EDUCATION MATHEMATICS ELEMENTARY

Kayak Design

GRADE LEVEL
6
K 1 2 3 4 5 6 7

Scientific Method and Statistical Analysis

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders is the result of a long-term collaboration. These supplemental math modules for grades K-7 bridge the unique knowledge of Yup'ik elders with school-based mathematics. This series challenges students to communicate and think mathematically as they solve inquiry-oriented problems. Problems are constructed with constraints so that students can explore and understand mathematical relationships, properties of geometrical shapes, place value understanding, conjecturing, and proofs. The modules tap into students' creative, practical, and analytical thinking. Our classroom-based research strongly suggests that students engaged in this curriculum can develop deeper mathematical understandings than students who engage only with a procedure-oriented, paper and pencil curriculum.

Also in this series for Grade 2

Going to Egg Island: Adventures in Grouping and Place Values Students learn to group objects, compose and de-compose numbers using the Yup'ik counting system (base 20 and sub-base 5), and use place value charts in multiple bases. Package includes a storybook, *Egg Island*; five posters; two CD-ROMs; and coloring book.

Picking Berries: Connections Between Data Collection, Graphing, and Measuring Students engage in hands-on activities that help them explore data, graphic representation, and linear measuring. Package includes two storybooks, *Big John and Little Henry* and *Berry Picking*; one poster; a CD-ROM; and coloring book.

Patterns and Parkas: Investigating Geometric Principles, Shapes, Patterns, and Measurement Students learn about the properties of squares, rectangles, triangles, and parallelograms. They learn a variety of ways to make these shapes and how Yup'ik elders use these shapes to create patterns on their traditional winter parkas. Through a hands-on approach to making these shapes, students learn about symmetry, congruence and proof. Package includes a DVD documenting elders making shapes and patterns; *Iluvaktuq*, a storybook about a famous warrior; and seven posters.

Also in this series for Grade 6-7

Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area
Following an elder demonstration, students must lay out a rectangular base and prove it is a rectangle. In so doing, they explore various properties of quadrilaterals, including measurements of perimeter and area. They further investigate the key relationship between area and dimensions of a rectangle when perimeter is held constant. Package includes three posters and a CD-ROM.

Salmon Fishing: Investigations into Probability
Students use activities based on subsistence and commercial fishing in Southwest Alaska to investigate various topics within probability, such as experimental and theoretical probability, the law of large numbers, sample space, and equally and unequally likely events. Package includes two posters, a CD-ROM, and an interactive Excel spreadsheet.

Building a Smokehouse: The Geometry of Prisms
Students learn to generalize properties of rectangles to three-dimensional rectangular prisms through constructing models. Further investigations into triangles and triangular prisms arise while designing roofs for the structures. The hands-on activities lead students through comparing and contrasting models to understand prisms in general. Package includes a CD-ROM and one poster.

Star Navigation: Explorations into Angles and Measurement
Students learn ways of observing, measuring, and navigating during the day and at night, including specific details of the location and orientation of the Big Dipper and Cassiopeia. They refine their understanding of angle measurements and how they differ from linear measures throughout the activities. Package includes *The Star Navigation Reader* with traditional stories and personal accounts related to navigating; a CD-ROM; and two posters.

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MCC

The *Supplemental Math Modules* curriculum was developed at the University of Alaska Fairbanks.

