders to determine if a shape is a rectangle and to find the center of the rectangle (see Figure 2.3). Using midpoint connectors is another method for finding the center of the rectangle (see Figure 2.4).

Next students lay out a full-size base for a smokehouse. This gives students the opportunity to use mathematical proof: They will use the properties they have learned to prove that their base is a rectangle (methods for physical proofs are explained in detail in the Math Note at the end of this activity). It is important for students to be able to verify that their base is a rectangle as this will ensure that their smokehouse model will be straight and sturdy. Also, it is essential that students grasp and apply the properties of rectangles in two dimensions so that they will be able to generalize these properties to three-dimensional rectangular prisms in Activity 3.

In this activity, students will start making posters that illustrate the methods their group used for constructing a smokehouse. They will continue this process throughout the module. See Teacher Note at the end of this activity for more information on the poster.

Goals

- To construct a rectangular base for a smokehouse
- To define, describe, and apply the properties of a rectangle
- To construct different types of quadrilaterals and compare their properties
- To distinguish among different types of quadrilaterals based on their properties

Materials

- Blocks
- Brads
- Handout, Properties of Quadrilaterals
- Handout, Shape Strips—one sheet per student plus three sheets per pair
- Rulers
- Scissors
- String
- Transparency, Elders Using Diagonals to Construct a Rectangle

Duration

Two to three class periods.

Vocabulary

Angle—a geometric figure formed when two lines, rays or line segments meet at a point. The meeting point is called the vertex of the angle. An angle is measured in degrees of rotation between the two lines, rays or line segments. An angle can measure between 0 and 360 degrees.

Center—a point that is equidistant from all points in a circle; a point that is the intersection of the diagonals of a square, rectangle, rhombus or parallelogram.

Congruence—the property of two shapes that are exactly equal in size and shape.

Congruent—geometric figures (or parts of figures) that are the same shape and size. Two shapes are congruent if one shape can be slid, flipped and/or rotated so that that shape fits exactly on top of the other one. Parts of a shape, for example, sides or angles may also be considered congruent.

Conjecture—a statement about a mathematical fact, relationship or generalization that is based on careful ob-servation or experimentation, but which has not been proven. (See Proof.)

Diagonal—a line joining two non-consecutive vertices of a plane or solid figure. For a quadrilateral, a line joining opposite vertices.

Generalize—to take one idea and apply it to a related topic in a different domain.

Kite—a quadrilateral with exactly one line of symmetry.

Line of symmetry—a line that is a property of a geometric figure (a shape, design or pattern); it divides the figure into two equal parts, such that when the figure is reflected about that line, the result is identical to the starting figure. For example, a square has four lines of symmetry: vertical horizontal, and diagonal lines through the center of a square are all lines of symmetry. An informal test for a line of symmetry in a two-dimensional shape is to fold the shape along a line through its center. If both sides match exactly after folding the line is a line of symmetry.

Teacher Note: Construction Groups

Establish construction groups today, letting students know that they will work together throughout the module. Below are some suggested group roles, either permanent or rotating.

Materials manager: gets materials for the group and puts unused materials away after the construction.

Recorder: summarizes information and records on poster when needed. Updates information about the construction on the group poster.

Reporter: reports results of groups to the class, acts as spokesperson.

All students are responsible for assembling components, building the smokehouse, completing individual assignments and assessment activities, and cleaning up.

Midpoint—a point that bisects a line segment into two equal lengths.

Midpoint connector—a line segment connecting the midpoints of opposite sides of a quadrilateral.

Nonstandard measures—a system of measurement using units that are derived from a specific situation and are not based on standardized units such as inches, centimeters, or kilogram. For example, measuring a length on the ground in terms of the length of an individual's foot, or the number of paces it takes to walk a distance.

Parallelogram—a quadrilateral with two pairs of parallel and equal sides.

Polygon—a closed plane figure bounded by three or more line segments.

Proof—a mathematical argument—based on logical reasoning—that demonstrates that a particular fact or relationship is true.

Quadrilateral—a four-sided polygon.

Rectangle—a four-sided polygon with four right angles and opposite sides congruent.

Rhombus (pl. rhombi)—a quadrilateral with four equal sides. Alternative definition: a parallelogram with four equal sides.

Square—a regular quadrilateral. All sides have the same length and all the angles are right angles. Alternative definition: a rectangle with all sides the same length.

Standard measures—a system of measurement using standardized units such as inches, meters, or kilograms.

Trapezoid—a quadrilateral with exactly two parallel sides.

Preparation

- Make a base for a model smokehouse using shape strips (Figure 2.1) and brads that is in the shape of a parallelogram without 90-degree angles.
- Form construction groups (see Teacher Note).
- Find a space to lay out the full-size smokehouse bases.

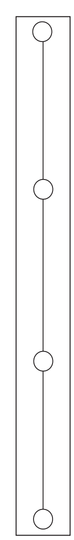


Fig. 2.1: Shape strip.

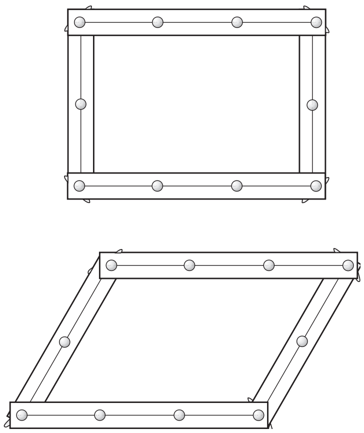


Fig. 2.2: Using shape strips to make a rectangle and a parallelogram.

Instructions

1. **Demonstrate.** Show students the non-rectangular model base you have built and explain that you want this to be the foundation for your smokehouse. Ask students what they think your smokehouse will look like. Add the corner posts. Have them compare the results to their predictions. Next ask students what they might do to make sure their smokehouse doesn't look like yours. (Students should describe a base that is a rectangle with equal opposite side lengths and right angles.) Make a list of properties of rectangles. Explain that today they will find ways to make sure their smokehouse base is rectangular, just as Yup'ik elders and builders do.

2. Hand out one sheet of shape strips, the brads, and the scissors. After demonstrating how to attach the strips with the brads through the circles, ask students to make a rectangle with their strips. See Figure 2.2 for examples. Students should verify that their shape is a rectangle using the properties brainstormed in Step 1.

3. Show the transparency of Yup'ik elders laying out the base of a smokehouse. Tell students that the elders in the picture are making sure that they have a rectangle by measuring from corner to corner. Ask students why they think this method works. Introduce the vocabulary word "diagonal." Pass out string. Ask students to prove their shape is a rectangle using diagonals (see Figure 2.3).

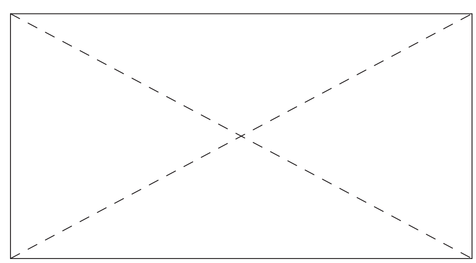


Fig. 2.3: A rectangle with diagonals of equal length.

Teacher Note

This proof of a rectangle will be used to help students apply what they have learned when they verify that their base is a rectangle.

Math Note

We focus on diagonals, a constructed feature of quadrilaterals, because they are useful for differentiating shapes and proving whether a shape is or is not a rectangle. Diagonals are discussed in the context of both two- and three-dimensional shapes. As students construct different quadrilaterals, encourage them to measure and compare the diagonals of each shape as this will give them insight into many of the properties of quadrilaterals. For example, if students form a parallelogram from their rectangles as shown in Figure 2.2, they will notice that the diagonals are no longer congruent and that the diagonal connecting the corners with the larger angles is shorter. Students can use this feature of diagonals to discover more about the angles of shapes and illustrate that parallelograms have opposite angles that are congruent. After students have filled out the handout, Properties of Quadrilaterals in step 6, they should be able to make comparisons between different types of quadrilaterals based on their diagonals.

4. Have students check the diagonals on their shape strip rectangle using strings attached to the brads. Use rulers to check diagonal length. Next ask students to prove to the person next to them that their shape truly is a rectangle. Pass out rulers and string for students to use to help them do this. Record students' proofs of their rectangles on the board. Pass out the Properties of Quadrilaterals handout and have them fill in the top row.

5. **Challenge**. Ask the students to find the center of their rectangles using their string but without using diagonals this time. Do the lines meet in the same place where the diagonals meet? **Teacher Note:** MCC supports teachers and students using a variety of math tools to help students investigate and understand mathematical relationships and properties. This could be a good place to use paper folding as another tool to explore the properties of rectangles and squares. Depending upon how well your students understand diagonals, midpoint connectors, lines of symmetry, and congruence, paper folding can reinforce and provide a different modality to learn these concepts. For example, by folding the shape in half they can prove that the opposite sides are equal. By folding into quarters they can prove that all four angles are equal. By cutting two congruent rectangles along opposite diagonals, they can rearrange the shapes to show that the lengths of the hypotenuses of all four triangles are equal. This type of exploration can be applied to other shapes.

6. **Challenge.** Have students work in pairs for this step. Pass out three more sheets of the shape strips to each of the pairs. Ask students to make three different quadrilaterals that are not rectangles. (Students can make parallelograms, rhombi, or trapezoids.) Students should record properties of each of these shapes using the Properties of Quadrilaterals handout. Once again, encourage the use of diagonals while students explore properties. **Teacher Note:** For some of these shapes, the strips will hang over the edge as shown in Figure 2.5. Further, encourage students to rip off ends to create sides of different lengths. They do not have to be limited to the original length of the shape strips.

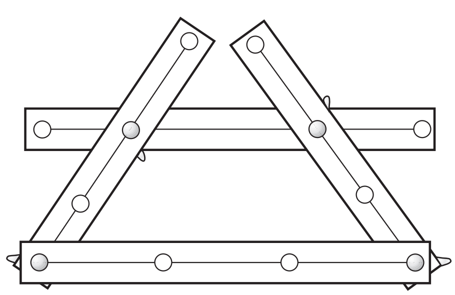


Fig. 2.5: Shape strip trapezoid.

Math Note

Another property of rectangles that students can explore with their shape strips is midpoint connectors (see Figure 2.4).

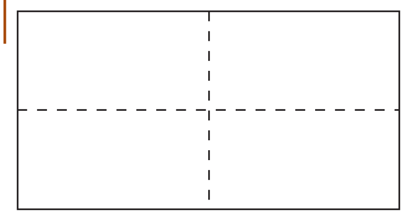


Fig. 2.4: A rectangle with midpoint connectors.

Math Note

This game not only gives students practice with the properties of quadrilaterals, it also introduces them to the idea of using counter examples and helps them to think about what properties are sufficient to determine a shape. This should help students to understand what it means to mathematically prove that a shape is a parallelogram, a rectangle, and so forth.

Teacher Note

To help students design the size of their smokehouse, provide them with the information included in this table.

2 to 4 people = 6 x 4
4 to 8 people = 12 x 8
8 to 12 people = 18 x 12

7. Share. Ask students to share the shapes they have made so that each student has a complete list of the six types of special quadrilaterals on their Properties of Quadrilaterals sheet. Ask students to point out differences among the shape properties. See Math Note, page 39.

8. Name That Shape game. Model the game first. Have a student model how to play. Students choose one of the quadrilaterals they made with the shape strips and as a clue give the class one of its properties. The class guesses which shape it is based on that property. If their guess fits the property but is not correct, the student gives another property that fits the shape, and the class takes another guess. If the shape doesn't fit the properties given, the student does not give another clue and the class guesses again. Repeat this process until the class figures out the shape. As an example, imagine that the student chooses a parallelogram and gives the property, "opposite angles equal." The class may guess a rhombus. The student counters, "No, it can't be a rhombus because all the sides of my shape are not congruent."

9. Plan and design. Put students in their construction groups. Explain to students that they will use their understanding of the properties of rectangles to build a sturdy base for a full-size smokehouse. Decide how many people are in the "extended family" and estimate the number of salmon to be smoked. Have students work in their construction groups and make a design for their smokehouse on paper. Some things for students to consider include: what they will smoke (type of meat or fish); the amount of food to be smoked; the shape and dimensions they prefer; where to place the fire, door, and racks.

Cultural Note

The dimensions of the smokehouse are determined by a number of factors. Yup'ik people often make the smokehouse the same size as the fish racks they use to dry fish so that the racks from the fish rack can be brought inside the smokehouse with the fish already on it.

Because much of the area where the Yup'ik live is tundra and lacks trees, another design consideration is the availability of materials, especially the size of the driftwood found.

Although a typical smokehouse dimension is rectangular, some are nearly square.

10. **Challenge.** When they have decided on their design, have construction groups lay out a full-sized smokehouse base outside or in the gym using something such as rocks or blocks to mark the corners and string to mark the sides. Remind students of the Yup'ik elders' emphasis on making sure their base is a rectangle so that the smokehouse will be sturdy. **Teacher Note:** While students are working on their base, prompt groups to find ways to prove that their base is rectangular, such as using diagonals.

Teacher Note

As modeled by the elders, diagonals are an effective way to verify that the base is truly a rectangle. A way to encourage the use of diagonals (or midpoint connectors) is to challenge students to find the center of their smokehouse base.

11. When the rectangle has been laid out, have construction groups verify that their base is a rectangle, as Yup'ik elders do, using the properties discussed during the paper cutting activity. **Teacher Note:** This is a good time to introduce the concept of "good enough." If students' side lengths are off by a small amount, do we call it a rectangle? See the Math Note at the end of the activity for more information on properties of rectangles.

12. When all the rectangles have been laid out, visit each base as a whole class and have the construction groups describe the layout of their smokehouse and their ideas for its function. They should explain their proof that the base is a rectangle and why they chose the size they did.

13. **Discuss.** Lead a discussion on the different types of rectangles used and the impact the different shapes would have on the use and function of the smokehouse.

14. Encourage students to visualize a real smokehouse from their base by saying where the door, racks, and stove would be and how many fish (or how much meat) they think their smokehouse would hold. Ask them to show these things in a drawing in their math notebooks.

15. **Literacy Counts.** As a prewriting activity to prepare for doing the poster in Step 16, give students Post-it® notes to write the steps they followed to construct the smokehouse model. Students can then arrange the sticky notes in the order they will use for their posters.

16. **Poster.** Have groups start their poster of guidelines for building a smokehouse by making their first entries. They should record how they constructed a rectangular base and framed the walls. The poster should also list properties of rectangles students found (see Teacher Note on page 36 for more information on the poster).

Fig 2.6: Students laying out a rectangle.

Math Note

Proving that their full-size base is a rectangle will bring up issues of the differences between mathematical constructions and real-world models. A mathematical rectangle is defined by congruent and parallel opposite sides and ninety degree angles, while a real-world rectangle such as the ones that students construct for their smokehouse base can never be exact. This reality can generate discussion about the differences between mathematical and real-world constructs as well as what is "good enough" in different contexts.

Teacher Note on Smokehouse Poster

The poster provides a means for students to synthesize their learning of three-dimensional geometry and can be used as an assessment tool. The idea of the poster is that it could be used by someone who had never built a smokehouse model to successfully build a model of their own.

Students will add information to the poster at different points of the module. Each poster entry should include both a drawing of the smokehouse construction and a written explanation of what was done and what was learned about the geometric shapes.

Entry 1 on Laying out the Base (Activity 2): Students should list the steps they took to lay out their base, specifically how they constructed and proved their rectangle. The poster drawing can include their ideas for where the smokehouse door, fire, and racks would be.

Entry 2 on Building the Body (Activity 3): Students should explain how to put together the body and the properties of rectangular prisms they found.

Entry 3 on Finding the Slope of the Roof (Activity 5): Students should explain how to calculate the slope of a roof and what slope they decided to use for their smokehouse and why.

Entry 4 on Attaching the Roof (Activity 6): Students should explain how to put together the roof and the properties of triangular prisms they found.

Figure 2.7 shows how the poster might look with example drawings.

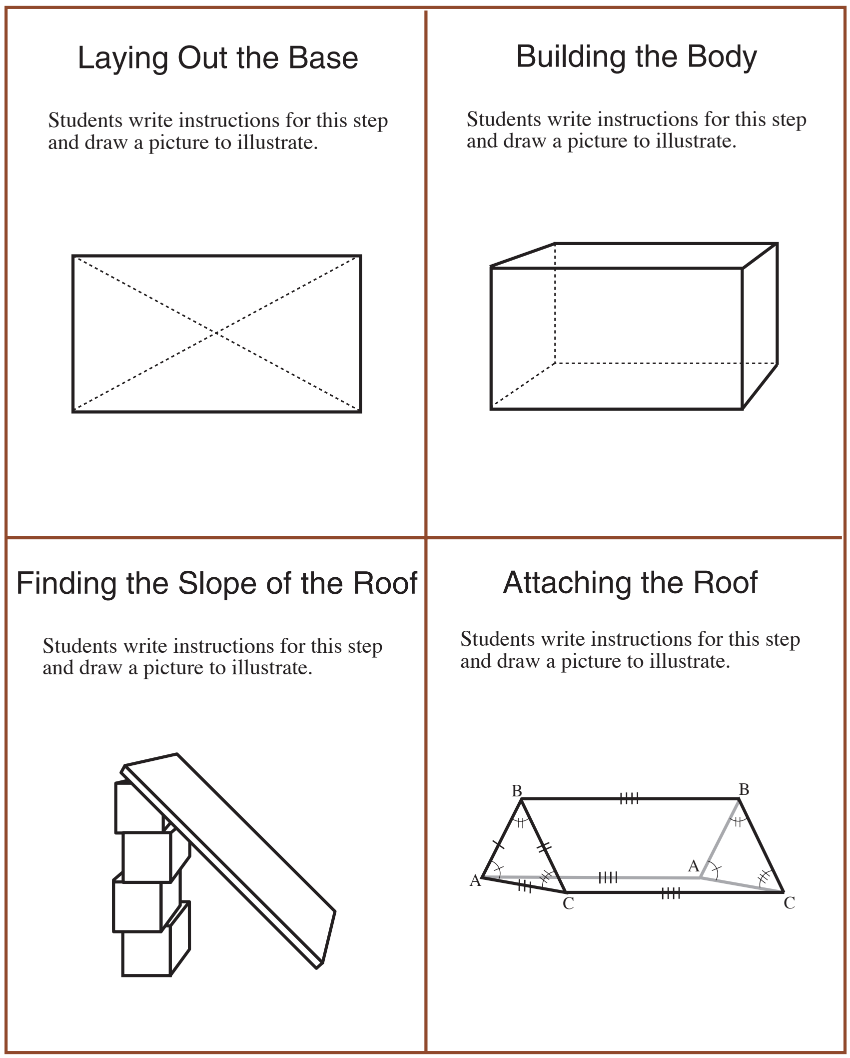


Fig. 2.7: Example of completed student poster.

Math Note: Properties of Rectangles (Do NOT Distribute to the Students)

There are several methods available to you and your students to verify whether or not a rectangular base has been made. The following methods involve an understanding of the basic properties related to rectangles. Although your students are not expected to create rigorous geometric proofs, they are expected to perform physical demonstrations.

Laying out the smokehouse base is the first time that students will practice modelling, which is central to this module. Constructing the smokehouse model is the way that students will learn properties of geometric shapes; therefore, it is important that students are guided to use their models to uncover mathematical properties.

Definition of a Rectangle

To design a smokehouse base, first pace the length and the width of the rectangle, marking out an L shape on the ground (Figure 2.8). Put markers at the endpoints of the L, then put in the remaining markers using the marked endpoints as a guide.

PHYSICAL PROOF: Use string to measure the width and check if the opposite side covers the same distance. Perform a similar operation for the length. Then estimate by sight, or with a book or some other object that has ninety-degree corners, to verify that the rectangle has at least two right angled corners.