Part of the series *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders©*

Kayak Design Scientific Method and Statistical Analysis

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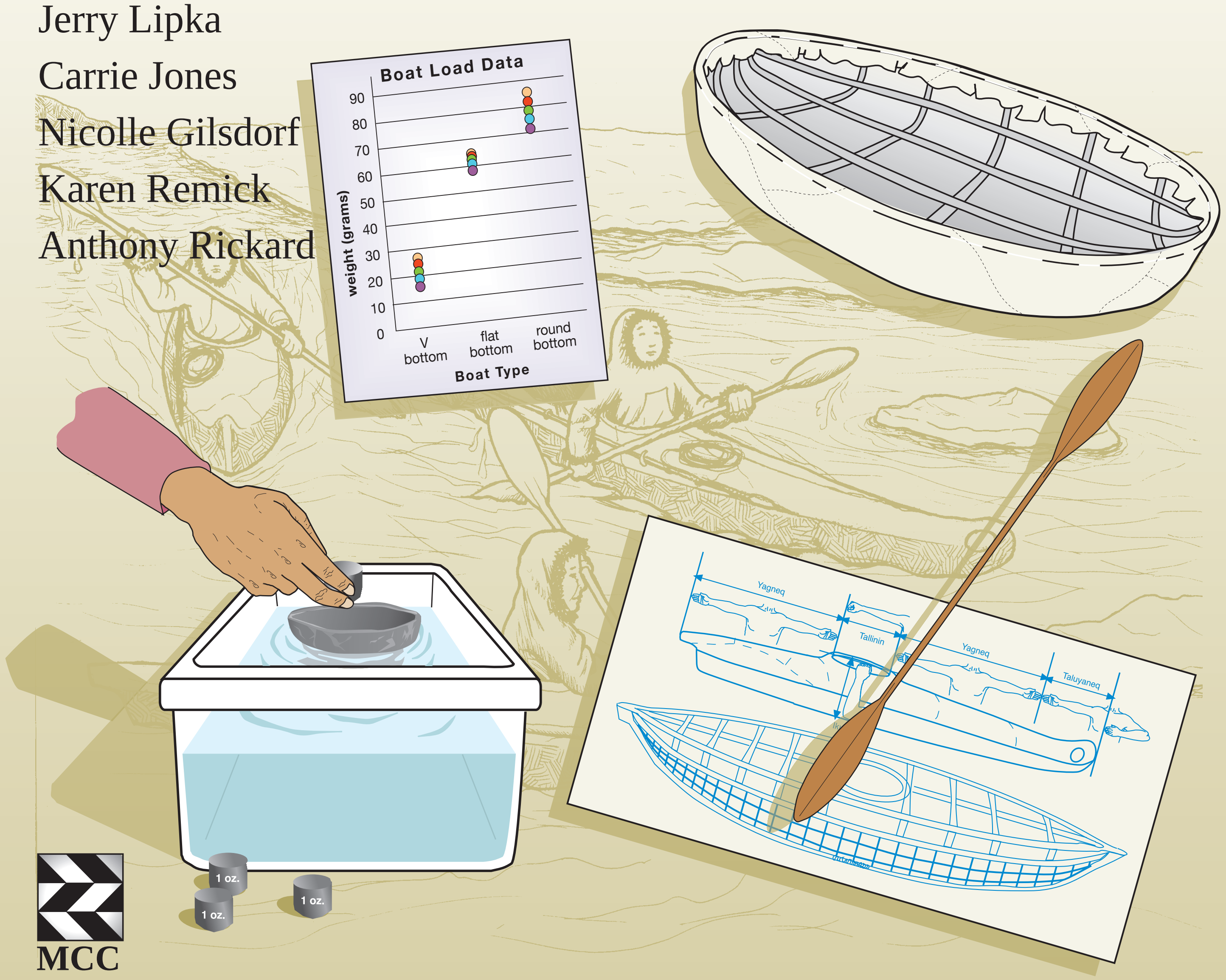

MCC

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Kayak Design:

Scientific Method and Statistical Analysis

Part of the Series

Math in a Cultural Context:

Lessons Learned from Yup'ik Eskimo Elders

Grade 6

Jerry Lipka
Carrie Jones
Nicolle Gilsdorf
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Anthony Rickard

Developed at University of Alaska Fairbanks, Fairbanks, Alaska

*Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo El*ders© was developed at the University of Alaska Fairbanks.

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Acknowledgements

From Jerry Lipka, Series Editor

The supplemental math series *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders* is based on traditional and present-day wisdom and is dedicated to the many elders who gave their time and energy to this project. The elders' wisdom, motivation, and curiosity have been an inspiration to many of us who carry this work forward to the future. This module is dedicated to an individual who made this entire project possible. It is with deep sadness and an equal amount of indebtedness that I dedicate this module to Gerald V. Mohatt, former dean of the College of Human and Rural Development and its predecessor, the College of Rural Alaska. As dean, Mohatt established a research fund with former Chancellor Pat O'Rourke. The Alaska Schools Research Fund was established to provide seed money for action research to improve schooling in rural Alaska. Mohatt's values were put into action, and I applied for funding to continue my work with a very dedicated group of Yup'ik teachers and elders. A few teachers who worked with me back in 1981 still do so today. Evelyn Yanez is one of those teachers who began this journey with me. She and I and a group of Yup'ik teachers began identifying pedagogical practices of Yup'ik teachers. Mohatt would attend those meetings, and often he invited some of his colleagues, such as Frederick Erickson, who had paved the way in videotape analysis in indigenous schools and communities. What makes a Yup'ik teacher Yup'ik? It was Evelyn's videotape, which she thought was "too Yup'ik," that revealed key elements of what a Yup'ik pedagogy would look like. This began a longstanding and ongoing action research agenda, eventually resulting in this project and this module.

Mohatt was a good man who did all he could to further indigenous education, knowledge, and practices. As fate would have it, Mohatt passed away on a day he was going to join us again analyzing videotapes, and Evelyn Yanez was going to be at this meeting. On the day of Mohatt's passing, Evelyn became a proud grandmother to a baby boy, who according to Yup'ik tradition and with his mother's smiling agreement, was named Dr. Mohatt in Yup'ik.

This module, *Kayak Design: Scientific Method and Statistical Analysis,* is also dedicated to the late George Moses of Akiachak Alaska. George Moses believed that this project was a positive force that valued Yup'ik knowledge and people. He shared his knowledge about the environment, search and rescue, and the kayak. George Moses and other elders such as Henry Alakayak of Manokotak, Annie Blue of Togiak, Frederick George of Akiachak, Sam Ivan of Akiak along with Mike Toyukuk and many others created a vivid picture of what it was like to live in Alaska when people relied on local resources and ingenuity. They shared little-known ethnographic and historical information about "round boats," log rafts, and canoes. They went into great detail about the specific environmental conditions when a particular boat was used. Their descriptions of specific watercraft used at different times of the year provided a practical way to develop curriculum that explored form and function. We thank all of the elders who provided this rich information.

Those of us who attended these meetings were privileged to be in the company of a group of elders whose curiosity rivaled any collaborative group of natural scientists. We all learned together so that we could bring this knowledge forward to future generations.

Accomplishing this vision required the cooperation and collaboration of many individuals beyond the elders. It took the expertise of physicists, mathematicians, mathematics educators, literacy specialists, cultural brokers (those working between the Yup'ik and school community), as well as a graphic artist, editors, and a dedicated staff. Therefore I would like to acknowledge and thank each of them for their efforts. Taking the elders' knowledge and presenting it as classroom mathematics curriculum is a rewarding and difficult endeavor.

It is also important to acknowledge the individuals who worked behind the scenes to make the MCC series an effective research-based curriculum. To that end, we need to thank Stan Triebenbach, Brandon Uzzell, and Anne Kornkven, who set up the database to receive thousands of test data for the efficacy study of the module. They cleaned and passed this data on to Rebecca Koskela, who placed it in a relational database, and finally to Ellen Kisker, who crunched the data.

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Our current University of Alaska Fairbanks faculty who finalized this module include Karen Remick, who refined the physics embedded in the module and who combed the module for inaccuracies in mathematics; Nicolle Gilsdorf, who brought her knowledge of teaching to ensure that the module would be teacher-friendly and inviting to sixth-grade students; and Carrie Jones, who sped up the final difficult steps to ensure that a final completed draft would be done before another decade of work on this module began!

I wish to thank the MCC adjunct faculty who observed classrooms implementing this module across Alaska: Lynn Williams, Janet Henderson, Gretchen Murphy, Pat McDonald, and LuAnne Nelson for their suggestions on improving this module; Annie Caulkins, project evaluator; and Kayt Sunwood for supporting the eLive classes.

The Oregon Museum of Science and Industry (OMSI) and their hands-on exhibits helped me formulate a structure for teaching the mathematics of this unit. The OMSI staff suggested a simple clay model in a time of need, after two or three different kayak models took too much time to construct or were too difficult for students. This simple solution allowed us to move forward and to complete this endeavor.