Fig. 2.8: Walking the "L" shape.

Diagonal Theorem

When Henry Alakayak, an elder from Manokotak, Alaska, was asked how he would construct a smokehouse base if he had to begin in the middle, he explained the following method. He began by laying down four logs of equal length beginning from the center and toward where he envisioned the four corners of his base to be (Figure 2.9) to form the diagonals. To do this, he already had in mind some approximate measurements of the length and the width of the rectangular base. Henry kept in mind the whole picture of a finished smokehouse and marked each corner before he started digging the corner post holes.

PHYSICAL PROOF: Use string to verify that the diagonals of the rectangle are equal in length. The diagonals also need to bisect each other.

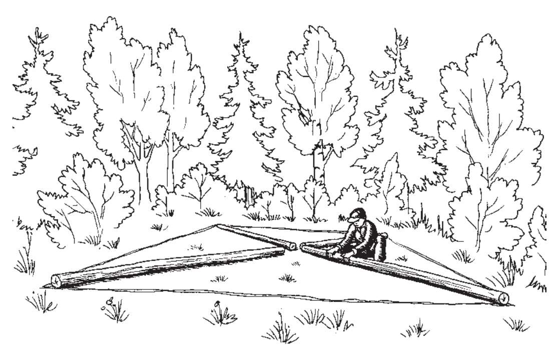


Fig. 2.9: Measuring the diagonals with logs.

Math Note (continued)

Perpendicular Bisector Theorem

PHYSICAL PROOF: Determine the midpoints of each side of the rectangle and use two pieces of string to connect each pair of opposite midpoints (Figure 2.10). These pieces of string should cross perpendicularly, and you can use another object with right angles to verify this.

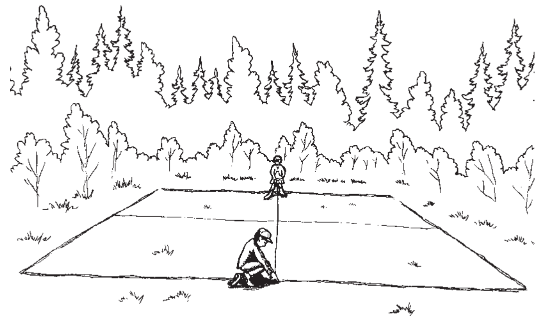


Fig. 2.10: Measuring bisectors with string.

Inscribed Rectangle Theorem

PHYSICAL PROOF: Someone stands in the center of the rectangle and holds a rope that stretches to the estimated corner of the rectangle. Someone else holds the other end of the rope taut and walks around the first person. The rope becomes the radius of a circle being traced on the ground that should intersect the rectangle at each corner (Figure 2.11). This works because the four corners of a rectangle are always equidistant from its center. This proof is a dynamic demonstration because it brings together a relationship between two different geometric shapes.

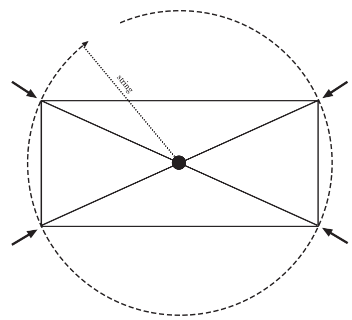


Fig. 2.11: A rectangle inscribed in a circle.

Midpoint to Corner Theorem

PHYSICAL PROOF: Use a string to determine that the midpoint B of line AC is an equal distance from points D and F (Figure 2.12). Likewise, verify that the midpoint E of line DF is an equal distance from points A and C. When the distances AE, CE, BF, and BD are all equal in length, the corners of a rectangle have been established.

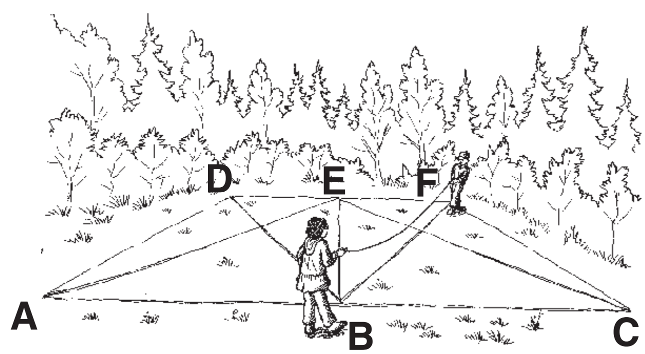
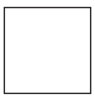
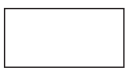
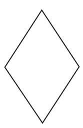
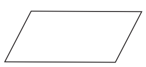
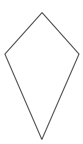
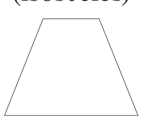
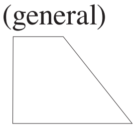


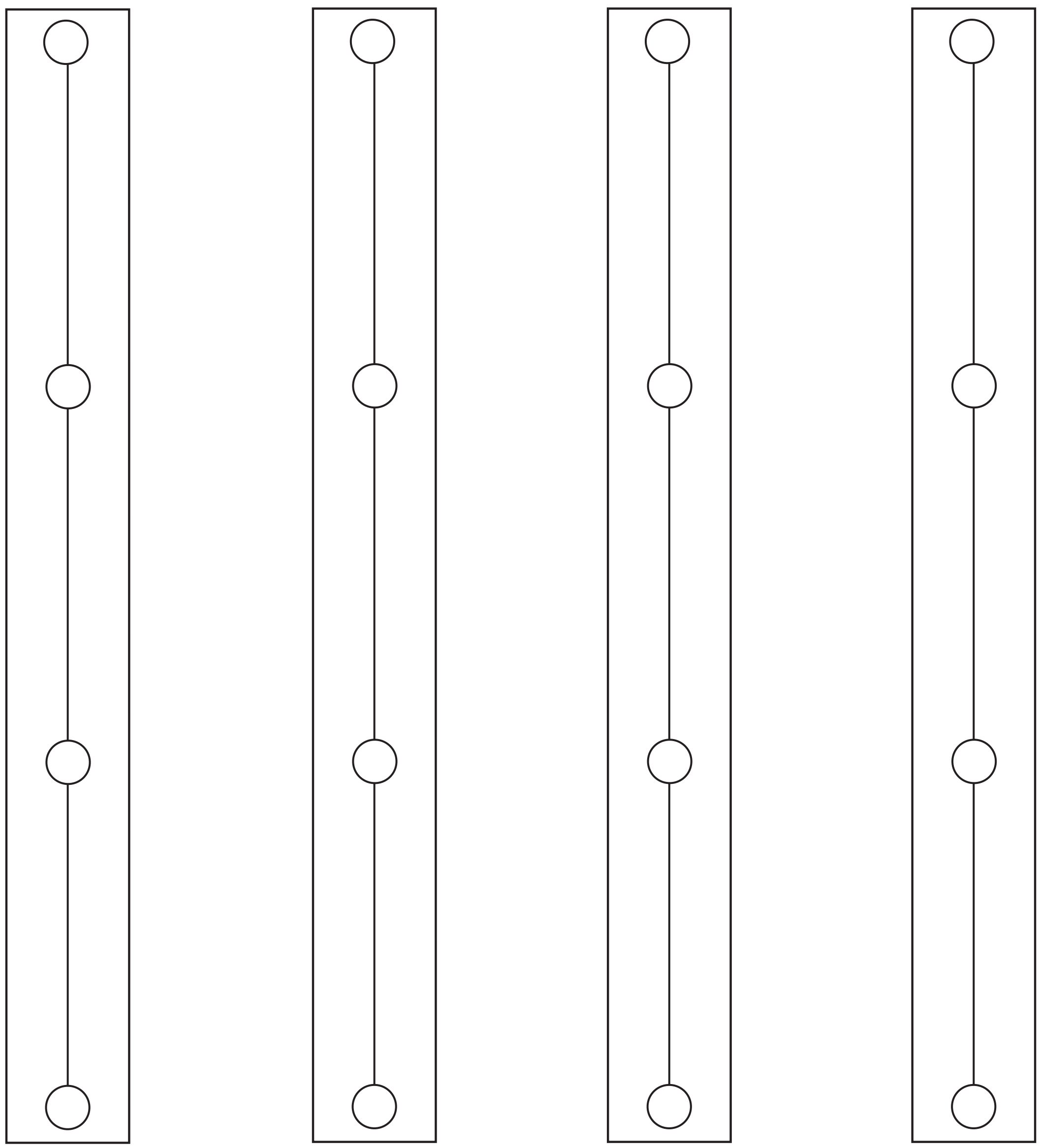
Fig. 2.12: Using string to determine the midpoint of the sides of a rectangle.

Math Note: Properties of Quadrilaterals

This table lists properties of shapes students might make with their shape strips. Another property that students could explore, not listed below, is the angle bisector. A nice comparison is to determine between rectangles and squares is if the diagonals form angle bisectors.

Shape	Sides	Angles	Diagonals			
			Congruent	Cross at Midpoint	Cross at Right Angle	Bisect Angles of Shape
Square 	All sides equal; opposite sides parallel	90 degrees	Yes	Yes	Yes	Yes
Rectangle 	Opposite sides equal and parallel	90 degrees	Yes	Yes	No	No
Rhombus 	All sides equal; opposite sides parallel	Opposite angles equal	No	Yes	Yes	Yes
Parallelogram 	Opposite sides equal and parallel	Opposite angles equal	No	Yes	No	No
Kite 	Two pairs of adjacent sides equal	One pair of opposite angles equal	Sometimes	No	Yes	One diagonal does
Trapezoid (isosceles)  (general) 	One pair of opposite sides parallel but not equal	Two pairs of angles; equal only in isosceles	Only in isosceles trapezoid	No	Sometimes	No

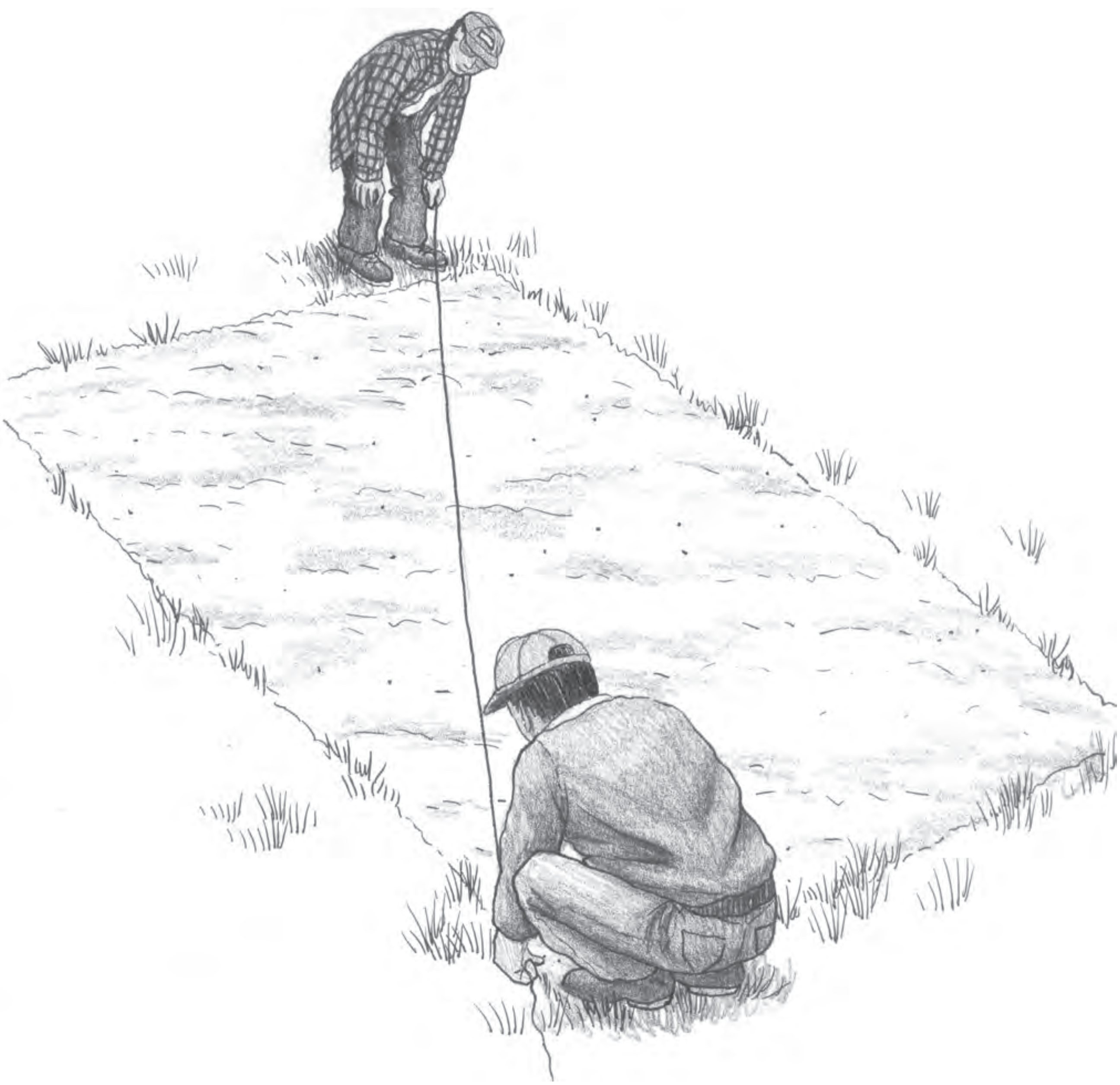
Shape Strips



Properties of Quadrilaterals

Sketch of Shape	Name	Sides	Angles	Diagonals	Other Properties

Elders Using Diagonals to Construct a Rectangle



Section 2

The Smokehouse Walls: Rectangular Prisms

Activity 3:
Framing the Walls

Today students will start building their smokehouse model. They start by making and proving a rectangle as they did in Activity 2; however, this rectangle forms the base of their desktop model smokehouse. Putting on the walls of the smokehouse model creates a three-dimensional shape, the rectangular prism. Like the previous lesson, this activity is designed to give students hands-on experience with rectangular prisms that they can draw on to understand their mathematical properties.

The activity is structured so that students first build their model, verifying that the base is a rectangle so that their model will be straight and sturdy, before constructing the walls to make a rectangular prism. They use the models they have built to discover and describe properties of rectangular prisms. Next students make conjectures about how the properties of a rectangle might apply to rectangular prisms and test these properties using their models.

Because students at this point are constructing an open model of a smokehouse, the focus of students' exploration of properties will be on the edges, vertices, and interior space of the prism. Activity 4 will give students the opportunity to explore the faces of rectangular prisms.

Goals

- To construct a rectangular prism
- To generalize properties of rectangles to rectangular prisms.
- To begin to formulate the definition of a prism.

Materials

- Butcher paper
- Math Notebooks
- Model building materials: straws and pipe cleaners or newspaper
- Scissors
- String
- Tape

Duration

Two to three class periods.

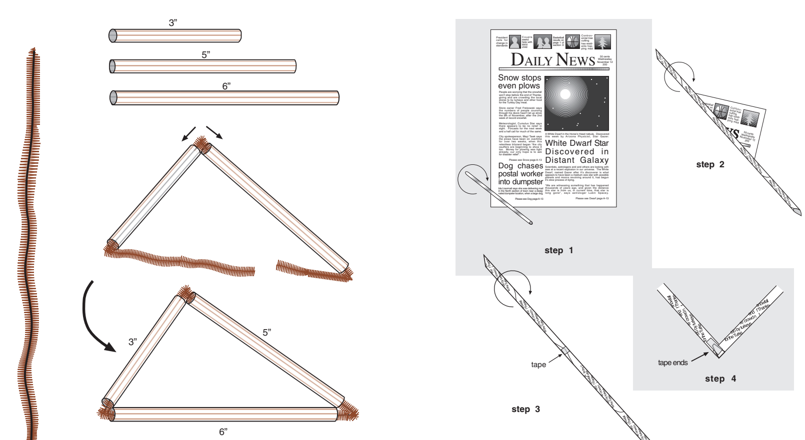
Vocabulary

Base—the bottom of a geometric shape, usually parallel to the ground or the bottom of a page.

Bisect—to cut or divide into two equal parts.
Bisector—something that cuts or divides onto two equal parts.
Cube—a solid figure in which all six faces are equal-sized squares.
Edge—a line segment formed by the intersection of two faces of a solid figure.
Face—a flat side of a solid figure.
Oblique prism—a prism whose faces are not perpendicular to its bases.
Perpendicular—two lines or planes intersecting at a right angle.
Prism—a solid figure that has two congruent parallel polygons as bases and rectangles or parallelograms for all its other faces. There are two types of prisms, right prisms, and oblique prisms.
Rectangular prism—a prism with rectangular bases.
Right prism—a prism whose faces are perpendicular to its bases.
Three-dimensional—a quality of space measured in three mutually perpendicular directions, such as length, width, and height.
Vertex (pl. vertices)—the point or points at which sides of a polygon, lines of an angle, or edges of a solid figure meet. Vertices of polygons are sometimes called corners.

Teacher Note

There are different ways to construct the model. An easy and effective way is to use straws and pipe cleaners. The pipe cleaners can be inserted into the straws at different angles as shown in Figure 3.1. Alternatively, paper can be rolled into dowels with or without something like a straw inside. These dowels can be folded at different angles and attached to each other with tape, also shown in Figure 3.1.



Option 1: Straws and pipecleaners   Option 2: Paper dowels

Fig. 3.1: Two options for constructing the smokehouse model.

Preparation

Collect materials for smokehouse models. Build a model smokehouse and then follow step 8 and determine properties of a rectangular prism. Read and become familiar with the Math Note on pages 52-56. Build a model yourself and use it to familiarize yourself with these properties. Determine which properties you were able to derive and which you were not through this exploration. This will assist you as you teach.

Kathleen Meckel, a teacher in Fairbanks, said, "Students loved this activity: the construction and the freedom to explore."

Instructions

1. Review the properties of rectangles from the previous activity.

2. Students will start construction of their smokehouse models today. Explain to students that as they build they will also be investigating properties of two-dimensional rectangles and three-dimensional rectangular prisms.

3. **Model.** Demonstrate how to use the available materials to make the edges of the smokehouse (see Figure 3.1).

4. Challenge students to construct a model base that will be the foundation of a sturdy smokehouse on their first try by using the properties of a rectangle learned in the last activity. As students work, build a model base for your own smokehouse. **Teacher Note:** Encourage students to visit you as you are working as well as visit other students. (See note on joint activity on page 6 of the introduction.)

5. Have students build the walls of their models by attaching pipe cleaners and straws or paper dowels to the base they constructed in step 4. As students work, frame the walls of your own smokehouse model. **Teacher Note:** Putting the walls on changes the shape of the model from a two-dimensional rectangle to a three-dimensional rectangular prism. The construction process should help students discover properties of prisms, such as the number of sides and the angles of the faces.

Joint Activity: Work on the walls of your smokehouse model, allowing students to observe.

Cultural Note

The height of the smokehouse is determined by the height of the user, who needs to be able to reach all the racks to hang the fish. Ferdinand Sharp of Manokotak said he buries the base about 6 inches in the ground.

Math Note

Encourage students to use properties they know about rectangles (as listed in step 1) to generalize to properties of right rectangular prisms. Generalizing is a key component to mathematics. The more we can encourage students to begin thinking in terms of generalizing, the more we can help students to think mathematically. However, be aware that it is easy and common to generalize incorrectly, so students should be encouraged to conjecture and prove their generalizations and continue until they find something that works consistently.

The properties of both diagonals and midpoint connectors generalize to rectangular prisms. However, given the greater complexity of the shape, these properties also become more involved. In the case of diagonals, there are two diagonals in a rectangle but four in the interior of a rectangular prism (see Figure 3.8). In both cases, the diagonals indicate the center of the shape. Midpoint connectors can also be used to find the center of the interior of the shape (see Figure 3.6). Students will need to use midpoint connectors in at least four of the faces of the rectangular prism in order to locate and attach the bisectors that go through the interior of the prism.