Much appreciation is due to my long-term colleagues and friends Evelyn Yanez and Dora Andrew-Ihrke for translating Yup'ik and attending more meetings than anyone could ever expect to get this and other modules published. I also wish to acknowledge Annie Blue for telling the epic Yup'ik story *Kukugyarpak* (which accompanies this module) and Evelyn Yanez and Dora Andrew-Ihrke for turning this story into a classroom version. I wish to acknowledge Eliza Orr for translating and transcribing at many meetings. Also, I wish to thank Ben Orr, folklorist, for the early work he did on documenting kayak nomenclature and finding archival photographs.

I would like to thank Paula Elmes for the layout. I wish to acknowledge Sue Mitchell, University of Alaska Fairbanks, for editing the final version and Sarah McGowan and Jana Canary for editing earlier versions. Special thanks goes to Carol Lewis, dean of the School of Natural Resources and Agricultural Sciences, who supports this work.

The MCC staff has been rock solid ensuring that projects come to completion. Also thanks to Annabelle Roland and Loria Chaddon for their support. I wish to thank Wendy Wood, MCC coordinator, for organizing countless meetings that eventually led to this finished product. To Putt Clark, MCC's graphic artist from day one who has produced master pieces and every single graphic found in this module and in this series: thanks, Putt. Flor Banks, MCC's project manager for the past decade or more, has shown fortitude, an unbelievable work ethic, and persistence until the job is done. She has contributed greatly to this project.

This has been a long-term and collaborative endeavor and I hope that we have met the expectations of the many elders who shared their knowledge so freely.

Last but not least, thank you to my loving wife, Janet Schichnes, who has stood behind and alongside of me offering support in countless ways that allowed me to complete this work, and to my now-grownup children, Alan and Leah.

Introduction

Math in a Cultural Context:

Lessons Learned from Yup'ik Eskimo Elders

Introduction to the Series

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders (MCC) is a supplemental math curriculum based on the traditional wisdom and practices of the Yup'ik Eskimo people of southwest Alaska. The kindergarten-to-seventh-grade math modules that you are about to teach are the result of more than a decade of collaboration between math educators, teachers, Yup'ik Eskimo elders, and researchers to connect cultural knowledge to school mathematics. To understand the rich environment from which this curriculum came, imagine traveling on a snowmachine over the frozen tundra and finding your way based on the position of the stars in the night sky. Or, in summer, paddling a sleek kayak across open waters shrouded in fog, yet knowing which way to travel toward land by the pattern of the waves. Imagine building a kayak or making clothing and accurately sizing them by visualizing or using body measures. These are a small sample of the activities in which modern Yup'ik people engage. The mathematics embedded in these activities formed the basis for this series of supplemental math modules. Each module is independent and lasts from three to eight weeks.

From 2001 through spring 2009, MCC has conducted numerous trials using quasi-experimental and experimental designs to understand how these modules performed (see Lipka, Webster, & Yanez, 2005; and Lipka et al., 2009). In almost every instance, teachers and students who used MCC's modules performed better at statistically significant levels than students who used only their regular math textbooks on MCC's tests. This was true for urban as well as rural students, both Caucasian and Alaska Native. We believe that this supplemental curriculum will motivate your students and strengthen their mathematical understanding because of the engaging content, hands-on approach to problem-solving, and the emphasis on mathematical communication. Further, these modules build on students' everyday experience and intuitive understandings—particularly in geometry, which is underrepresented in school.

A design principle used in the development of these modules is supporting student autonomy through mathematical explorations that can often occur successfully in small groups. Through the use of hands-on materials, students can "physically" prove conjectures, solve problems, and find patterns, properties, shortcuts, or generalizations. The activities incorporate multiple modalities and can challenge students with diverse intellectual needs. Hence, the curriculum is designed for heterogeneous groups with the realization that different students will tap into different cognitive strengths. According to Sternberg (1997, 1998), when you engage students creatively, analytically, and practically, they develop a more robust understanding of math concepts. This approach allows for shifting roles and expertise among students rather than only privileging those students with analytical knowledge.

The modules explore the everyday application of mathematical skills, such as grouping, approximating, measuring, proportional thinking, informal geometry, and counting in base twenty, and then present these skills in terms of formal mathematics. Students move from the concrete and applied to more formal and abstract math. The activities are designed to meet the following goals:

- Students learn to solve mathematical problems that support an in-depth understanding of mathematical concepts.
- Students derive mathematical formulas and rules from concrete and practical applications.
- Students become flexible thinkers because they learn that there is more than one method of solving a mathematical problem.
- Students learn to communicate and think mathematically while they demonstrate their understanding to peers.
- Students learn content across the curriculum, since the lessons comprise Yup'ik Eskimo culture, literacy, geography, and science.

Curriculum Design

Beyond meeting some of the content (mathematics) and process standards of the National Council of Teachers of Mathematics (2000), the curriculum design and its activities respond to the needs of diverse learners. Many activities are designed for group work. One of the strategies for using group work is to provide leadership opportunities to students who may not typically be placed in those roles. Also, the modules tap into a wide array of intellectual abilities—practical, creative, and analytical. We assessed modules that were tested in rural and urban Alaska and found anecdotal evidence that students who were only peripherally involved in math became more active participants through the use of these modules.

Students learn to reason mathematically by constructing models and analyzing practical tasks for their embedded mathematics. This enables them to generate and discover mathematical rules and formulas. In this way, we offer students a variety of ways to engage with the math material through practical activity, spatial/visual learning, analytical thinking, and creative thinking. They are constantly encouraged to communicate mathematically by presenting their understandings, while other students are encouraged to provide alternate solutions, strategies, and counterarguments. This process also strengthens their deductive reasoning.

The curriculum design includes strategies that engage students:

- cognitively, so that students use a variety of thinking strategies (analytical, creative, and practical);
- socially, so that students with different social, cognitive, and mathematical skills use those strengths to lead and help solve mathematical problems;
- pedagogically, so that students explore mathematical concepts and learn to reason and communicate mathematically by demonstrating their understanding of the concepts; and
- practically, as students apply or investigate mathematics to solve problems from their daily lives.

The organization of the modules follows seven distinct approaches to teaching and learning that converge into one pedagogical system.

Pedagogical Approaches Used in the Modules

Because this project represents an extraordinarily long-term collaboration with Yup'ik elders and expert Yup'ik teachers as well as mathematicians, math educators, and educators, we have come to understand alternative ways of organizing and teaching math in the elementary school classroom. The concept of a third space has become a place where we develop creative ways of teaching mathematics. Third space refers to a dynamic and creative place among school-based knowledge, everyday knowledge, and knowledge related to other nonmainstream cultural groups. Third space also includes local knowledge, such as ways of measuring and counting that are distinct from school-based notions, and brings these elements together in a creative, respectful, and artful manner. Within this creative and evolving space, pedagogical forms can develop creatively from both Western schooling and local ways. In particular, this module pays close attention to expert-apprentice modeling, joint productive activity (where teachers and students engage in similar classroom tasks), and cognitive apprenticeship (where teachers voice how they may solve a particular math problem, making their thinking accessible to all students). Math in a Cultural Context also incorporates a reform-oriented approach, includes familiar and unfamiliar contexts for the students as well as utilizes multiple intelligences.

Expert-Apprentice Modeling

The first approach, expert-apprentice modeling, comes from Yup'ik elders and teachers and is supported by research in anthropology and education. Many lessons begin with the teacher (the expert) demonstrating a concept to the students (the apprentices). Following the theoretical position of the Russian psychologist Vygotsky (cited in Moll, 1990) and expert Yup'ik teachers (Lipka and Yanez, 1998) and elders, students begin to appropriate the knowledge of the teacher (who functions in the role of expert), as the teacher and the more adept apprentices help other students learn. This establishes a collaborative classroom setting in which student-to-student and student-to-teacher dialogues are part of the classroom fabric.

Joint Productive Activity

In addition, we have observed experienced teachers use joint productive activity—the teacher works in parallel with students modeling an activity, a concept, or a skill. When effectively implemented, joint productive activity appears to increase student ownership of the task as well as responsibility and motivation. The typical authority structure surrounding classrooms changes as students take on more of the responsibility for their learning. Social relations in the classroom become more level. Also, the connections between out-of-school learning and in-school learning are strengthened through pedagogical approaches, such as expert-apprentice modeling and joint productive activity, that are used in the community.