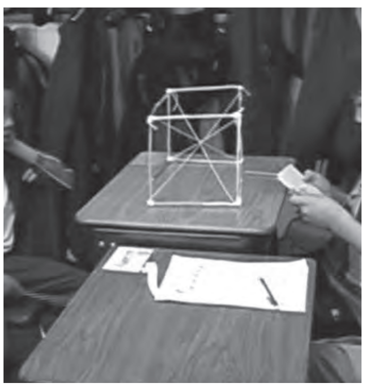


Fig. 3.2: Students verifying diagonals on their smokehouse model.

6. When construction is complete, ask students to describe the shape they have constructed. Introduce the concept of a rectangular prism as well as other vocabulary words that help students to describe their models such as vertices, edge, and face.

7. **Discuss.** Ask students to describe what is the same between rectangles and rectangular prisms and what is different. Use this discussion to make a list of possible properties of rectangular prisms. **Teacher Note:** If students don't mention properties such as diagonals or midpoint connectors, introduce these into the discussion. Each of the properties of rectangular prisms is described in the Math Note at the end of this activity.

8. **Challenge.** Give students string to test if the properties of a rectangle apply to rectangular prisms and discover properties of their rectangular prism (smokehouse). (See Figure 3.2 for an example of students using their model to test diagonals.)

9. After students have conducted tests on their models, revise the list of properties of rectangular prisms listed in Step 7 to include only those that stood up to students' tests. Have students record these properties in their math notebooks.

Fig. 3.3: Student-made smokehouse model.

Teacher Note

It is okay if students have not found how all the properties of rectangles apply to rectangular prisms at this point as they will continue their explorations in the next activity; however, make sure that students understand how the diagonals and midpoint connectors work in three-dimensional figures as such explorations will not be practical in the next activity. Save the list of properties the class generates for use in the next activity.

10. To extend students' understanding of three-dimensional objects, ask students to describe what shape they would get if they started with a square smokehouse base, making the height of the smokehouse the same length as the base. Students can draw or make this shape with straws as needed. **Teacher Note:** Ask students whether this shape would make a good smokehouse. Introduce the vocabulary word cube and make sure students understand that each of the faces will be a square with edges the same lengths as those of the base.

11. **Poster.** Students should make their second entries on the poster, describing the how they built the smokehouse body and the properties of rectangular prisms they were able to show using the model.

Homework: List one property that generalizes from a two-dimensional rectangle to three-dimensional rectangular prism. Using drawings and words, show how this property is the same and different between the two types of shapes.

Generalize properties from rectangles to rectangular prisms

Math Note: Generalizing from Two Dimensions to Three Dimensions

After reviewing properties and proofs of rectangles, students should be able to conjecture about which properties may generalize to three-dimensional right rectangular prisms. To make this process less overwhelming, we have reduced the explorations in several ways. First, students will consider only rectangular and triangular prisms. Second, they will use what they already know in two dimensions as a starting point for properties in three dimensions. Finally, students are not to prove that the three-dimensional shape is really a rectangular prism as they did with the two-dimensional rectangle. Rather students will assume that they have a rectangular prism and will attempt to determine and prove which properties, when generalized from two-dimensional to three-dimensional, still hold.

Properties such as diagonals become much more involved in a three-dimensional shape than in its two-dimensional counterpart. The following discussion pertains to rectangles and rectangular prisms; we will use the same process to generalize to triangles and triangular prisms later in the module.

Generalizing Properties: Properties and Dimensions

To begin the generalizing process you will want to work with your students to first develop comparable vocabulary between rectangles and rectangular prisms. One key idea is that there are more properties and more complexity when we move from two to three dimensions. For example, some of the most basic properties of a rectangle are that it has four sides and four vertices. Sides in two dimensions generalize to faces in three dimensions. Vertices joining two sides in two dimensions generalize to vertices joining three edges in three dimensions. More specifically, a side is a one-dimensional length but a face is a two-dimensional shape. A vertex of a rectangle is the intersection of two sides and is a point which is zero-dimensional (0-D). In a prism, adjacent pairs of faces intersect to form one-dimensional edges. Further, three intersecting edges form a vertex, which is a point (0-D). So, although with a rectangle there is only one form of intersection, there are now two forms of intersections among the basic properties for a rectangular prism: two faces connect to form one-dimensional edges and three edges connect to form zero-dimensional vertices. Note that the faces of a rectangular prism come in parallel pairs, thus the edges are parallel and of equal length.

Properties of shapes can be thought of in three categories: **Inherent Properties**, those that seem most obvious from the definition of the object; **Calculated Properties**, those that can be calculated based on other properties of the shape; and **Constructed Properties**, those that involve additional constructions such as diagonals and midpoint connectors.

Figure 3.4 is an illustration of the **Inherent Properties** of a rectangular prism:

- 3 pairs of opposite rectangular faces;
- 1 pair of faces defines the base
- 12 edges
- 8 vertices

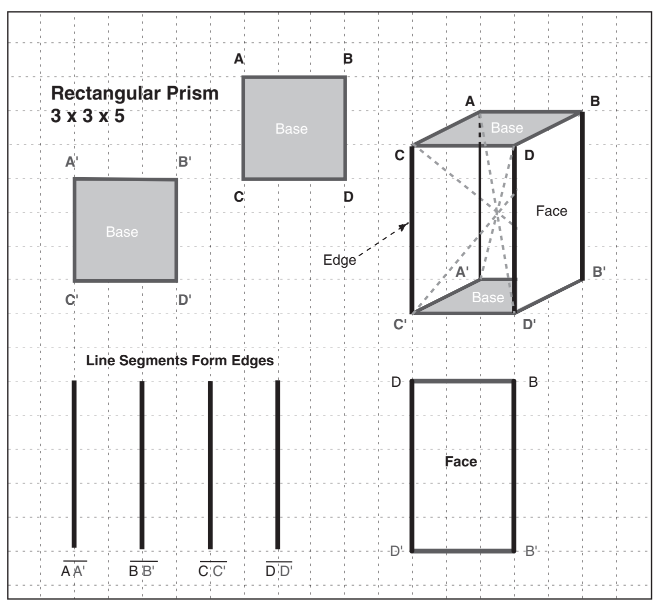


Fig. 3.4: Inherent properties of rectangular prisms.

Math Note (continued)

Rectangle	Rectangular Prism
Inherent Properties	**Inherent Properties**
two-dimensional (2-D)	three-dimensional (3-D)
length and width	length, width, and height
four sides at right angles; opposite sides congruent and parallel sides	six faces at right angles; opposite faces congruent and parallel
four vertices, each joining two sides meeting at right angles	twelve edges, each joining two faces meeting at right angles
four sides with opposite pairs of sides congruent and parallel to each other	eight vertices, each joining three edges meeting at right angles
	six faces with opposite pairs of faces being congruent shapes (same shape and same size) and parallel to each other
Calculated Properties	**Calculated Properties**
perimeter: sum of lengths of four sides	surface area: sum of areas of six faces
area: product of length, width	volume: product of length, width, height
Constructed Properties	**Constructed Properties**
two diagonals—line segments connecting opposite vertices. Diagonals meet in the center and bisect each other	four interior diagonals—line segments connecting opposite vertices that meet in the center of the prism;
midpoint connectors—two perpendicular line segments that connect the midpoints of opposite sides, They also meet in the center and bisect each other.	twelve exterior diagonals—line segments connecting opposite vertices in each of 6 faces. These diagonals meet in the center of each face.
center—a point in the middle of the rectangle; can be found by using diagonals, midpoint connectors, or circumscribing a circle around the rectangle.	midpoint connectors—three lines all perpendicular to each other that connect the centers of opposite faces. They meet in the center of the prism and bisect each other. The centers of each face are defined by the midpoint connectors in each face.
	center—a point that is in the middle of the rectangular prism; can be found by using interior diagonals, midpoint connectors, or circumscribing a sphere around the rectangular prism.
	angle bisectors—lines or planes that bisect each vertex; in the special case of the cube the angle bisectors and the diagonals are equivalent; however, in non-cube rectangular prisms the angle bisectors are not helpful or useful.

The chart on the previous page is provided as a summary, comparing properties of two-dimensional rectangles and three-dimensional rectangular prisms, broken down into the three categories of properties.

Below is an example of a *Constructed Property* that can be generalized from the two-dimensional properties of midpoint connectors of a rectangle.

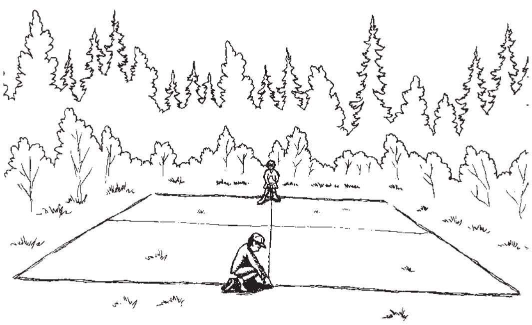


Fig. 3.5: Measuring connectors with string.

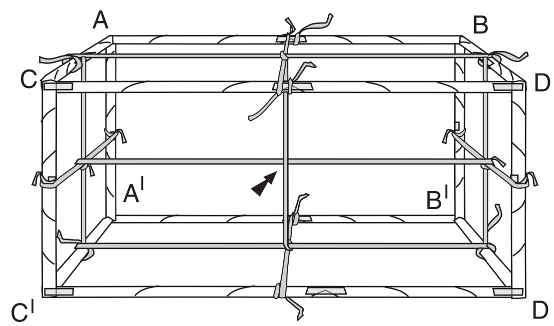


Fig. 3.6: Midpoint connectors in a rectangular prism intersect in the middle of the prism.

Interestingly, finding the midpoint connectors in 3-D becomes a two-step process. First, the midpoint connectors for each face are found. These reside in just one plane or one rectangle. Then, using the centers of each face—the point where the midpoint connectors meet—additional midpoint connectors are created that reside in the interior of the prism. Thus, in 3-D, the process has more steps and there are more midpoint connectors to find. This is another example of the expanding complexity of properties when moving from 2-D to 3-D.

Later we will consider similar properties for triangles and triangular prisms. Although most of these properties will generalize in a similar way, the uniqueness of each will become apparent as well.

Difficulties with Generalizing Properties

Note that it can be easy to generalize between properties of rectangles and rectangular prisms in ways that may not be useful or entirely correct. For example, when drawing a rectangle, the focus is on creating the four sides. When drawing a rectangular prism, the focus is on creating the twelve edges. Because both the sides of the rectangle and the edges of the rectangular prism are 1-D lines, we might conclude that the four sides of the rectangle generalize to the twelve edges of the rectangular prism. However, this generalization does not hold. It is the sides of a rectangle that define its exterior and interior and determine its perimeter and area. The comparable attribute of a rectangular prism is the six faces since they define the exterior and interior of the shape. The edges define the faces, but it is the faces that fully define the rectangular prism. Using this process, we see that the edges in 2-D should generalize to the faces in 3-D.

You can apply this understanding of the relationship between sides and faces to generalize between calculated properties as well. It is easy to assume that the area of a rectangle would generalize to the surface area of a rectangular prism given the similarities in names. However, area is a calculation of the space inside of a rectangle which generalizes to volume, or the calculation of the space inside of a rectangular prism.

Generalizing Proofs

Proofs

Now that students have investigated comparable 3-D properties of the more well known 2-D properties, they can conjecture about which proofs may generalize between 2-D rectangles and 3-D rectangular prisms. It is important to realize that proofs are directional in nature. This means that they can usually be written as if-then statements. For example, in Activity 2 students investigated properties of a rectangle and found that every rectangle has two diagonals. Further, they found that these diagonals are the same length and they bisect each other in the center of the rectangle. Another way to state that finding is to say students proved that "**If** a shape is a rectangle **then** it has two diagonal lines of equal length that intersect in the center of the shape."

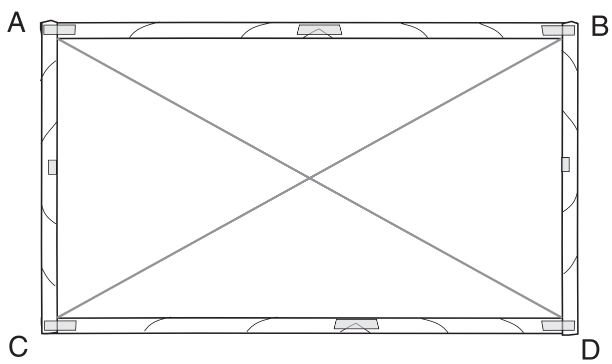


Fig. 3.7: Smokehouse model using diagonals as proof of a rectangle.

To generalize proofs, students will consider what they have learned about the diagonals of a rectangle discovered in Activity 2, and will conjecture whether a similar property of diagonals holds true for a rectangular prism and in what way. One conjecture could be that if it is a rectangular prism then it should have three diagonals of equal length. This is a valid conjecture given that three-dimensional objects generally have more and complex properties when compared to related two-dimensional objects.

When testing that conjecture using the smokehouse model, students will find that the rectangular prism has four interior diagonals (plus twelve more, two in each of six rectangular faces). All four interior diagonals of the rectangular prism are of equal length and intersect in the center, similar to the diagonals of a rectangle. Thus, they are verifying that "**If** a shape is a rectangular prism **then** it has four interior diagonals of equal length that intersect in the center of the prism." This fact does not mean the converse is true: "**If** the four diagonals of a shape have equal length and intersect in the center **then** the shape is a rectangular prism." Proving whether this is or is not true is beyond the scope of this module. As mentioned above, we are having students show rather than prove properties in the rectangular prism given the increased complexity of the three-dimensional shape.

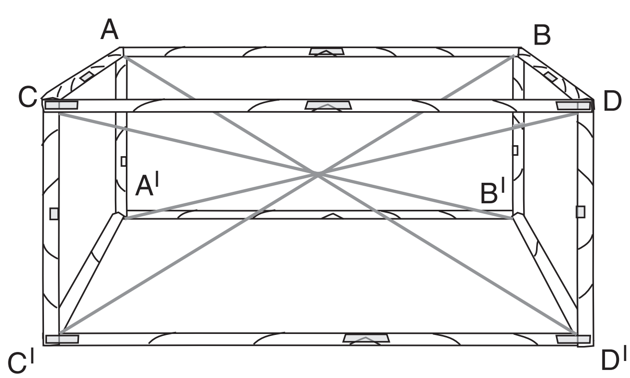


Fig. 3.8: Diagonals in a rectangular prism are equal length, meet in the center of the prism, and bisect.

Keeping students aware of the fact that the proof may not work in the other direction should aid in clarity of what they can say. It is beyond the scope of this module to prove that three-dimensional shapes are of a certain type, whereas proofs involving two-dimensional shapes are accessible at this level. (In *Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area*, students had the knowledge to say that because a four-sided closed figure had two equal diagonals that bisected each other, and they met in the center of the figure then the figure MUST be a rectangle. With three-dimensional figures, this is much more difficult.)

Terminology

The rectangular prisms we deal with in this module are called right rectangular prisms, meaning that the rectangular bases are perpendicular to all of their adjacent faces. This is the most common representation of a rectangular prism that we see, such as any typical type of box. However, rectangular prisms can also be oblique prisms, whose bases are not perpendicular to the other faces. This type of prism is what we asked you as the teacher to build at the beginning of Activity 2 as a demonstration of why it is important to verify that the base is a rectangle. Some modern structures might have a non-rectangular prism shape, but it's not a practical way to build a utilitarian structure such as a smokehouse or shed. For simplicity in this module, we will only consider right rectangular prisms and call them "rectangular prisms" although you should be aware of this distinction in case students create or find examples of these other cases.

Finally, there are different terms for special cases of rectangles and rectangular prisms. In two dimensions, when a rectangle has all sides of equal length, it is called a square because of its special properties. Likewise, in three dimensions, when a rectangular prism has six congruent faces, then each face must be a square and the object is called a cube.

Activity 4: Using Plane Shapes to Make Rectangular Prisms

In this activity, students construct rectangular prisms from index cards or old manila folders based on their smokehouse models. Making closed rectangular prisms in this way helps students visualize the prism as a closed shape, making it easier to see and explore properties involving the faces. Cutting out the faces from index cards guides students toward exploring congruence as they discover which faces are the same size and shape. In addition, students can distinguish the faces as two-dimensional objects within the three-dimensional prism, as compared to the one-dimensional sides of a rectangle that is only in two dimensions. Attaching the pieces of index cards together to create the prism reinforces the idea that faces in a right rectangular prism are at 90-degree angles. These experiences lead students to refine their understanding of properties of prisms.

In the extension to this activity, students have the opportunity to make their smokehouse models more realistic by adding racks for hanging the fish, and, if desired, other "accessories" such as a fire pit and even paper fish. Doing this will help students to relate the model they are constructing to the real smokehouses used by the Yup'ik. There is additional information to share with students about how smoke flows in the smokehouse to cure the fish.

Goals

- Construct a rectangular prism using faces rather than edges
- Discover and describe properties of rectangular prisms
- Practice the use of terminology such as face, edge, vertex, diagonal, and concepts such as congruence

Materials

- Handout, Additional Practice Problems for Activity 4 (optional)
- Handout, Vocabulary Map
- Index cards or old manila folders, enough so that each student can construct his or her own model
- Math Notebooks
- Rulers
- Scissors
- Tape
- Transparency, Flow of Air Through a Smokehouse
- Transparency, Smokehouse with Two-Tiered Racks

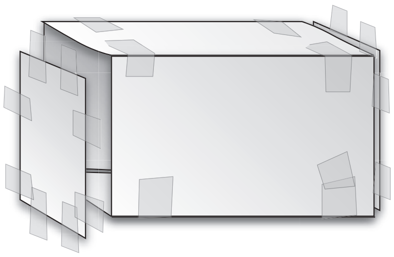


Fig. 4.1: A taped-together rectangular prism made with index cards.

Duration

Two class periods.

Vocabulary

Right angle—an angle that measures 90°, or one fourth of a full rotation. This is the angle found in squares, rectangles, and right prisms.

Instructions

1. Show a shoe box, book, or another object that is in the shape of a rectangular prism. Ask students how this shape relates to that of their smokehouse models (emphasize that they are both three-dimensional figures). Have students point out other rectangular prisms in the room. Ask students how the box type of rectangular prism is the same or different than the one they constructed in activity three. (The boxes have closed faces while the smokehouse model is open.) Explain that today they will construct rectangular prisms that will help them to visualize an enclosed prism, like a smokehouse (see Figure 4.1 for an example).

Fig. 4.2: Teacher Thad Keener using a tissue box as a rectangular prism.

Fig. 4.3: Students constructing rectangular prisms with index cards.

2. **Challenge.** Hand out index cards and tape. Ask students to cut out the faces needed to construct rectangular prisms that approximate the shape of their smokehouse models. Students may work in their groups and use their open-faced model to compare it to their closed-faced model. **Teacher Note:** Approximating the proportions of the smokehouse models will encourage students to compare the closed-face rectangular prisms they are making from index cards with the open one of the model.

3. **Discuss.** Ask students how they used the properties of rectangular prisms discussed in the previous activity to construct their index card prisms. For example, students might have used the fact that all angles in a rectangular prism are right angles to correctly attach one index card

Math Note

The process of cutting out the shapes should focus students' attention on the fact that opposite faces are congruent in rectangular prisms. If students pursue strategies that show understanding of congruence, such as cutting out two congruent faces at the same time or measuring the dimensions of one to make the second congruent face, have them share their strategies with the class and explain why they work.

Melissa Lowinske, a teacher in Akiachak, said, "Students got really into this activity and made some great prisms. It was interesting to see different approaches to making them."

to another. Further, students may have noticed how faces intersected to form edges and vertices, creating the three-dimensional object.