Cognitive Apprenticeship

A third pedagogical approach based on expert-apprentice modeling is particularly relevant to Alaska's indigenous students as well as for any student that learns through observation. Cognition is far more accessible when expert-apprenctice modeling is coupled with cognitive apprenticeship. As Carol Lee (1995) states, "in a cognitive apprenticeship, the goal is to make visible and explicit complex thinking strategies that experts use in particular domains." Teachers from kindergarten through high school have often effectively voiced how they might solve an unexpected problem, making what is internal and private external and social. Students do not have enough of this type of experience to understand how another person may understand a problem, how they might solve it, and how they might work through it. Expert-apprentice modeling provides a way to include this in your teaching.

Reform-Oriented Approach

Another pedagogical approach emphasizes student collaboration in solving "deeper" problems (Ma, 1999). This approach is supported by research in math classrooms and particularly by recent international studies (Stevenson et al., 1990; Stigler and Hiebert, 1998), strongly suggesting that math problems should be more in-depth and challenging and that students should understand the underlying principles, not merely use procedures competently. The modules present complex, two-step, open-ended problems that require students to think more deeply about mathematics.

Also, pedagogical approach has to do with familiarity. The modules attempt to build familiar and engaging educational context that is at once inviting and challenging. The project has attempted to reduce excessive "noise" and disincentive for students to learn, particularly second-language learners and Alaska's rural indigenous students. Attempting to teach students challenging mathematics concepts while also presenting unfamiliar background or contextual information may only deter students from engaging and may create a sense of being lost before they even begin. This project has gone to great lengths to create a more engaging classroom climate.

Familiar and Unfamiliar Contexts Challenge Students' Thinking

Curriculum designers and teachers face a difficult balance between familiar and unfamiliar contexts and knowledge. The background knowledge that students possess can be cancelled out by presenting new concepts when the context in which it is placed is unfamiliar. Simultaneously, if the knowledge and context are both too familiar, students may not engage. Thus we attempt to achieve a balance between familiar and unfamiliar context and knowledge.

For example, building on students' cognitive strengths such as spatial abilities, in this context, while building on existing knowledge structures student have, places students in a better position to learn. Work by Grigorenko et al. (2004) in this cognitive domain provided some empirical evidence supporting this principle.

By working in unfamiliar settings and facing new and challenging problems, students learn to think creatively. They gain confidence in their ability to solve both everyday problems and abstract mathematical questions, and their entire realm of knowledge and experience expands. Further, by making the familiar unfamiliar and by working on novel problems, students are encouraged to connect what they learn from one setting (everyday problems) with mathematics in another setting. For example, most sixth-grade students know about rectangles and how to calculate the area of a rectangle, but if you ask students to go outside and create or construct the four corners of an eight-foot-by-twelve-foot rectangle without using rulers or similar instruments, they are faced with a challenging problem. As they work through this everyday application (which is needed to build any rectangular structure) and as they "prove" to their classmates that they do, in fact, have a rectangular base, they expand their knowledge of rectangles. In effect they must shift their thinking from considering rectangles as physical entities or as prototypical examples to understanding the salient properties of a rectangle. Similarly, everyday language, conceptions, and intuition may, in fact, be in the way of mathematical understanding and the precise meaning of mathematical terms. By treating familiar knowledge in unfamiliar ways, students explore and confront their own mathematical understandings and begin to understand the world of mathematics. These major principles guide the overall pedagogical approach to the modules.

Multiple Intelligences

These modules tap into multiple intelligences. While some students may learn best from hands-on, real-world problems, others may learn best when abstracting and deducing. This module provides opportunities to use both methods of learning. Robert Sternberg's work (1997, 1998) made us more mindful of how important it is to tap into different aspects of intelligence—creative, practical, and analytic. His research has consistently found that students who are taught so that they use their analytic, creative, and practical intelligences will outperform students who are taught using a single modality, most often analytical. Therefore, we have shaped our activities to engage students in this manner. In fact, in conducting research with Sternberg et al. (2006), we found the MCC students outperformed the control groups in line with Sternberg's theory. Sternberg noted that MCC was an excellent example of his theory in practice (personal communication, 2002).