4. Show students the list of properties of rectangular prisms from the previous activity. Ask them to verify the properties on their new models and list new ones. Note that faces and angles may be more accessible than diagonals and midpoint connectors with this closed-face model. Add or modify your list of properties based on students' discoveries. **Teacher Note:** Save your list of properties of rectangular prisms for use in Activity 6.

5. Have students label their models with the vocabulary words face, vertex, and edge to reinforce understanding and use of these terms. Explain that they will save these prisms to use again in a future activity.

6. In their math notebooks, have students write a working definition of prisms in general, based on what they have learned about prisms from constructing the smokehouses and paper models of rectangular prisms. Have students share their definitions with the class and construct one definition from their responses that the class can agree on. Tell students that they will continue to work on refining their definitions as the module continues. **Teacher Note:** It is okay if this definition is incomplete or inaccurate as it will be revised. For example, if students decide that a prism has six faces based on their experience with rectangular prisms, leave that in for now, but make sure to discuss and correct the discrepancy when you explore triangular prisms (which only have five faces) in the next section.

7. Synthesize. Draw on students' understanding of prisms developed in the last two activities to help them generalize between rectangles and rectangular prisms. This is a good time to bring up the idea of perimeter and area in two-dimensional objects and ask students to generalize similar measurements of three-dimensional objects. (See Math Note on generalizing in Activity 3 for an explanation of this topic.)

8. **Math Notebook.** Have students draw and label a rectangular prism in their math notebooks using the vocabulary you have introduced.

9. **Optional.** An additional tool for vocabulary development is included here but can be used throughout the module as appropriate. Demonstrate how to use a Vocabulary Map (blackline master) using the example on the next page or insert another word. Have students fill out their own maps for one of the vocabulary words associated with the properties of rectangular prisms such as edge, diagonal, face, etc. Have students with different words compare vocabulary maps.

Math Note: Vocabulary

As students are talking about two-dimensional and three-dimensional shapes, vocabulary will become important. When looking at a rectangle in two dimensions, we don't call it a plane, but the rectangle is understood to be in a plane. However, when looking at a rectangular prism in three dimensions, the six faces that make the prism are each plane shapes that are rectangles.

Teacher Note: Vocabulary Map

The vocabulary map can be used to learn the vocabulary words presented in the module. Each box provides students with a different way to think about the word in order to build a deeper understanding of the word.

In addition to rewriting the definition in their own words, students can also provide examples of the word, either in writing or drawing.

You may also ask students to locate objects in the room or bring in objects from home that might demonstrate a term. These can be assembled in a *vocabulary artifact exhibit* that includes placards with the vocabulary words displayed by the object(s).

Because many words have different meanings depending on the discipline and context, we have included a box titled *Common Meaning*. This is aimed at accessing prior knowledge of a word that may be known in one context as meaning one thing, but in another it may have a different meaning altogether. Learning to differentiate between the multiple meanings of words can aid in reading comprehension across content areas. When learning new vocabulary words, it is good to begin with the *common meaning* and then move to the mathematical meaning.

Finally, the *Category* box can help students to connect a word with a group of words or a larger concept. This helps to build a conceptual understanding of the word and its relationship to other words that are key to the concept within mathematics. Refer to Figure 4.4 for an example.

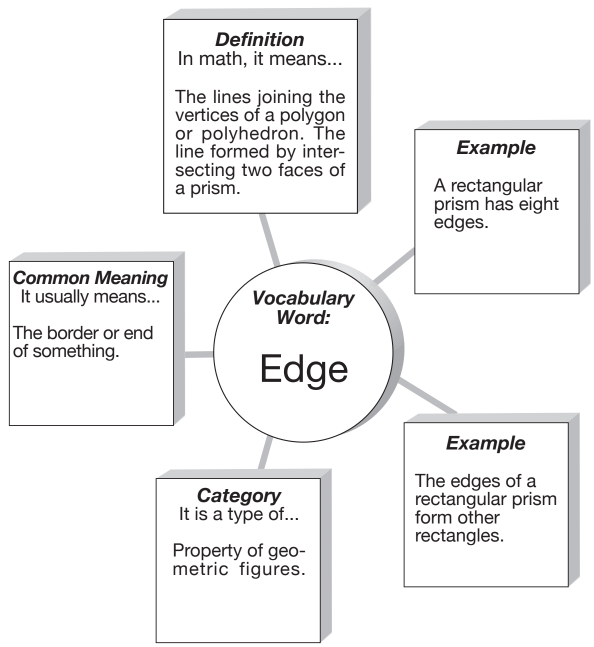


Fig. 4.4: Example of a vocabulary map.

Extension: Fish Racks

If you would like a more authentic smokehouse model, you can add fish racks to the interior to hang the fish on. If the students are going to put in more than one of level of fish racks in the smokehouse, they will need to add rack supports. The students should try to make their racks as realistic as possible.

Duration

One class period.

Materials

- Gumdrops or marshmallows
- Poster, Smokehouse (from Activity 1)
- Scissors

- Tape
- Toothpicks
- Transparency, Flow of Air Through a Smokehouse
- Transparency, Smokehouse with Two-Tiered Racks
- Transparency, Yup'ik Smokehouse (from Activity 1)

Instructions

1. Tell the students that today they will be putting racks into their smokehouses. Explain how a smokehouse is used or have students describe their own experiences if appropriate.

2. Show the transparencies, Smokehouse with Two-Tiered Racks and Flow of Air Through a Smokehouse, so students will understand how to construct their racks. If needed, read the Cultural Note on smoking salmon on page 63 to your students to help explain the process.

3. Hand out toothpicks and gumdrops for students to construct their racks. Encourage students to design their fish racks for their smokehouse models to maximize the function of their smokehouses. If desired, students can also add a fire pit and paper "fish."

4. Ask students to explain the location of their fire pits and discuss the air flow given the size and shape of their smokehouses and design of their fish racks.

Additional Practice Problems for Activity 4

The additional practice problems on the handout Additional Practice Problems for Activity 4 can be used to reinforce and extend students' understanding of the topics presented in this activity. To promote mathematical discussion and get the most from these problems, we recommend that students work on them in pairs or in their construction groups. To assist students as they solve these additional practice problems have various tools available for students such as shape strips, models of prisms, and index cards or card stock to help them visualize shapes and their properties. The solutions to these problems are below.

Materials

- Cubes of any size, all equal
- Handout, Additional Practice Problems for Activity 4 (one for each group)

Duration

One class period.

Solutions to Additional Practice Problems

1. Students should draw six-sided three-dimensional figures with opposite sides being parallel, congruent rectangles. Students should be able to cite properties of rectangular prisms in their drawing to establish whether they meet the criteria for a rectangular prism.
2. A correct drawing for the conditions of the problems would be any six-sided prism that has one pair of opposite faces that are quadrilaterals but not rectangles. For examples, a prism that has two opposite faces that are trapezoids (and other four sides would be rectangles) would be a six-sided prism that is not a rectangular prism.
3. (a) Students should construct 2 by 2 by 2 cubes with the eight unit cubes. The cube meets all the criteria of a rectangular prism (i.e., a cube is a rectangular prism that happens to have all of its faces congruent).
   (b) There are four small (or unit) squares that make up each face of the 2 by 2 by 2 cube.
   (c) Each of the eight unit cubes that make up the 2 by 2 by 2 cube has exactly three of its faces exposed.
4. (a) Students should make 3 by 3 by 3 cubes. As in 3 (a) above, a cube meets all of the criteria for a rectangular prism.
   (b) There are nine small (or unit) squares that make up each face of the 3 by 3 by 3 cube.
   (c) Eight cubes have three faces exposed (i.e., the cube at each corner of the 3 by 3 by 3 cube); twelve cubes have two faces exposed (i.e., the middle cubes of each edge of the 3 by 3 by 3 cube); six cubes have one face exposed (the cube at the center of each face of the 3 by 3 by 3 cube); and one cube has zero faces exposed (i.e., the cube at the center of the 3 by 3 by 3 cube that is surrounded by the other twenty six cubes here).
   (d) Most students are surprised that the unit cubes have different numbers of faces exposed when used to make a 3 by 3 by 3 cube. However, one way to make sense of the numbers of exposed faces for different cubes is to note that a 3 by 3 by 3 cube has eight corners (these are the cubes with three faces exposed), twelve edges (the cubes in the middle of each edge are the cubes with two faces exposed), six faces (the cubes in the center of each face of the 3 by 3 by 3 cube have one face exposed), and one center (the cube in the center of the 3 by 3 by 3 cube is the cube with zero faces exposed).

Cultural Note: Smoking Salmon

Smoking fish is an activity that has to be done properly. To be preserved, smoked fish needs to be completely dried and free from any moisture. However, it cannot be cooked so it is important that the fire in the smokehouse smolder with no hot flames.

Some smokehouses have two levels of fish racks that are used at different stages in the smoking process (see Figure 4.5). In the first stage, fresh salmon are hung on the lower level near the fire pits for a number of days to dry and cure. They are then moved forward on the lower level as new fish are hung close to the fire. As they dry further, they are moved to the upper level and finally back to the lower level along the periphery of the smokehouse for storage and slightly more smoke. This rotation of the salmon to different levels and locations allows it to dry and smoke properly without cooking, because the smoke flows around the fish (see Figure 4.6).

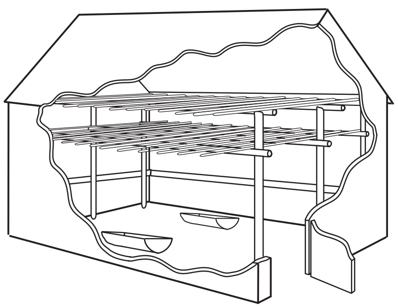


Fig. 4.5: Two-level smokehouse.

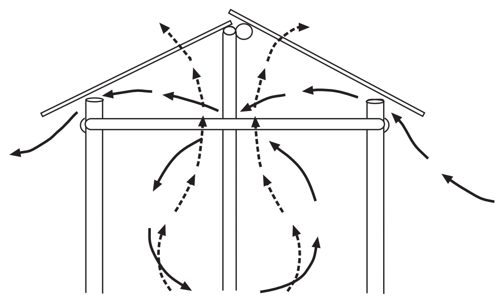
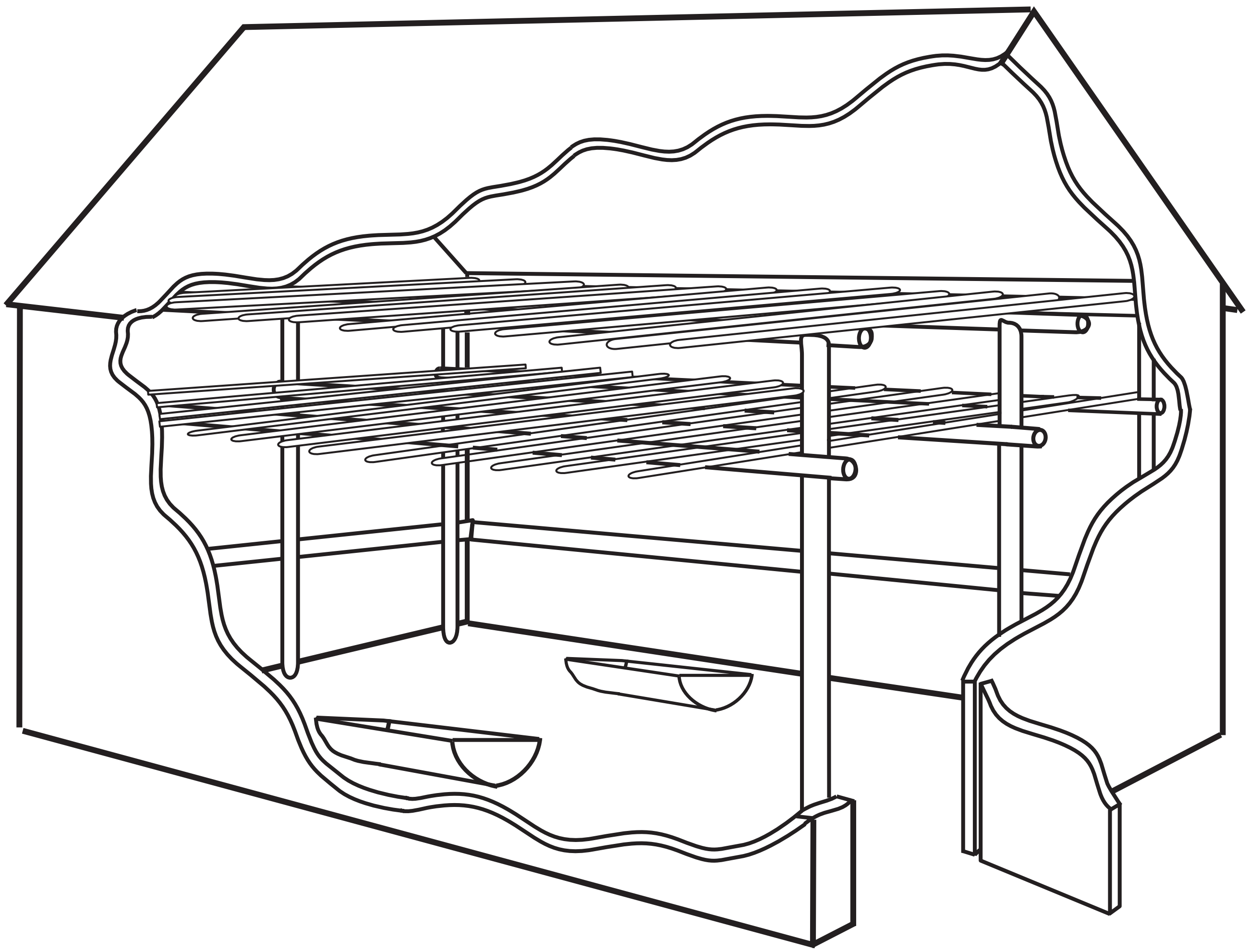


Fig. 4.6: Flow of air through a smokehouse.

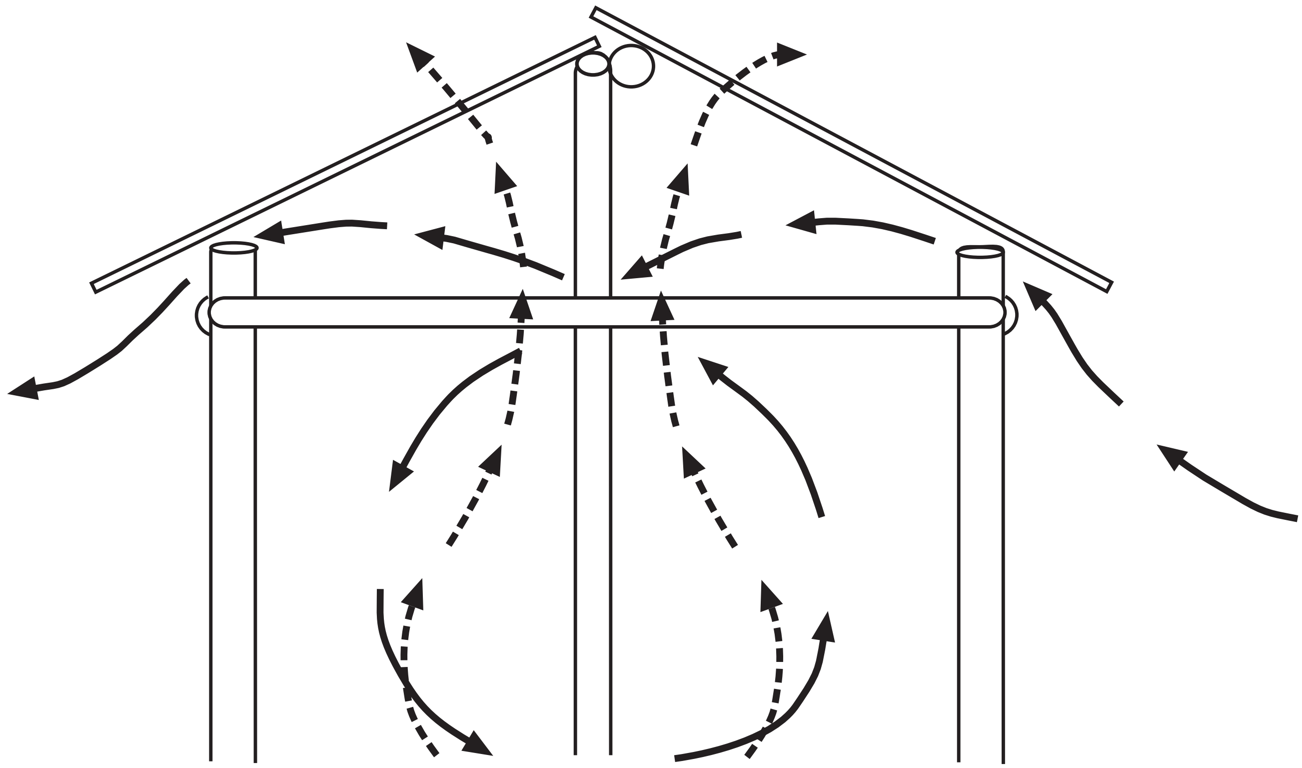
Ferdinand Sharp of Manokotak describes the method he and his wife Nancy use:

> *To get the fish ready for smoking, we cut up our catch of the day, soak them in salted brine, and hang them on the fish rack.*
>
> *We prefer to smoke in the cool of the morning or evening; this helps prevent the fish from getting cooked. We like to hang them with the skin on the outside because the wet skin is very sticky and it would stick to the pole.*
>
> *Our smokehouse is just one level, not two. When we put the fish inside the smokehouse, the skin has to be on the outside and we put the fish as close together as possible. We put the fish as far to the far wall as possible. We put the fish in the smokehouse in the order they were cut in.*
>
> *My wife checks them as they smoke. When we first put them in, the skin is silver-colored; when they turn gold, my wife decides to take them out. Or she looks at the meat and if it looks kind of dark—it gets darker and darker as it smokes—that's when she decides when to take them out. If they are oversmoked, they will taste bitter, so we try to smoke them with just the right amount of smoke.*

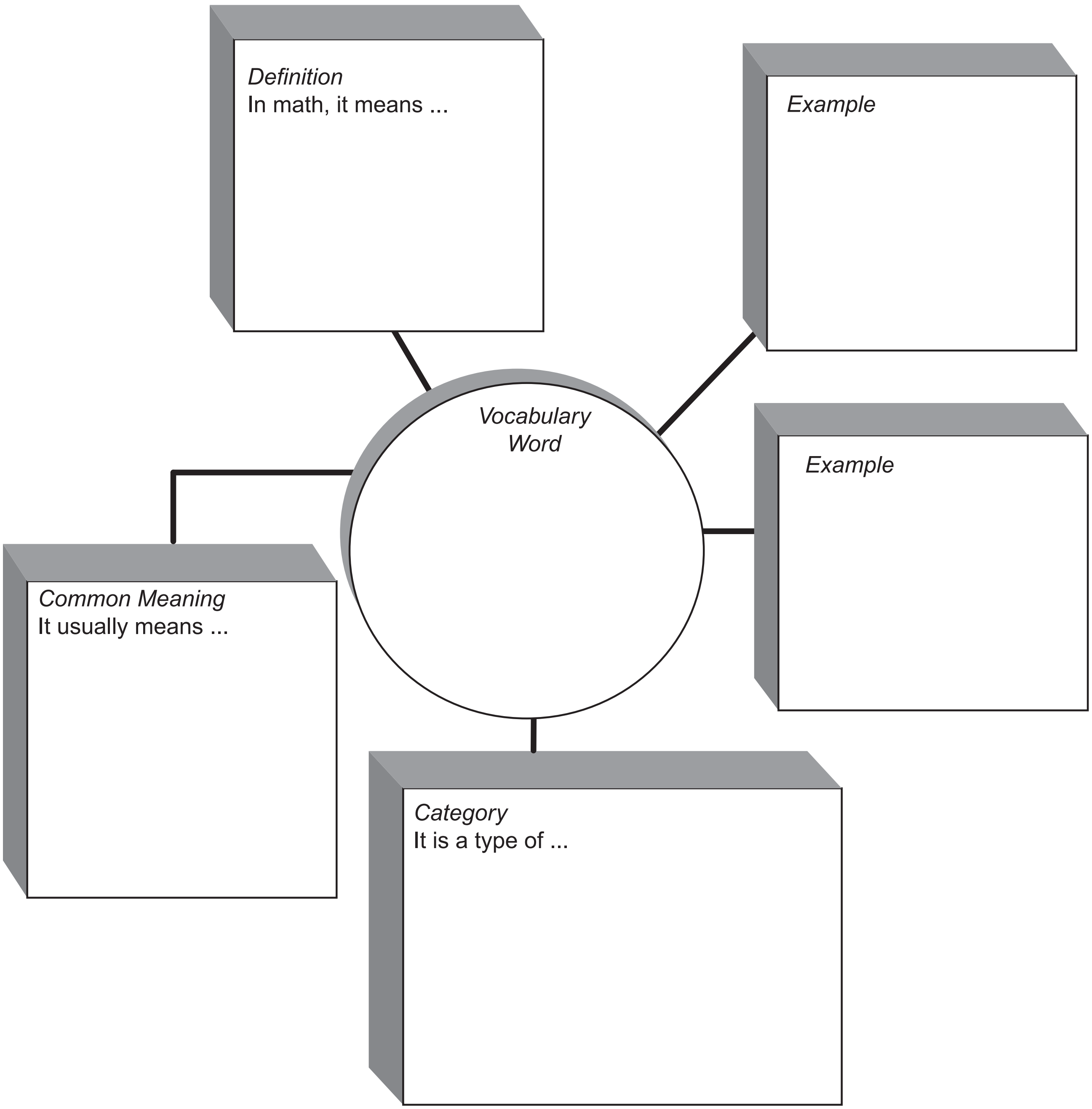
Smokehouse with Two-Tiered Racks



Flow of Air Through a Smokehouse



Vocabulary Map



Additional Practice Problems for Activity 4

1. Draw a rectangular prism. Explain why you think your drawing is a rectangular prism.

2. Draw a prism that has six faces but is not a rectangular prism. Explain why your drawing is a six-faced prism but not a rectangular prism.