Mathematical Argumentation and Deriving Rules

The purpose of math communication, argumentation, and conceptual understanding is to foster students' natural abilities. These modules support a math classroom environment where students explore the underlying mathematical rules as they solve problems. Through structured classroom communication, students learn to work collaboratively in a problem-solving environment in which they learn both to appreciate alternative solutions and strategies and to evaluate these strategies and solutions. They will present their mathematical solutions to their peers. Through discrepancies in strategies and solutions, students will communicate with and help each other to understand their reasoning and mathematical decisions. Mathematical discussions are encouraged to strengthen students' mathematical and logical thinking as they share their findings. This requires classroom norms that support student communication, learning from errors, and viewing errors as opportunities to learn rather than to criticize. The materials in the modules (see Materials section) constrain the possibilities, guide students in a particular direction, and increase their chances of understanding mathematical concepts. Students are given the opportunity to support their conceptual understanding by practicing it in the context of a particular problem.

The Organization of the Modules

The curriculum includes modules for kindergarten through seventh grade. Modules are divided into sections—activities, explorations, and exercises—with some variation between each module. Supplementary information is included in Cultural Notes, Teacher Notes, and Math Notes. Each module follows a particular cultural storyline, and the mathematics connects directly to it. Some modules are designed around a children's story, and an illustrated text is included for the teacher to read to the class.

The module is a teacher's manual. It begins with a general overview of the activities ahead, an explanation of the math and pedagogy of the module, teaching suggestions, and a historical and cultural overview of the curriculum in general and of each specific module. Each activity includes a brief introductory statement, an estimated duration, goals, materials, any preclass preparatory instructions for the teacher, and the procedures for the class to carry out the activity. Assessments are placed at various stages, both intermittently and at the end of activities.

Illustrations help to enliven the text. Yup'ik stories and games are interspersed and enrich the mathematics. Transparency masters, worksheet masters, assessments, and suggestions for additional materials are attached at the end of each activity. An overhead projector is necessary. Blackline masters that can be made into overhead transparencies are an important visual enhancement of the activities, stories, and games. Such visual aids also help to further classroom discussion and understanding.

Resources and Materials Required to Teach the Modules

Materials

The materials and tools limit the range of mathematical possibilities, guiding students' explorations so that they focus on the intended purpose of the lesson. For example, in one module, latex sheets are used to explore concepts of topology. Students can manipulate the latex to the degree necessary to discover the mathematics of the various activities and apply the rules of topology.

For materials and learning tools that are more difficult to find or that are directly related to unique aspects of this curriculum, we provide detailed instructions on how to make those tools for the teacher and students. For example, in *Going to Egg Island: Adventures in Grouping and Place Values,* students use a base-twenty abacus. Although the project has produced and makes available a few varieties of wooden abaci, detailed instructions are provided for the teacher and students on how to make a simple, inexpensive, and usable abacus with beads and pipe cleaners.

Each module and each activity lists all of the materials and learning tools necessary to carry it out. Some of the tools are expressly mathematical, such as interlocking centimeter cubes, abaci, and compasses. Others are particular to the given context of the problem, such as latex and black-and-white geometric pattern pieces. Many of the materials are items a teacher will probably have on hand, such as paper, markers, scissors, and rulers. Students learn to apply and manipulate the materials. The value of caring for the materials is underscored by the precepts of subsistence, which is based on processing raw materials and foods with maximum use and minimum waste. Sometimes, we use food as part of an activity. In these instances, we encourage minimal waste.

DVD's

To more vividly convey the knowledge of the elders that underlies the entire curriculum, we have produced DVDs to accompany some of the modules. For example, the *Going to Egg Island: Adventures in Grouping and Place Values* module includes videos of Yup'ik elders demonstrating some traditional Yup'ik games. We also have footage and recordings of the ancient chants that accompanied these games. The videos are available on DVD and are readily accessible for classroom use.

Yup'ik Language Glossary and Math Terms Glossary

To help teachers and students get a better feel for the Yup'ik language, its sounds, and the Yup'ik words used to describe mathematical concepts in this curriculum, we have developed a Yup'ik glossary on CD-ROM. Each Yup'ik word is recorded in digital form and can be played back. The context of the word is provided, giving teachers and students a better sense of the Yup'ik concept, not just its Western "equivalent." Pictures and illustrations often accompany the words for additional clarification.