3. Using eight cubes, make a rectangular prism that has six congruent faces. Then answer these questions:
   (a) Describe the shape of the prism you have made and why you think it is a rectangular prism.

   (b) How many small squares make up the surface of each face of the prism?

   (c) How many faces of the eight cubes make up the surface of the entire prism?

4. Using twenty seven cubes, make a rectangular prism that has six congruent faces. Then answer these questions:
   (a) Describe the shape of the prism you have made and why you think it is a rectangular prism.

   (b) How many small squares make up the surface of each face of the prism?

   (c) How many faces of the twenty seven cubes make up the entire prism? For example, how many of the cubes have no faces on the surface? How many have one face on the surface? How many have two? How many have three? Do any cubes have more than three faces on the surface of the prism?

   (d) Did your findings from part (c) surprise you? Why or why not?

Section 3

The Smokehouse Roof: Triangular Prisms

Activity 5
Designing the Roof: An Exploration of Slope

In this activity, students design a roof for their smokehouse including an appropriate slope. In Activity 6, they construct the roof following their design. The math of this activity addresses the mathematical concept of slope as a way to describe lines, a departure from the focus on geometric shapes of the previous activities. However, this topic has ties to the broad study of geometry (as described in the Math Note at the end of the activity) and follows the theme of learning math topics through the construction of the smokehouse model.

The activity is set up to build on students' intuitive understanding of slope, which they know as the steepness of a hill, roof, or stairs, to bring them to a mathematical understanding of slope as a ratio of the rise over the run (as shown in Figure 5.1) through experimentation and hands-on exploration of the relationship between the length and height of various roof model slopes. Students will apply their understanding of slope in the next activity when they build the roof on their smokehouse model with the same slope as the one they design in this activity. This grounding in an authentic use of slope will help students to understand the concept of slope within algebraic expressions when they encounter that topic in future math.

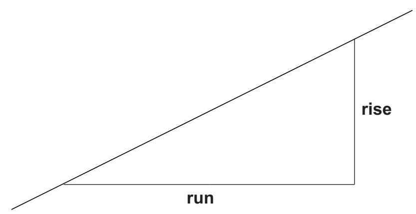


Fig. 5.1: Example of rise and run of a sloping line.

Goals

- To introduce the concept of slope as the ratio of rise over run
- To calculate slope using the rise:run ratio
- To design a roof for the smokehouse model that will perform well

Materials

- Corrugated cardboard (old boxes work well) cut into two sets of squares of two different sizes, one of each size per construction group. (**Note:** This activity works best if the sides of the smaller square are half the size of the bigger one.) For example, each group should have a 2 by 2 inch and a 4 by 4 inch piece.
- Cubes of equal size (i.e., unit cubes) or grid paper if no cubes are available—all students should use the same size blocks
- Handout, Slope Data Table
- Math Notebooks
- Pennies
- Transparency, Slope Data Table

Duration

Two class periods.

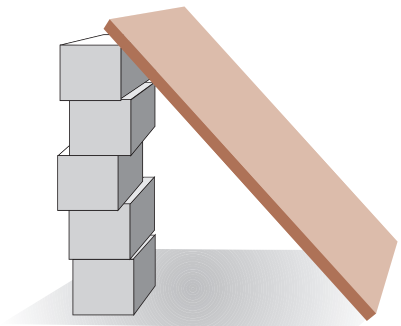


Fig 5.2: Cardboard square roof model.

Vocabulary

Horizontal—parallel to the horizon or to a baseline.
Ratio—a comparison of two quantities by division.
Rise—the height measured between two points on a line segment (or plane surface), used to calculate its slope.
Run—the horizontal distance measured between two points on a line segment or plane surface, used to calculate its slope.
Slope—the ratio of the rise divided by the run of a line segment or plane surface. A measure of its steepness.
Vertical—perpendicular to the plane of the horizon or to a primary axis.

Math Note

Depending on students' preparation, different strategies can be used to construct the same pitch. The simplest verification is done by sight, lining up the two pieces of cardboard to see if the slopes match. To push the investigation further, ask students why they needed fewer blocks with the smaller piece of cardboard and how they might predict how many blocks to use. If students are ready, encourage them to look at the relationship between rise and run using either the grid paper or more blocks on the table (see Fig 5.3 below).

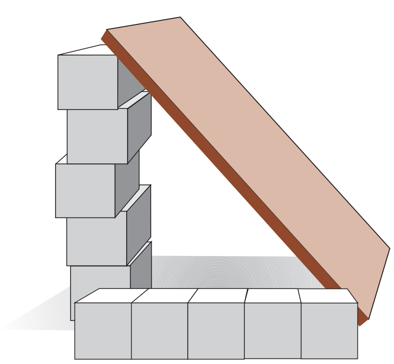


Fig 5.3: Measuring the rise and run with blocks.

Preparation

Cut out cardboard squares, two for each construction group.
Review the concept of slope and how to demonstrate varying ratios by altering the pitch of the slope. Review the slope data table.

Instructions

1. Tell students they are going to start designing their roofs today. Ask students to describe smokehouse roofs they've seen (if applicable) and to brainstorm a list of things that are affected by how steep any roof in general is (snow and rain removal, how the fish smoke, etc.). Tell students that the math word for steepness is slope. (**Note:** The concept of slope will be further developed mathematically through this activity.) Today's activity will help students to design a roof with a slope that will meet their needs.

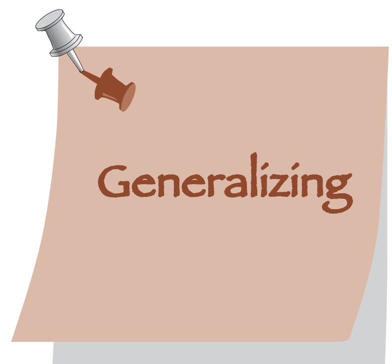
2. **Model.** Demonstrate how to set up the cardboard square roof model with blocks (see Figure 5.2). Explain that you are making a simpler model of the smokehouse roof that students can use to design their real roofs. Show how a penny can be used to simulate the snow load on a roof and demonstrate how to slide it down your "roof." Use the Slope Data Table transparency to model filling in the data for each of the columns.

3. Have students get into their construction groups and hand out one Slope Data Table for each group, plus the cardboard for the roof.

4. **Challenge.** Tell students they should experiment with a variety of slopes until they find cases where the penny: (1) slides down quickly, (2) slides and stop part way down, and (3) doesn't slide at all by

experimenting with putting different amounts of cubes under their cardboard as you modeled. These results should be recorded on the Slope Data Table. Have students decide which slope they would like for their smokehouse model and keep their cardboard piece at that slope.

5. **Discuss.** Have groups share their results. Facilitate organizing all findings based on steepness of slope, such as no slide, slides part way, and slides quickly. Discuss the Slope Data Table as a class, asking students to describe the patterns they see.



6. **Challenge.** Challenge students to position a new piece of cardboard which is half the size of the piece they used in Step 3, but has exactly the same slope. Students should use blocks, pennies, and their eyes to verify that their new model has the same slope. As a class, discuss students' strategies for determining that the slope of their two pieces of cardboard are the same. **Math Note:** Students should see that their data shows that the run and rise increase or decrease proportionally and can be described as a pattern. For example, if you double the rise, the run also doubles. Students may also point out that you can use some multiple to get the run from the rise. Once students understand the pattern, ask them to predict what the run would be if you give them the rise and vice versa.

7. Have students use their discoveries in the previous step to come up with a way to get the same slope no matter the size of the cardboard. **Teacher Note:** To do this, students will need to calculate slope as a

Math Note

Slope is the ratio of the rise (vertical distance between two points on a line) divided by the run (horizontal distance between the same two points). Two points with the same ratio can be modeled as a line, such as the edge of the cardboard in Figure 5.4. Placing any two points on the x and y axes on grid paper produces a line with the ratio rise/run remaining constant. To create the same slope with a smaller piece of cardboard, the amount of rise needs to be shortened proportionally to the run. This can be demonstrated by having students put a second set of blocks under their bigger piece of cardboard. If they add a set of blocks at the midpoint, it will use half the number of blocks (see Figure 5.4).

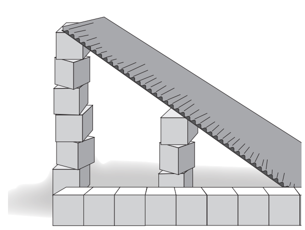


Fig. 5.4. Three-dimensional model of roof.

Math Note

In Figure 5.5, two triangles are formed using the same line. These triangles are called similar triangles, the ratio of their sides are the same. The slope is the ratio of the rise over the run or the vertical distance divided by the horizontal distance.

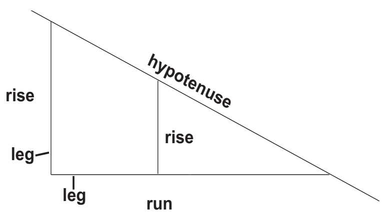


Fig. 5.5. Two-dimensional drawing of roof.

ratio. Once they know the slope ratio of their chosen roof, they choose values for the rise and run that will fit their piece of cardboard.

8. Draw a two-dimensional picture for students of the roof as in Figure 5.5 to reinforce slope as a characteristic of a line. Ask students to explain why the slope of the line stays the same no matter where you measure the rise to make sure they understand slope as a ratio. Students should draw this illustration in their math notebooks.

9. **Discuss.** Ask students to connect the calculation of the slope of each of the roofs on their Slope Data Table to what they observed. Encourage students to reduce fractions or convert to decimals or whatever numeric value helps them understand the ratio.

10. **Assess.** Have students use their cardboard pieces to demonstrate the answers to the following questions (and others you pose). How can you get a roof with a slope ratio of 1? How does the slope of a roof with a rise that's greater than the run compare to a roof with a slope of 1? How does the slope of a roof with a rise that's smaller than the run compare to a roof with a slope of 1? Which of the following ratios would give you the steepest roof: 2/3, 3/1, 3/2? Discuss strategies for answering these types of questions.

Math Note: Use of Slope and Mathematics

Slope is an important concept in the field of mathematics and so is taught at many different levels. Students usually first learn about slope in beginning algebra in reference to the slope of a line and calculate it by using the ratio of rise over run. They usually see it in the equation of a line $y = mx + b$ where m is the slope, b is the y-intercept, and (x, y) are the ordered pairs that fall on the line. As students progress through mathematics, they next learn in trigonometry that the slope concept can also be discussed using triangles. Using the rise and run, a triangle can be formed by connecting the two legs with a straight line forming the hypotenuse of the triangle (as shown in Figure 5.5). The slope is related to the angle between the hypotenuse and its horizontal leg and is equivalent to the tangent of the angle. Further, when students enter calculus, the slope topic arises once again connecting the derivative of a function at a point to the slope of the tangent line. This slope value or function has physical meaning depending on the equation being analyzed. For example, if the equation represents the location of an object being dropped as a function of time then the derivative determines the velocity of the object at any point in time.

Although we do not ask students to focus on the angle of the roof during this lesson, measuring the rise and run does later give way to a whole field of mathematics called trigonometry. Trigonometry is the study of angles and relationships between them and tends to concentrate on angles within triangles which closely links it to geometry. Trigonometry centers more on the actual measurements of angles and sides of a triangle, whereas geometry focuses on establishing relationships without using measurement. For more details on this topic, see the website: http://www.sparknotes.com/math/trigonometry/angles/summary.html

11. **Math notebooks.** In their notebooks, have students write down which slope they think would make the best roof and why.

12. **Smokehouse Poster.** Students should add their third entry to the poster on determining the slope. The graphic should show how the students calculated the slope and the writing should explain the slope chosen for the model.

Additional Practice Problems for Activity 5

These additional practice problems can be used to reinforce and extend students' understanding of the topics presented in this activity. To promote mathematical discussion and get the most from these problems, we recommend that students work on them in pairs or in their construction groups. Hand out the additional problems.

Materials

- Handout, Additional Practice Problems for Activity 5

Duration

One class period.

Solutions to Additional Practice Problems

1. Students should draw a triangle (the typical shape used to represent a roof) that has a rise of three for a run of five. Note that students may present valid drawings that have a different rise and run than three and five, respectively. For example, a student who draws a roof that has a rise of nine and a run of fifteen has a correct solution – i.e., the ratio of the rise to the run of nine to fifteen is equivalent to 9/15 = 3/5 (i.e., as long as the ratio of the rise to the run is 3/5 the student's solution is valid).
2. (a) Students should sketch a right triangle that has a base of six and height of nine. Students who mistakenly label the hypotenuse of their triangle as having a length of six or nine should be reminded that the base and height are perpendicular to each other, so neither the base nor the height can be the hypotenuse.

(b) Depending on how students have their triangle oriented, the rise can either be six or nine and the run would be nine or six, respectively (i.e., if students say the rise is six then the run must be nine; if they say the rise is nine then the run must be six).

(c) Depending on how students have chosen the rise and run, the slope of the hypotenuse is either 6/9 or 9/6.

3. (a) Most students will draw the diagonal that goes from the lower right corner of the rectangle to the upper right corner, but drawing the other diagonal is also correct.

(b) Depending on how students have drawn their rectangle (i.e., two by five or five by two), the rise will be two and the run will be five, or vice versa. Because of how the diagonals of a rectangle are drawn, one side of the rectangle is the rise and the adjacent side is the run.

(c) Depending on how students have drawn their rectangle, the slope of the diagonal will be either 2/5 or 5/2.

(d) The slope of the other diagonal would be the same (**Math Note**: Technically, the slope of the other diagonal would be of the opposite sign from the first, since it slopes in the opposite direction. In particular, lines that go upward from left to right have positive slope and lines that go down from left to right have negative slope. However, for the purposes of this module, students do not need to worry about the sign, but observe only that the two diagonals slope are in different directions).

4. (a) Yes, you can find the slope of the diagonal of a square without knowing the edge length of the square. This is because, for the diagonal of a square, the rise and the run are always the same because the adjacent edges of the square (which are equal) define the rise and run.

(b) Since, for the diagonal of the square, the rise and run are equal, the slope will be 1.

(c) The slope of the diagonal of a smaller or larger square will still be 1 because the ratio of the rise to the run (i.e., the slope) will always be 1 because the rise and run are always the same.

(d) The slope of the diagonal of any square is 1. Regardless of the size of the square (i.e., whatever edge length is has), the rise and run will always be equal for the diagonal so the slope will always be 1. Algebraically, if the edge length of the square is x, then the rise = run = x for the diagonal, so the slope = rise/run = $x/x = 1$. Students should not be expected to come up with an algebraic justification, but should note that since the edges of any square are equal, the rise and run will always be equal for the diagonal, so for any square the slope if its diagonal is 1.

Slope Data Table

# of Blocks Used to Hold Up Cardboard	# of Blocks on the Table	Slope Description and Value: $\frac{\text{Rise}}{\text{Run}}$	What Happened to the Penny? Why?
EXAMPLE: 1	10	very little 1/10	did not move small slope

Additional Practice Problems for Activity 5

1. Make a drawing on grid paper that shows a roof that has a slope of 3/5. Be sure to label the rise and the run of the roof and explain why the slope of the roof is 3/5.

2. A right triangle has a base of six and a height of nine.
   (a) Make a drawing of the triangle on grid paper.
   (b) If the hypotenuse of the triangle represents the roof of a house, what is the rise and what is the run of the roof?
   (c) Using your answers from part (b), what is the slope of the roof represented by the hypotenuse of the right triangle?
3. Draw a rectangle on grid paper so that one side has a length of two and an adjacent side has a length of five.
   (a) Draw in one of the diagonals of the rectangle.
   (b) What are the rise and the run of the diagonal that you have drawn? How do you know?
   (c) What is the slope of the diagonal that you drew? Explain how you found your answer.
   (d) If you drew in the other diagonal of the rectangle, would its slope be the same or different than the slope of the first diagonal? Make a conjecture. Can you prove that your conjecture is true for diagonals of any rectangle?
4. Draw a square and then draw in one diagonal of the square going from the lower left corner to the upper right corner.
   (a) Can you find the slope of the diagonal of the square without knowing the edge length of the square? Explain your reasoning.
   (b) What do you think the slope of the diagonal of the square is? Explain how you found your answer.
   (c) If you drew in the same diagonal for a smaller or larger square, would the slope of the diagonal be the same or different than the slope of the diagonal of your square that you found in part a)? Explain your reasoning.
   (d) Based on your work in this problem, make a conjecture about the slope of the diagonal of any square? Can you prove this conjecture?

Activity 6
Framing the Roof

This activity builds on the previous one as students apply slope or a ratio to construct a roof for their smokehouse model. This gives them further practice with the concept of slope and how it is used in designing and building.

The activity also introduces students to triangular prisms. As with the rectangular prisms, students develop an understanding of the properties of triangular prisms through the construction process. Because they have previously learned about rectangular prisms, students should be able to generalize properties from rectangular to triangular prisms as well as between the two types of triangular prisms introduced, isosceles triangular prisms (Figure 6.1) and right triangular prisms (Figure 6.2).

Goals

- To apply understanding of slope to a real-world context
- To define and describe the properties of triangular prisms
- To generalize properties of rectangular prisms to triangular prisms
- To compare properties of various types of triangular prisms

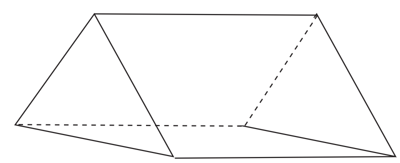


Fig. 6.1: Example of an isosceles triangular prism.

Materials

- Group posters
- Model building materials
- Rulers
- String
- Transparency, Types of Smokehouses
- Your own model with part of a shed roof constructed

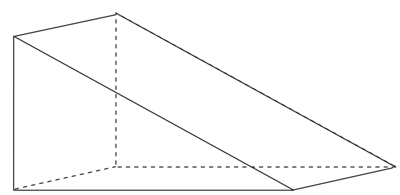


Fig. 6.2: Example of a right triangular prism.

Duration

Two to three class periods.

Vocabulary

Centroid—the center of mass of a figure. The centroid of a triangle is the intersection of the medians. The centroid of a rectangle is the intersection of the diagonals.

Hypotenuse—the side opposite the right angle in a right triangle.

Isosceles triangle—a triangle with at least two sides the same length. The base angles of an isosceles triangle are also equal.

Isosceles triangular prism—a prism whose bases are isosceles triangles.

Median—a line connecting a vertex of a triangle with the midpoint of the opposite side.

Right angle—an angle that measures 90˚, or one fourth of a full rotation. This is the angle found in squares, rectangles, and right prisms.

Right prism—a prism whose faces are perpendicular to its bases.
Right triangle—a triangle with one right angle.
Right triangular prism—a prism with right triangle bases.
Triangle—a polygon with three sides.
Triangular prism—a prism with triangular bases.

Preparation

Start to put a shed roof on your own model by attaching the pipe cleaners for the rise of the roof.
Read the Math Note on pages 83-85.

Math Note

You may want to review properties of triangles, in particular, right and isosceles triangles. Encourage students to be specific about what makes these triangles special (lengths of sides and size of angles). Ask them to further investigate midpoints, medians, and the centers of both types of triangles as these properties may be useful when generalizing to triangular prisms. (See Math Note on generalizing properties of triangles and triangular prisms at the end of this activity for more information on the background knowledge students will need.)

Instructions

1. Start by showing students the beginning of the roof on the model that you have started. Ask students what shape they think your roof will be when it is finished. Have them make a sketch in their math notebooks and discuss their ideas.

2. Referring back to the last activity, ask students how they would determine the slope of your roof.

3. Show students the different types of roofs (use transparency, Types of Smokehouses). Introduce the term triangular prism. Ask students to outline on the transparency a triangular prism (isosceles triangular prism is used for gable roof and right triangular prism is used for shed roof). Introduce the specific vocabulary and discuss the properties of isosceles and right triangles. Discuss why a flat roof is not ideal for a smokehouse (water and snow could collect and leak into the structure). If appropriate, this is a good time for students to discuss types of smokehouse roofs they have seen. Have construction groups decide which type of roof they would like to make, either a gable or shed roof.

4. **Challenge.** Tell construction groups to frame a roof on their model, either gable or shed, that has close to the same slope as the one they decided to be ideal in the previous activity. Encourage them to use the strategies they developed in Activity 5 to construct and verify their slope. As students work on their models, encourage them to visit other groups and observe your work as you put a roof on. **Teacher Note:** If all the groups favor one type of roof, you may want to go back and change your roof to the other way so that you have examples of both gable and shed roofs. Also, due to construction constraints, it is difficult to exactly copy the desired slope; this can be another place to discuss the difference between mathematical constructs and real-life models.

Joint Activity: Work on your own smokehouse model, allowing students to observe and further learn if necessary

5. When students have finished their constructions and posters, visit each of the models to see their constructions and have groups explain their choice of roof type and how they calculated and constructed the slope of their roof to match the cardboard model from Activity 5. Introduce the vocabulary words isosceles triangular prism and right triangular prism to differentiate the two types of roofs.

6. As a whole class, review the properties of rectangular prisms (from Activities 3 and 4). Have students make conjectures about which of these properties might also be true for the triangular prism of their roof as well as properties they think the triangular prism might have. List these on the board.

7. Introduce the terms isosceles triangular prism and right triangular prism to describe the two types of prisms used for the roofs. Ask students why they think these prisms are called isosceles or right triangular prisms and how to tell the difference between the two. Make sure students know that it is the base that identifies the type of prism, just as in the rectangular prisms and cubes seen in previous activities.

8. **Discuss.** Have students discuss what is the same and different between rectangular and triangular prisms. **Teacher Note:** It is important that students can explain why we call both the triangular and rectangular solids they have constructed "prisms" despite their differences. This is a preliminary discussion of triangular prisms that should be continued after students construct index card triangular prisms in the next activity. (See Math Note at the end of this activity for more information on the properties of triangular prisms.)

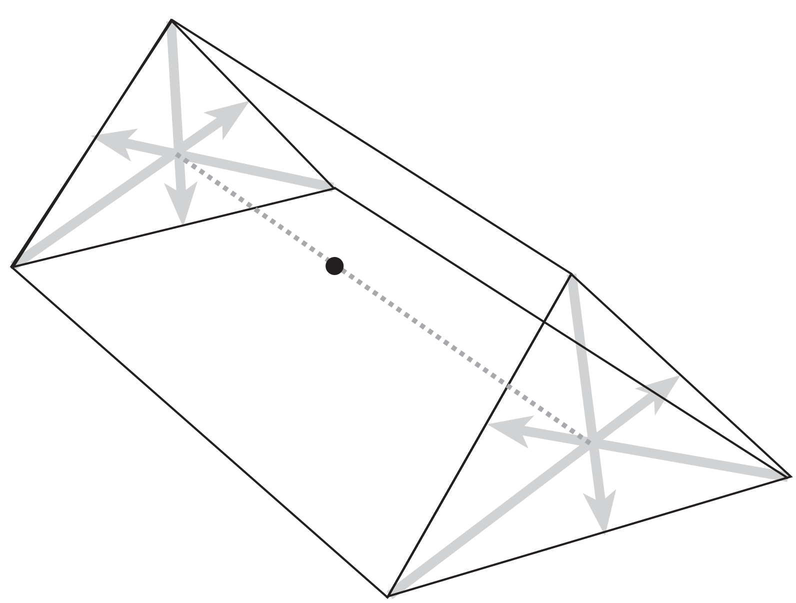


Fig. 6.3: Finding the center of a triangular prism.

Math Note

This activity should demonstrate differences between prisms where all the faces are quadrilaterals and those where the bases are shapes other than rectangles, such as in triangular prisms with their triangular bases. If students would like to see how diagonals or midpoint connectors work in a triangular prism, let them know that constructed properties of triangular prisms are more complex than for rectangular prisms. Triangles, being three-sided, do not have diagonals, as students will quickly discover. It is possible, on the other hand, to find the center of a triangular prism using midpoint connectors as illustrated in Figure 6.3.

9. **Literacy Counts.** To prepare for adding information about the roof to their posters, including the type of roof and slope chosen and why, students can present their model as well as their reasoning about their choices for the roof to the class. For example, students might choose a steep roof in an area where there is heavy snowfall. An essential element of the presentation is a question and answer period after their presentation where students can defend their design choices.

Math Note: Generalizing Properties of Triangles to Triangular Prisms

Properties of Triangles

Using the ideas outlined in the Math Note on Rectangles and Rectangular Prisms, we can generalize to properties of 2-D triangles and 3-D triangular prisms. Transitioning from triangles to triangular prisms can be more complicated than from rectangles to rectangular prisms. It may be helpful to use properties of rectangles as a starting point to conjecture and prove which ones can be applied to triangles. Students should also investigate the specifics of how the property works with a three-sided figure in contrast to a four-sided figure.

Inherent Properties

The most basic inherent properties found in all triangles include three sides and three angles with the sum of the angles totaling 180 degrees. Unlike rectangles, which have only 90-degree angles, triangles are further grouped into different classifications based on the measure of these inherent properties:

- Isosceles—two equal sides and two equal angles,
- Right—one angle is exactly 90 degrees,
- Acute—all angles less than 90 degrees,
- Obtuse—one angle more than 90 degrees, and
- Equilateral—all sides equal and all angles equal.

For our purposes, we will work only with right and isosceles triangles in generalizing to 3-d prisms and classify the remaining types in an “Other” group. In triangles, the length of the sides and the size of the angles always relate to each other. Therefore, the isosceles triangle can also be defined as having two angles of equal size and the equilateral triangle can be defined as having all equal angles, each 60 degrees.

Calculated Properties

The Calculated Properties (perimeter and area) for a triangle are identical to those of the rectangle and are calculated in a similar fashion. The perimeter is found by adding the side lengths and the area is found by determining the number of square units that cover the inside. The area formula for a triangle is ½ base x height since a triangle can always be described as half of a rectangle (or a parallelogram which can then be made into a rectangle).

However, the base and height of a triangle are *constructed properties* since you can define a triangle without them. The base of a triangle can be any of its sides. The height of the triangle must then be defined as a distance along a line segment perpendicular to the base joining the base to the opposite vertex. Drawing the base and height may look different for acute, right, and obtuse triangles as shown in Figure 6.4, but the height is always perpendicular to the base. For a right triangle, the base and height are usually defined as the two sides that form the 90-degree angle. If the hypotenuse is used as the base, the height is a line segment from the right angle vertex, perpendicular to the hypotenuse.

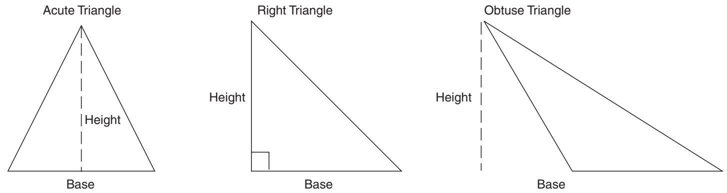


Fig. 6.4: Examples of three types of triangles and how to define the base and height of each.

Constructed Properties

We will start with the properties we discovered with rectangles to investigate how, if at all, they work with triangles.

Diagonals: The diagonals of a rectangle are the line segments connecting opposite vertices. Since triangles have only three vertices, each is adjacent to the other two. Thus there are no opposite vertices and so no diagonals.

Midpoint connectors: The midpoint connectors of a rectangle were defined as the lines connecting the midpoints of opposite sides. In triangles, we can find the midpoint of each side. But there is no opposite side, so instead we use the opposite angle or vertex. The corresponding construction, connecting a vertex to the opposite side, is called a median of the triangle. The medians intersect at a center of the triangle. This point is also called the centroid, or center mass of the triangle.

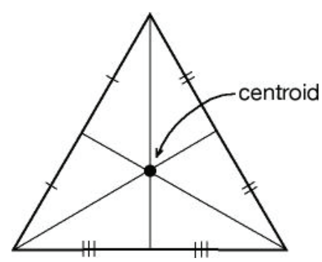


Fig. 6.5: Finding the midpoint connectors and the centroid of a triangle.

Center points: In a rectangle there is only one center point (as is the case with many other quadrilaterals and n-gons with more than four sides). In the triangle there are several different center points each found in a unique way and each having a unique role in applications. We discuss only three kinds of center points in triangles, although only the centroid is useful for finding the center of a triangular prism.

Centroid: The three medians meet in the centroid or center of mass (center of gravity). The centroid divides each median in a ratio of 2:1. The centroid is the point on which the triangle will balance.

Incenter: The three angle bisectors of a triangle meet in one point called the incenter. It is the center of the incircle, the circle inscribed in the triangle.

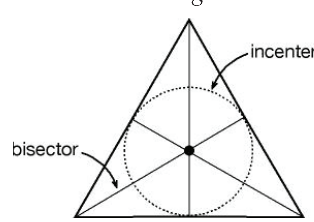


Fig. 6.6: Finding the angle bisectors and the incenter of a triangle.

Generalizing to Triangular Prisms

With all the information gathered at this stage, we can now generalize properties of triangular prisms from properties of both their two-dimensional counterparts in triangles and three-dimensional rectangular prisms. The following chart summarizes the properties of triangles and triangular prisms.

Triangle	Triangular Prism
Inherent Properties	**Inherent Properties**
two-dimensional (2-D) shape	three-dimensional (3-D) shape
three sides (1-D)	five faces (2-D), two congruent, parallel triangular bases
three vertices each joining two sides	nine edges, each joining two faces
	six vertices, each joining three faces
Calculated Properties	**Calculated Properties**
perimeter: sum of three side lengths	surface area: sum of five face areas
area: base x height	volume: area of base triangle x prism height
Constructed Properties	**Constructed Properties**
triangle height is perpendicular to the base	prism height is perpendicular to the triangular base
diagonals—do not exist	interior diagonals—do not exist
	six diagonals, two on each quadrilateral face
angle bisectors—lines that bisect each angle and connect to the opposite side, they intersect at the centroid	angle bisectors—lines or planes that bisect each vertex
three medians connect the midpoints of a side to the opposite vertex; they meet in the centroid and divide each other in a ratio of 2:1	midpoint connectors—four lines: one joins the centroids of the bases, the other three are perpendicular to the other three faces connecting the midpoints of each face to the opposite edge. They meet in the center of the prism.
centroid—the center of the mass of the triangle that is the intersection of three medians. (see Figure 6.5)	centroid—the center of mass of the triangular prism found by bisecting the line segment joining the centroids of the bases.

Triangular prisms focused on in this module are actually right right-triangular prisms and right isosceles-triangular prisms as shown below in Figures 6.7 and 6.8. We will call them just right triangular and isosceles triangular prisms to make it easier. The distinction refers to the base of each, being either a right triangle or an isosceles triangle. Since these types of triangles are the most prevalent in the real world, we do not include the other types of triangular prisms. Also, both the right triangle and isosceles triangle are the most common examples of roofs, especially for smokehouses. Keep in mind that other triangular prisms without these types of bases do exist, in case students create or ask about them.

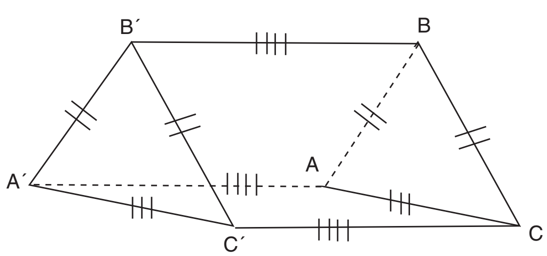


Fig. 6.7: Isosceles triangular prism.

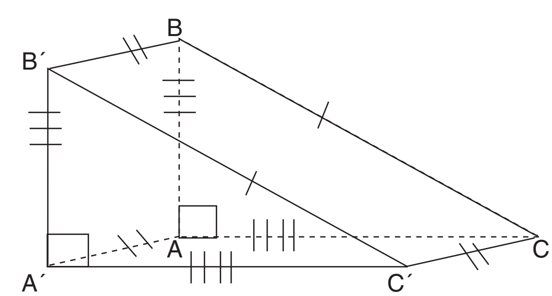
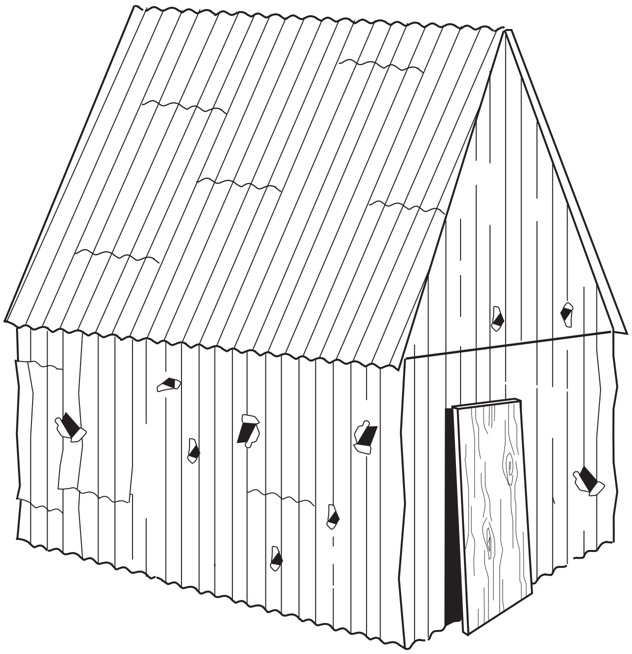
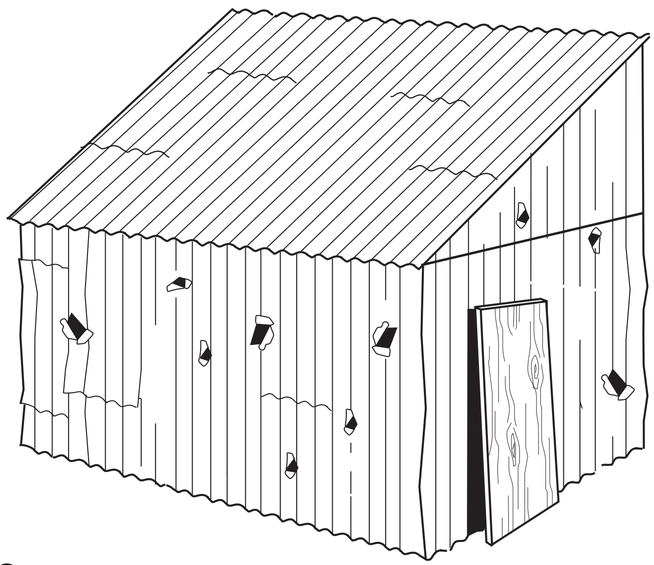


Fig. 6.8: Right triangular prism.

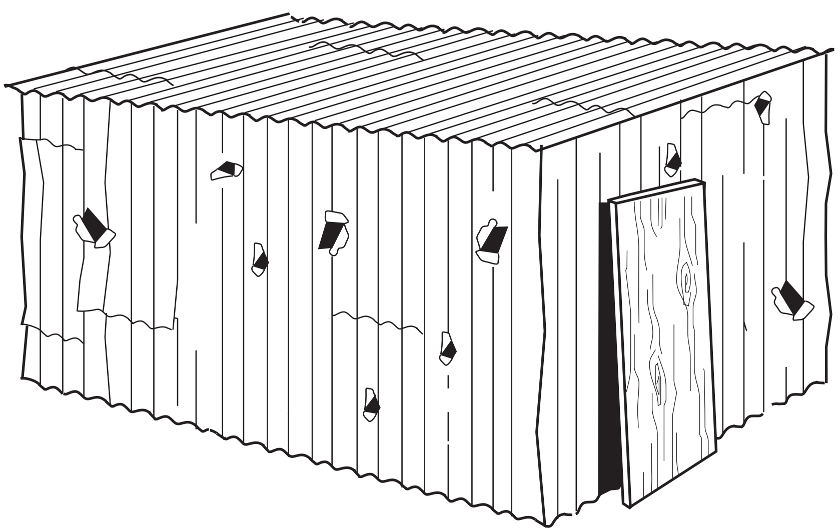
Types of Smokehouses



Gable roof



Shed roof



Flat roof

Activity 7
Using Plane Shapes to Build Triangular Prisms

This activity helps students further develop their understanding of the general definition of prisms, and the properties of triangular prisms in particular. Building models of triangular prisms with index cards focuses students on the faces of the prism, which are hard to discern with the open smokehouse model roof constructed in Activity 6, and guides them toward understanding properties associated with the faces. Also, the activity gives students experience with constructing both types of triangular prisms, isosceles and right, to see how they differ.

Students also construct a new type of three-dimensional solid, the pyramid, as a way to further explore the differences between this type of shape and prisms. Students will refine their understanding of the definition of a prism.

Goals

- To construct a triangular prism using faces rather than edges
- To discover and describe properties of triangular prisms
- To practice the use of terminology such as face, edge, vertex, median, centroid, and concepts such as congruence

Materials

- Handout, Plane Shapes
- Index cards or old manila folders
- Rulers
- Scissors
- Tape

Duration

One class period.

Fig. 7.1: Using shapes to create a prism.

Vocabulary

Congruent—geometric figures (or parts of figures) that are the same shape and size. Two shapes are congruent if one shape can be slid, flipped and/or rotated so that that shape fits exactly on top of the other one. Parts of a shape, for example, sides or angles may also be considered congruent.

Edge—a line segment formed by the intersection of two faces of a solid figure.

Face—a flat side of a solid figure.

Isosceles triangle—a triangle with at least two sides the same length. The base angles of an isosceles triangle are also equal.

Median—a line connecting a vertex of a triangle with the midpoint of the opposite side.

Pyramid—a solid figure with one base that is a polygon and triangles for its other faces.

Right angle—an angle that measures 90˚, or one fourth of a full rotation. This is the angle found in squares, rectangles, and right prisms.

Right triangle—a triangle with one right angle.

Triangular prism—a prism with triangular bases.

Preparation

Use the Plane Shapes handout and construct a pyramid. Identify properties that distinguish a prism and a pyramid. Pay attention to what problems students may have.

Instructions

Teacher Note

By cutting out the shapes, students should discover which sides of a triangular prism are congruent and the types and number of each of the faces.

1. Ask students to name common items that are triangular prisms like the roofs of their smokehouse models (for example, tents, wedges for splitting wood). As a review, students can classify these objects as right or isosceles triangular prisms.

2. For today's lesson, students will be assembling both types of triangular prisms from index cards (as they did with rectangular prisms in Activity 4). Tell students that they need to come up with one or more new properties of triangular prisms that they will explain to the class at the end of the activity.

3. **Challenge.** Have students work in their construction groups. Have students make a triangular prism using index cards that looks like the one they used to make their roof, using their smokehouse model as a guide for the shapes that make up the faces of their triangular faces. The pieces should be taped together to form a closed prism. Next students should build the other type of triangular prism. Encourage students to look at the smokehouses of other groups.

Teacher Note

Students should save the prisms they have constructed for use in Activity 8.

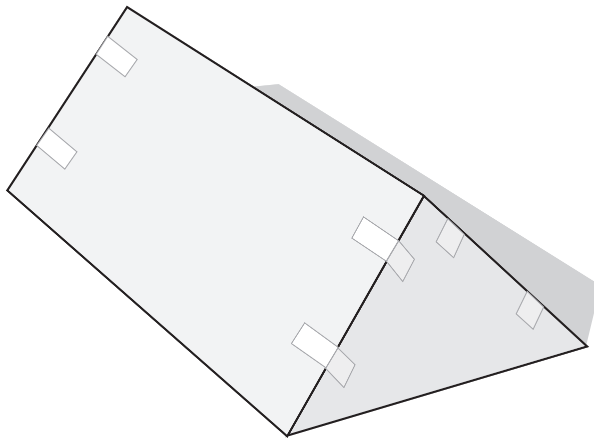


Fig. 7.2: Cardstock triangular prism.

4. **Discuss.** Have each group demonstrate and explain one of the properties of triangular prisms they have found. During this discussion, introduce the rest of the vocabulary of triangular prisms as they relate to your discussion. Make sure students understand which faces are congruent in triangular prisms. If needed, refer back to the Math Note on Generalizing Properties of Triangles to Triangular Prisms in Activity 6 on pages 83-85.

5. Use students' triangular prism models to discuss bases of prisms by displaying the prisms at different orientations. Verify that students know which face is the base no matter which way the prism is oriented. Use this discussion to reinforce the idea that a prism has a polygon base that determines the type of prism it is (so far we have seen rectangular and triangular prisms as well as cubes).

6. Give students copies of the handout, Plane Shapes, and ask them to use all the shapes to construct a closed, three-dimensional shape that is not a prism. **Math Note:** Students should form a pyramid with a square base. Unlike right prisms, pyramids only have one base and their faces, which are triangles, do not form right angles. Seeing a pyramid and exploring differences between it and a prism should help students to refine their understanding of the definition of a prism.

7. **Discuss.** Ask students to explain why this shape is not a prism using specific properties of prisms that this shape violates. Introduce the vocabulary word pyramid to describe the shape they made.

8. Have students compare the properties of the triangular prisms with the rectangular prism. **Teacher Note:** Encourage students to construct charts of the important properties that define a prism.

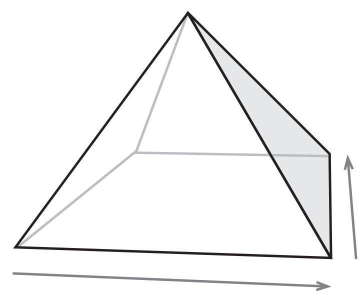


Fig. 7.3: A pyramid does not have a pair of opposite parallel sides. Therefore, it is not a prism.

Math Note

Property analysis tables are a visual way to compare the properties of different items. To complete the table, use a "+" to indicate that the property is present and a "-" if it is not. Making a prism properties table should help students to see what properties are essential to prisms and to develop their general definition of a prism.

Here is an example of a three-dimensional shape properties analysis table for you to reference. It contains only some of the possible properties students may use. Note that "two parallel bases" is a key property that differentiates the pyramid from the prisms.

	Three Dimensional	5 Faces	6 Faces	Contains at least 1 rectangular face	All faces are rectangular	Two Parallel Bases
Cube	+	–	+	+	+	+
Rectangular Prism	+	–	+	+	+	+
Isosceles Triangular Prism	+	+	–	+	–	+
Right Triangular Prism	+	+	–	+	–	+
Pyramid	+	+	–	+	–	–

9. In their math notebooks, have students modify and/or add to their definition of a prism. This new prism definition should hold for cubes, rectangular prisms, and triangular prisms. Assess students' understanding of prisms by pointing out discrepancies between the definition of prisms—rectangular, triangular, cubic. **Teacher Note:** Students will have the opportunity to revise their definitions in Activity 11, so it is okay to have either missing parts or inaccuracies in this definition, but students should be able to demonstrate that their definition applies to the three types of prisms studied so far.

Math Notebook: Have students draw and label isosceles and right triangular prisms in their math notebooks using the vocabulary you have introduced.

Homework: List one property that generalizes between types of triangular prisms and one that does not. Using pictures and words, show how this property is the same and different between the two types of shapes.

Additional Practice Problems for Activity 7

These additional practice problems can be used to reinforce and extend students' understanding of the topics presented in this activity. To promote mathematical discussion and get the most from these problems, we recommend that students work on them in pairs or in their construction groups. Have the students present their solution and solution strategies. Have groups help each other achieve understanding.

Materials

- Handout, Additional Practice Problems for Activity 7

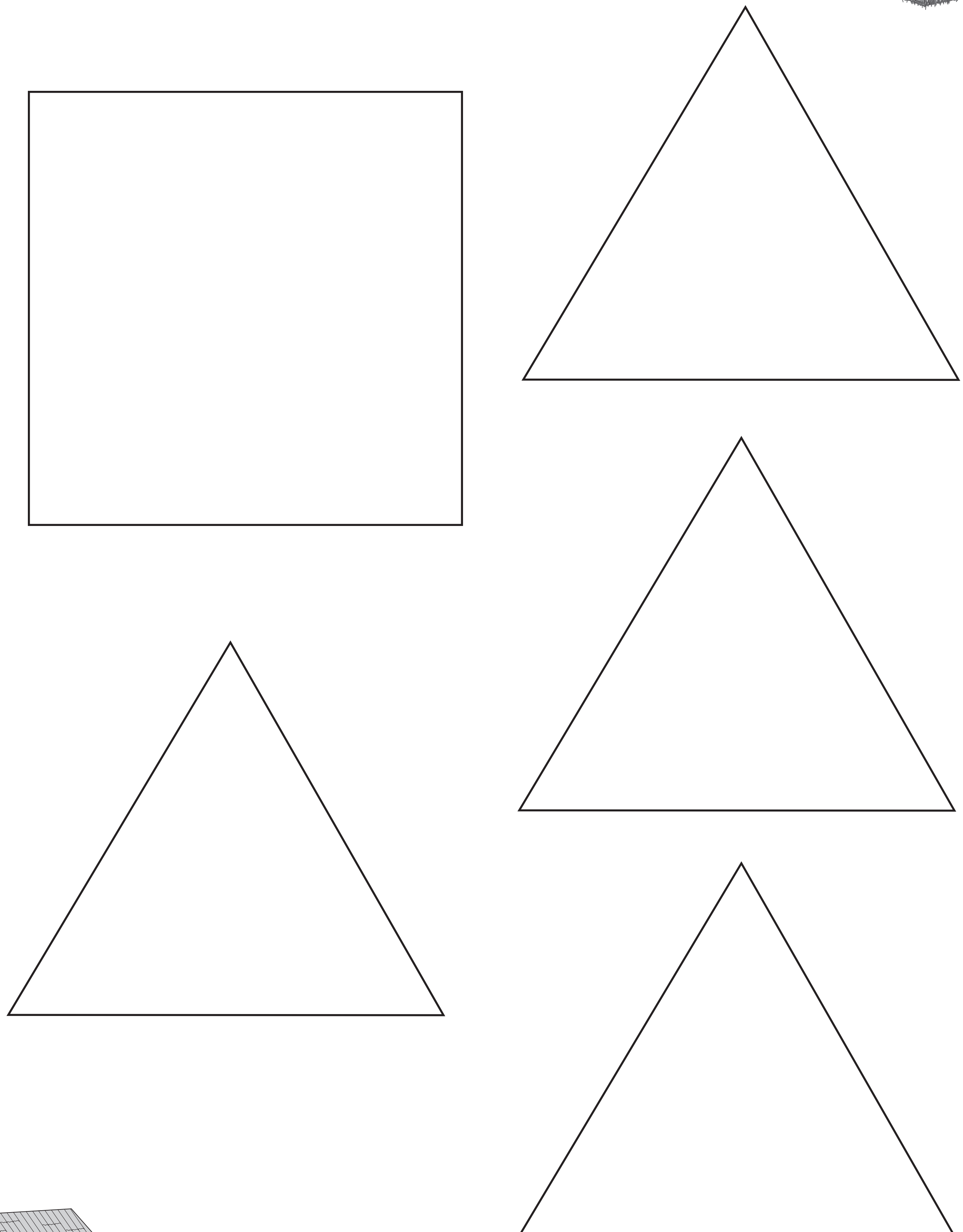
Duration

One class period.

Solutions to Additional Practice Problems

1. Students should draw a prism that has one pair of congruent, opposite, parallel faces that are triangles.
2. No, the figure is not a triangular prism: it's actually a pyramid. Among the reasons why the figure is not a triangular prism is that opposite triangular faces are not parallel and there are more than one pair of triangular faces. The figure could be a square pyramid, a triangular pyramid, or have a completely different base.
3. Yes, the figure is a triangular prism. The 3-D figure has one pair of congruent triangular faces that are opposite and parallel.
4. No, a triangular prism cannot have six faces. A triangular prism has exactly five faces (two faces are the opposite and parallel congruent triangular faces, and the other three are the rectangular faces that join them).
5. Any triangular prism has nine edges: the three edges for each of the two triangular faces and then three more edges formed by the rectangular faces, for a total of nine edges.

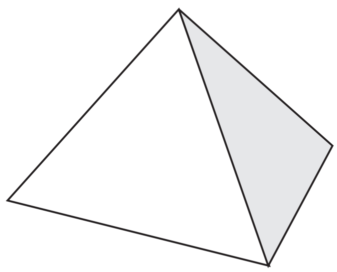
Plane Shapes



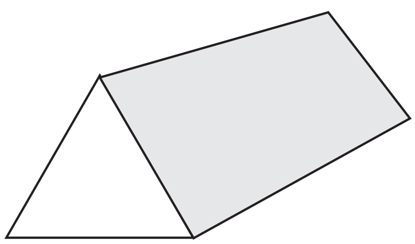
Additional Practice Problems for Activity 7

1. Draw a triangular prism. Explain why you think your drawing is a triangular prism.

2. Is the figure below a triangular prism? Explain why or why not:



3. Is the figure below a triangular prism? Explain why or why not:



4. Is it possible to draw a triangular prism that has six faces? Explain why or why not.

5. How many edges does a triangular prism have?

Section 4
Covering the Smokehouse: Surface Area

Activity 8
Covering the Smokehouse With New Materials

Now that students have built their models and investigated rectangular and triangular prisms, this activity gives them the opportunity to make their smokehouses more realistic by attaching siding to their models. This simulates covering the smokehouse and roof with new materials through a hands-on exploration. Doing this introduces students to the concept of surface area as a measure of "covering" a three dimensional object. It is not necessary to use the mathematical term "surface area" in this activity; an understanding of this term will be further developed in Activities 9 and 10.

Goals

- To develop a practical understanding of surface area and explain their understanding
- To approximate surface area

Materials

- Scissors
- Siding material: aluminum foil, newspaper, or butcher paper
- Tape
- Transparency, Akiachak Smokehouse
- Transparency, Flow of Air Through a Smokehouse (from Activity 4)

Instructions

1. Show the overhead, Akiachak Smokehouse, to provide students with a picture of a finished smokehouse. Ask students what kind of siding they have seen on smokehouses or other structures. Discuss the purpose and placement of holes and door in the siding of a smokehouse. As a reminder, show students the Air Flow Through a Smokehouse transparency from Activity 4.

2. Have available a variety of materials for siding. Have students cover the sides and roof of their smokehouses with the siding material, attached with tape. Students should also add smoke holes and a door. During this activity, emphasize the Yup'ik value of not wasting materials. Put siding on your model as students work on theirs.

3. When models are covered, visit each of the groups and have them explain how they covered their smokehouse. Ask groups to explain approximately how much siding material they used and strategies they employed to not waste materials as well as any challenges they faced in siding their smokehouses.

Fig. 8.1: Putting siding on the smokehouse model.

Teacher Note

Students may use their understanding of congruent faces to help them estimate and cut their siding material. Have students share these strategies, because they will help develop an understanding of how to calculate surface area in the next activity.

Bernardo Untalasco, a teacher from Kalskag, said, "The groups were very creative. One added a 'barrel' to catch the rain as it collected and an overhang over the doorway to keep the snow and rain away from the door so it won't rot the fish"

Cultural Note

A smokehouse is constructed to properly smoke fish so that they are thoroughly dried (fish that is even slightly moist is susceptible to spoiling) but not bitter, the result of over-smoking.

Smokehouses have holes in the siding to promote the flow of air through the smokehouse so that the smoke will cure all the salmon. Although there may be a gap under the eaves, there are no holes in the roof itself as this would allow the smoke to escape.

All smokehouses have doors. Some people use a regular door from a building, but anything on a hinge will work. The smaller the door, the less smoke will escape when it is opened—you often have to duck to enter a smokehouse! People also hang burlap behind the door as an added screen to keep in the smoke.

Akiachak Smokehouse

Activity 9
Covering Prisms to Find Surface Area

Now that students have developed a practical understanding of surface area, this activity gives them a more formal understanding of surface area and how it is calculated. Students start by computing the surface area of the index card rectangular and triangular prisms made in Activities 4 and 7 by either covering them with grid paper or using area formulas, depending on their preparation. Students next are guided to use this hands-on experience with surface area to generalize to mathematical formulas for computing the surface area of rectangular and triangular prisms. This activity also reinforces students' understanding of the properties of prisms as they explore relationships among the faces to determine surface area.

Goals

- To calculate the areas of rectangles and triangles
- To calculate the surface area of prisms

Materials

- Handout, Grid Paper
- Index card prisms from Activities 4 and 6
- Rulers
- Tape
- Transparency, Grid Paper (one per group, optional)

Duration

Two class periods.

Vocabulary

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth.

Height—the distance from the base of an object to its highest point.

Surface area—the sum of the areas of all the faces, or surfaces, of a solid figure.

Instructions

1. Ask students how they might be able to predict how much siding they would need before they started covering their smokehouse models, using strategies they developed in the previous activity. Have students talk over this problem with their groups and share their strategies with the whole

class. Introduce the vocabulary word surface area when students describe needing to measure the area of each of the surfaces to be sided.

2. Give students their paper rectangular prism models from Activity 4 and grid paper transparencies or handouts (or rulers if students are comfortable with the algorithms for finding the areas of rectangles and triangles). Ask them to find the surface area of their prism in square units (i.e., squares on the grid paper or square inches or square centimeters). Have groups report their strategies to the whole class.

3. Challenge students to come up with a general rule in words for finding the surface area of all the rectangular prisms in the room (for example, "add up the areas of each of the faces"). Have them write this in their math notebooks.

Teacher Note

This process leads students from the general concept of surface area to a formula. If you feel the formula is not essential for students at this time, feel free to skip to step 6.

4. Next ask students what adding all the areas would look like as an equation. Once again, it's okay if students stay general so that the equation will be something like: area of face one plus area of face two plus area of face three, etc. = surface area. This should be written in their math notebooks. If it wasn't originally constructed this way, ask students to combine areas in the equation if they are the same. (For example, two times the area of face one plus two times the area of face three plus two times the area of face five.) Students should write this equation in their math notebooks.

Generalizing: Surface area of rectangular prisms to surface area of triangular prisms.

5. Now rewrite the equation replacing the words "area of" with the algorithm for each area. Have students use their paper rectangular prism model to make sure they know the side length they are using (height, width or length) when describing the area of each face. Next have students combine the area algorithms for the congruent faces to come up with the final algorithm for surface area. **Math Note:** Surface Area of a Rectangular Prism = 2(length x width) + 2(width x height) + 2(height x length). Opposite faces in rectangular prisms are congruent, so the areas of these faces can be combined.

6. For practice using the surface area equation, have students trade index card rectangular prisms and calculate surface areas of the other group's prisms. Have groups check each other's calculations.

7. **Discuss.** Ask students to conjecture how to find the surface area equation of a right triangular prism by generalizing on what they learned about the surface area of rectangular prisms. **Teacher Note:** If needed, review how to find the area of a triangle, Area = ½ bh. Students can draw in and measure the perpendicular to find the height on the isosceles triangle. If students haven't yet learned how to find the area of a triangle, they can use the grid transparencies to count the squares. With a right triangle,

it is easy to show that the triangle is half the area of the rectangle, as shown in Figure 9.1.

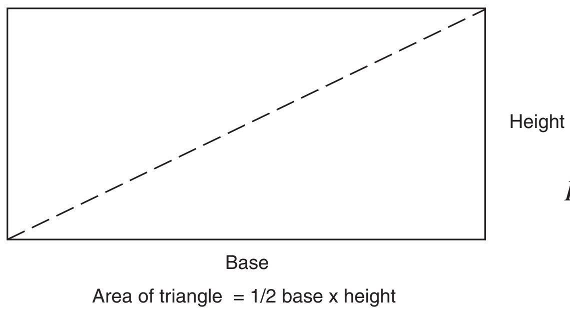


Fig. 9.1: Area of a right triangle.

8. Have groups prove their conjectures by finding the surface areas for their index card right triangular prism, using the following steps:
    1. Break apart the prism and tape each face to a piece of paper
    2. Label each side with the appropriate measurement.
    3. Find the surface area by adding together the areas of each face either by using the area algorithms or the grid transparencies or handouts
    4. Write a general surface area equation for right triangular prisms and compare to your conjecture.
    5. Trade triangular prisms with another group and check each other's calculations to practice manipulating the equation

9. Repeat steps 7 and 8 for the isosceles triangular prism.

10. Ask students to describe any differences in the calculations of surface area between the two types of triangular prisms. Use this discussion to reinforce the different types of faces of the prisms they have constructed. Have students write down the general equations for surface area for each of the triangular prisms in their math notebooks.

Math Note

The differences between the right and isosceles triangular prisms are reflected in the surface areas of these prisms. It is easy to calculate the area of the right triangle face because one of the sides of the triangle is also the height of the triangle as well as the side length of one of the faces. Also, with the right triangular prism, the surface area of the two bases can be calculated by using the rectangle formed by the two triangles with A = base x height of the triangle. To find the surface area of the isosceles triangular prism, students need to draw in the perpendicular for the height and find the lengths of the base and height of the triangle.

11. Ask students how they can apply what they just learned to calculate the surface area of their smokehouse. This discussion can help students to see differences between models and mathematical definitions. **Teacher Note:** Use this problem as a way to emphasize that surface area is the sum of the areas of the two-dimensional faces of the three-dimensional figure. In the smokehouse model, because the bottom of the triangular prism and both the top and bottom of the rectangular prism are not part of the outside of the model, the surface area of the model is different than the sum of the surface areas of the triangular and rectangular prisms.

Grid Paper

Activity 10
Covering the Smokehouse With Recycled Materials

In this activity, students will simulate using old oil barrels to cover their smokehouses, a common practice in rural Alaska where new materials can be difficult and expensive to transport. This activity introduces students to the surface area of a new three-dimensional shape, a cylinder. Students also find the surface area of their smokehouse in the activity, reinforcing the concept of surface area built in the previous activities.

Goals

- To cover the smokehouse models with simulated oil drums to further explore the concept of surface area
- To compare prisms to cylinders and note common and dissimilar properties
- To find the surface area of a cylinder
- To find the surface area of the smokehouse model

Materials

- Scissors
- Soda cans (1 per group) to simulate oil drums (paper cylinders can also be used if preferred)
- Simulated oil drums with a different diameter and height than the soda cans (optional)
- Transparency, Flattening an Oil Drum
- Transparency, Grid Paper (optional), one per group

Duration

Two class periods.

Vocabulary

Circumference—the length of the boundary of a circle.

Cylinder—a solid figure with two parallel circular bases.

Diameter—a line segment that goes through the center of a circle and whose end points are on the circle.

Pi—a constant represented by the Greek letter π that expresses the ratio of the circumference to the diameter of a circle. It appears in many mathematical expressions and its value is approximately 3.14159.

Preparation

Prepare the simulated oil drums, either by collecting soda cans or by taping rectangular pieces of paper together to make a cylinder. If you use paper, use a size so that approximately five of them will cover the roof. Use a different-sized can or piece of paper for the optional challenge activity (step 8).

Teacher Thad Keener of Fairbanks said: "One student shared his experience using a grinder to cut a barrel. We discussed that a torch could also be used."

Instructions

1. Hold up a soda can and explain that you are using it to simulate the 55-gallon drums that are often used to make siding for a smokehouse in rural Alaska. Explain that this shape is called a cylinder in mathematics Ask students how people might turn this common object into siding. Use students' ideas to demonstrate how to flatten your simulated oil drum as shown in Figure 10.1, using scissors instead of a saw and emphasizing safety considerations. Show students the transparency, Flattening an Oil Drum, after the discussion.

2. Ask students how they could figure out how much siding material they can get from the model barrel. To help students understand this problem, pass out the soda cans (or paper cylinders) and have students flatten them as shown in Figure 10.1. Students should put tape over the rough edges. **Teacher Note:** Although this step should only be done if you feel that your class can safely cut apart the cans, the advantage of having students cut the can apart is that they will physically experience

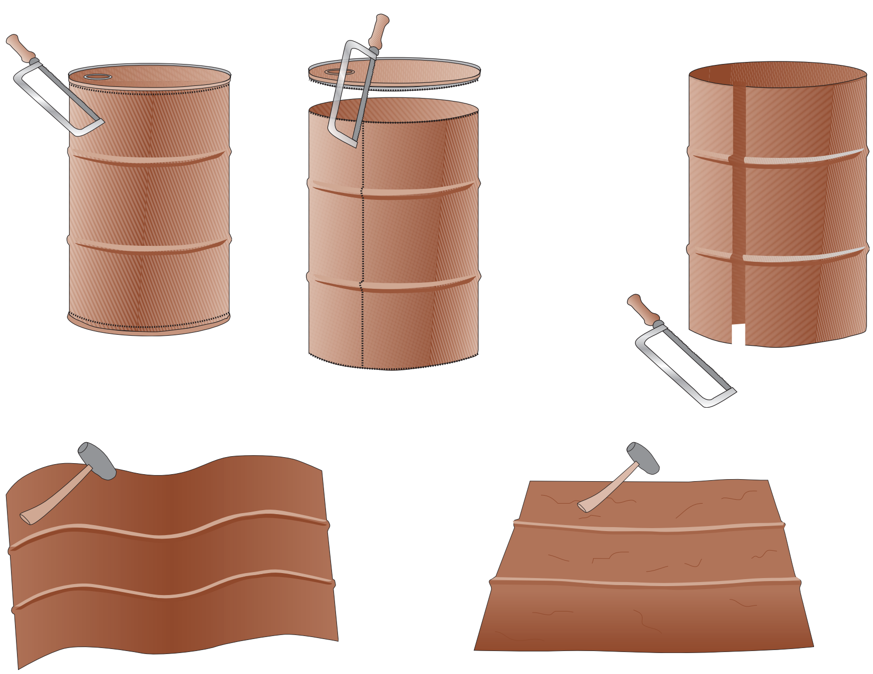


Fig. 10.1: Flattening a 55-gallon drum.

the relationship between the circumference of the can and the width of the rectangle as they cut the top and bottom.

Math Note

The flattened cylinder becomes a rectangle whose length is the height of the tin can and whose width is the circumference of the tin can. Once you know these two measurements, you can calculate the area of this rectangle.

3. Demonstrate how the flattened can might be used as siding for your smokehouse model. Ask students to figure out how many cans would be needed to cover their smokehouse. Remind students of the importance of not wasting; leftover pieces can be used on different parts of the smokehouse. **Teacher Note:** It is not necessary to actually cover the smokehouse with the simulated oil drums; students should be able to estimate from the one "drum" they prepare.

4. Have groups share their estimates and the strategies they used to find the total number of drums needed.

5. Have students calculate the surface area of their smokehouse model (minus the bottom as this does not get sided) from the number of cans it took to cover their smokehouse. Students can do this by multiplying the area (length x width) of their cans by the number of cans needed to cover the model.

6. To check their estimate of surface area of the model, have groups switch models. Have them calculate the surface area of their partner group's model by finding the sum of the areas of the faces, using the strategies developed in Activity 9. After completing the calculations, the groups should compare their respective surface areas and see if they can reconcile any differences.

7. **Discuss.** Remind students that a soda can is a model of a cylinder. Ask students to consider whether a cylinder is a prism or not and back up their answers mathematically. **Teacher Note:** Allow time for students to discuss in small groups or as a whole class the similarities and differences between cylinders and prisms.

Math Note

Your students may disagree on whether or not a cylinder is a prism. Allow students to conjecture and prove their ideas. Both ideas are acceptable and used as long as the definitions remain consistent (in a sense, a cylinder is a generalization of a prism to a solid shape with circular parallel bases.).

If the definition of a prism includes at least one set of parallel, congruent faces then a cylinder is an example of a prism, called a circular prism. In this case, it becomes a bit more difficult to define the faces, edges, and vertices. One possible resolution is that the cylinder has one rectangular face other than the two bases as demonstrated by flattening the can. Encourage students to work on defining these attributes if they take this stance

On the other hand, if the definition of a prism includes at least one set of parallel, congruent polygons, then a cylinder would not be a prism since the bases are circles and not polygons. (This is the standard definition of a prism.)

8. **Optional Challenge Step:** Show students a simulated drum with different dimensions and ask them to determine the number of these drums that would be needed to side their models, this time without opening the drum. **Math Note:** Students will need to measure the height and circumference of the drum to find the area of the rectangle. They can then use the surface area they have already calculated to find how many of these rectangles it will take to cover their smokehouse.

Additional Practice Problems for Activity 10

These additional practice problems can be used to reinforce and extend students' understanding of the topics presented in this activity. To promote mathematical discussion and get the most from these problems, we recommend that students work on them in pairs or in their construction groups.

Materials

- Handout, Additional Practice Problems for Activity 10

Duration

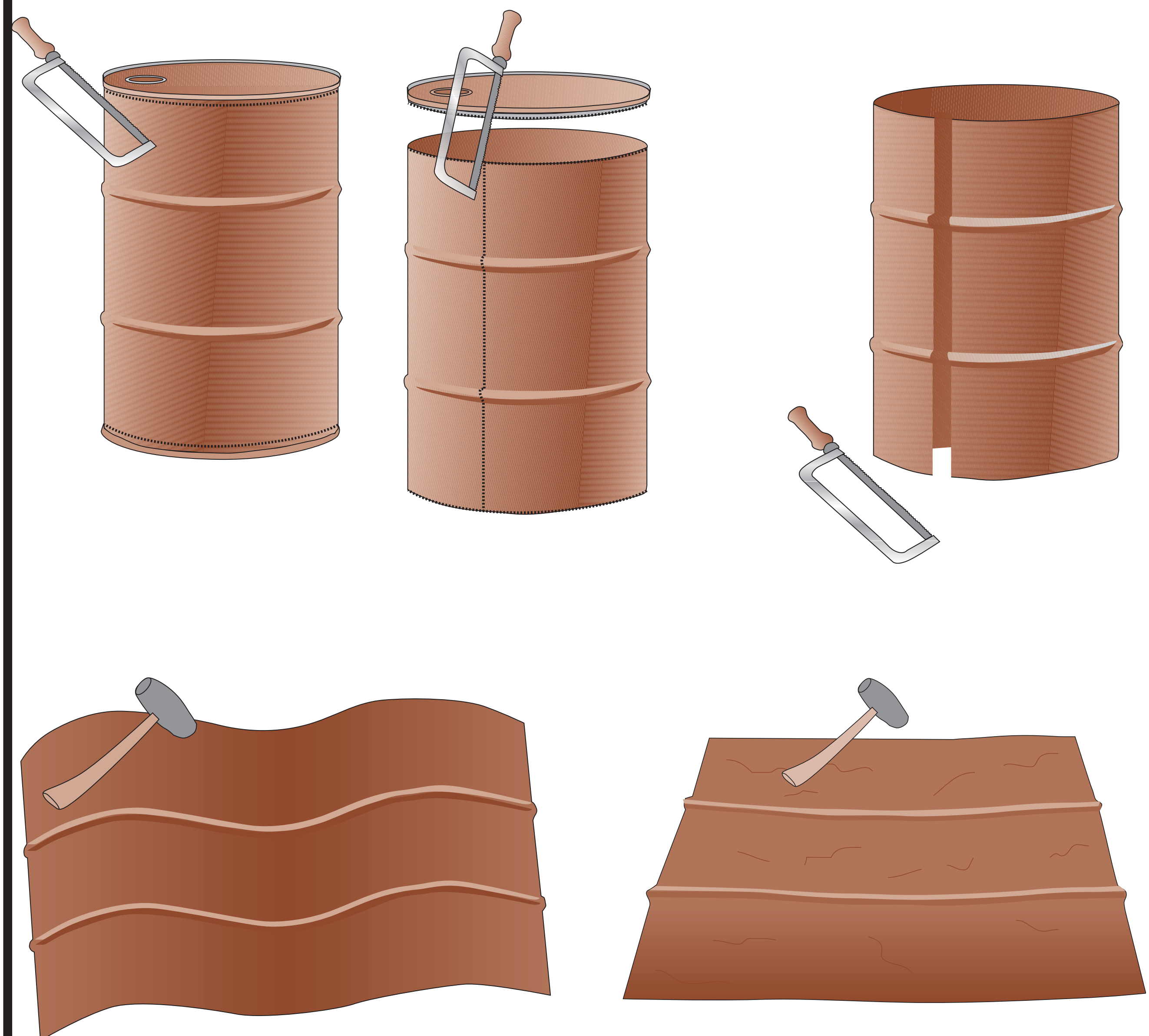
One class period.

Solutions to Additional Practice Problems

1. The surface area of the 4 by 4 by 4 cube is 96 cm^2. One way students may find the surface area is to note that each face has an area of 4 cm by 4 cm = 16 cm^2 and, since there are six faces of the cube, the surface area would be 6 by 16 cm^2 = 96 cm^2.
2. The surface area of a cylinder is the total area of the circular top and bottom (in this case 2 by 50 cm^2 = 100 cm^2) together with the surface area of the sides, which, in this case, is a rectangle with a length of 25 cm (i.e., the circumference of the base) and a height of 11 cm (i.e., the height of the cylinder), so the lateral surface area of the cylinder = 25 cm by 11 cm = 275 cm^2. Therefore, the surface area of the cylinder is 100 cm^2 + 275 cm^2 = 375 cm^2.
3. The surface area of the hexagonal prism will be the area of the hexagonal top and bottom = 2 by 36 cm^2 = 72 cm^2; since there are five rectangular faces, each with area 45 cm^2, the side surface areas of the hexagonal prism will be 5 by 45 cm^2 = 225 cm^2. Therefore, the total surface area of the hexagonal prism will be 72 cm^2 + 225 cm^2 = 297 cm^2.
4. One pair of congruent opposite faces will each have area 4 cm by 4 cm = 16 cm^2. The other four faces will each have area 4 cm by 10 cm = 40 cm^2. Therefore the surface area of the rectangular prism will be (2 by 16 cm^2) + (4 by 40 cm^2) = 32 cm^2 + 160 cm^2 = 192 cm^2.

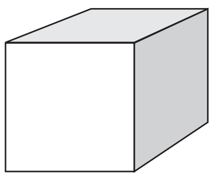
5. (a) The dimensions of the new rectangular prism would be 20 cm by 8 cm by 8 cm.
   (b) The surface area of the new prism will be 768 cm^2 (i.e., two faces that each have area 64 cm^2 and four faces that each have area 160 cm^2).
   (c) The surface area of the new prism with doubled dimensions has exactly four times the surface area of the prism in Problem 4 (i.e., 192 cm^2 by 4 = 768 cm^2). In general, increasing the dimensions of a prism by a factor of x will increase the surface area by a factor of x^2 (in this case, increasing the dimensions by a factor of 2 increases the surface area by a factor of $2^2 = 4$).

Flattening an Oil Drum



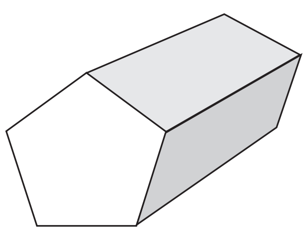
Additional Practice Problems for Activity 10

1. Each edge of the cube below has a length of four centimeters. Find the surface area of the cube and explain how you got your answer:

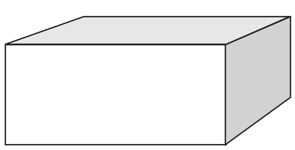


2. The area of the base of a cylinder is about 50 cm^2 and the circumference of the base is about 25 cm. If the cylinder has a height of 11 cm, what is the surface area of the cylinder? Explain how you got your answer.

3. The area of each of the two ends of the hexagonal prism below is 36 cm^2. If the area of one of the rectangular sides is 45 cm^2, what is the surface area of the entire hexagonal prism? Explain how you found your answer:



4. The rectangular prism shown below has length of 10 cm, a width of 4 cm, and a height of 4 cm. Find the surface area of the prism and explain how you found your answer:



5. Suppose the prism from problem 4 above has each of its dimensions doubled.
   a) What are the dimensions of the new prism?

   b) What is the surface area of the new prism?

   c) How does the surface area of the prism from problem 4 compare with the surface area of the new prism with doubled dimensions? Explain your reasoning.

Activity 11
Creating New Uses For a Smokehouse

This activity gives students the opportunity to apply what they have learned about prisms and building to a more creative project. Students will build a "smokehouse" out of toothpicks and gumdrops (these materials will make the building much quicker than their smokehouse model) whose body has the shape of a different type of prism than the rectangular prism made during the module. Students will have to prove that their shape is a prism and explain why they picked the shape they did. Encourage students to get creative with this one!

Goals

- To construct a new type of prism
- To use the definition of a prism they have developed to prove their shape is a prism

Materials

- Gumdrops or marshmallows
- Math notebooks
- Toothpicks (you can use skewers if you want larger constructions)

Duration

One class period.

Preparation

Build a pentagonal prism smokehouse such as shown in Figure 11.1 with gumdrops and toothpicks. For the demonstration, leave off the roof and discuss just the shape of the body.

Vocabulary

Hexagonal prism—a prism with hexagonal (6-sided) bases.
Heptagonal prism—a prism with heptagonal (7-sided) bases.
Octagonal prism—a prism with octagonal (8-sided) bases.
Parallel—in a plane, lines that do not intersect; In three dimensions, planes that do not intersect.
Pentagonal prism—a prism with pentagonal (5-sided) bases.

Math Note

A prism is defined as a three-dimensional shape with one set of opposite and congruent polygons that are parallel faces.

Joint Activity
Build your own creative smokehouse

Instructions

1. Show students your pentagonal smokehouse body made with toothpicks and gumdrops. Ask students whether or not your model is a prism and have them justify their answer. On the basis of this discussion, the class should decide whether their definition of prism holds for this one. Modify and add to the definitions needed.

2. Tell students they need to build a smokehouse that has a body made from a different type of the prism than the rectangular prism they built for their model with gumdrops and toothpicks. Tell students that they will have to both prove that the body of their construction is a prism (the roof can be any shape) and that they will also have to explain how their smokehouse would function (these "smokehouses" can be used for whatever students want; encourage creativity!) and why they chose the design they did.

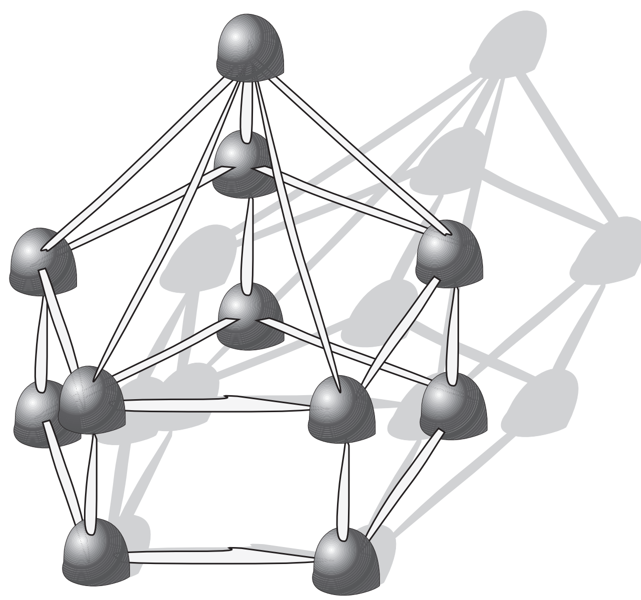


Fig. 11.1: Gumdrop pentagonal prism.

3. Pass out the materials. As students work on their smokehouses, build another model yourself.

4. When groups have completed their constructions, visit each model as a class and have students explain how they can prove that the body is a prism. Use this discussion to further refine students' understanding of what a prism is. Groups should also explain why they chose the shape they did and how their smokehouse would function. **Teacher Note:** Use models that are not prisms to serve as counterexamples to help develop the definition of a prism (make your own non-prism models as demonstrations if students don't do so on their own).

5. **Optional.** Ask students to describe the relationship between the polygon used as the base of the prism and the total number of faces in the prism, using what they have seen in the different types of smokehouses constructed in this activity. Have students use this relationship to predict the number of faces for various types of prisms that were not constructed in this activity. **Math Note:** The number of faces of any prism can be found by taking the number of sides of the polygon base and adding 2. For example, a triangular prism consists of five faces since the base is a triangle with three sides and then the top and bottom bases are added. For a rectangular prism, a rectangle has four sides, so the prism has six faces. If the prism had a pentagonal base, there would be five sides plus two bases for a total of seven faces.

6. As a class, come up with a final definition of a prism based on what was learned through building different types of prisms. Have students record the definition in their math notebooks. Finish the poster.