Cultural Notes

Most of the mathematics used in the curriculum comes from our direct association and long-term collaboration with Yup'ik Eskimo elders and teachers. We have included many Cultural Notes to describe and explain more fully the purposes, origins, and variations associated with a particular traditional activity. Each module is based on a cultural activity and follows a Yup'ik cultural storyline along which the activities and lessons unfold.

Math Notes

We want to ensure that teachers who may want to teach these modules but who feel unsure of some of the mathematical concepts will feel supported by the Math Notes. These provide background material to help teachers better understand the mathematical concepts presented in the activities and exercises of each module. For example, in *Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area*, the Math Notes give a detailed description of a rectangle and describe the geometric proofs one would apply to ascertain whether or not a shape is a rectangle. *Building a Smokehouse: The Geometry of Prisms* explores rectangular prisms and the geometry of three-dimensional objects; the Math Notes include information on the geometry of rectangular prisms, including proofs, to facilitate the instructional process. In every module, connections are made among the "formal math," its practical application, and the classroom strategies for teaching the math.

Teacher Notes

The main function of the Teacher Notes is to focus on the key pedagogical aspects of the lesson. For example, they provide suggestions for how to facilitate students' mathematical understanding through classroom organization strategies, classroom communication, and ways of structuring lessons. Teacher Notes also make suggestions for ways of connecting out-of-school knowledge with schooling.

Math in a Cultural Context Web-based Resources

Recently, MCC has developed extensive web-based resources for teachers and administrators. These resources are located at http://www.uaf.edu/mcc/. At the website, there are a variety of resources from a list of all the modules and stories, journal articles about everything from classroom case studies to more quantitatively based research, podcasts and video downloads about the program, and most importantly a page called MCC Construction Videos. These podcasts show how Dora Andrew-Ihrke, a long-term consultant to the project, constructs a square out of uneven material and how she transforms a square into a circle. This important work begins to describe how solutions to some Yup'ik everyday problems/activities incorporate mathematical solutions.

Language and Literacy Connections in MCC

Elders who have informed this project say that when visiting or even coming to project meetings, "always come prepared to tell a story." Because of this underlying Yup'ik value, MCC has increased the role of literacy in this "math" series. Yup'ik elders such as Annie Blue from Togiak, Alaska, continue to tell stories at project meetings and Summer Math Institutes, even though she is in her nineties. These stories provide a wonderful window into Yup'ik culture and to the past as elders like Annie tell stories that they heard in their youth. The inclusion of culturally based stories has proven to contribute to students' engagement with the math modules as well as provide cultural grounding for the module activities. MCC modules have also made use of literacy-based activities, such as journaling/math notebooks, to further students' understanding of math concepts and vocabulary.

Assessment

Assessment and instruction are interrelated throughout the modules. Assessments are embedded within instructional activities, and teachers are encouraged to carefully observe, listen, and challenge their students' thinking. We call this active assessment, which allows teachers to assess how well students have learned to solve the mathematical and cultural problems introduced in the module.

Careful attention has been given to developing assessment techniques and tools that evaluate both the conceptual and procedural knowledge of students. We agree with Ma (1999) that having one type of knowledge without the other or not understanding the link between the two will produce only partial understanding. The goal here is to produce relational understanding in mathematics. Instruction and assessment have been developed and aligned to ensure that both types of knowledge are acquired; this has been accomplished using both traditional and alternative techniques.

The specific details and techniques for assessment (when applicable) are included within activities. The three main tools for collecting and using assessment data follow. Teachers who have participated in our workshops and have agreed to use MCC's tests receive detailed feedback from the project on students' mathematical knowledge in terms of their overall math score (module specific), sub-scale scores, and suggested instructional strategies based on this assessment.

Notebooks

In recent years, the National Council of Teachers of Mathematics (NCTM) has promoted standards that incorporate math journals as part of math instruction. Journaling has most often occurred as a tool for reflecting on what was learned. In contrast, math notebooks are used by students to record what they are thinking and learning about math concepts before, during, and after the activities in the modules. Through the use of math notebooks, students build their content knowledge while at the same time developing their literacy skills through reading, writing, drawing, and graphic representations. Math notebooks also play an important role in helping students develop math vocabulary.

Observation

Observing and listening to students lets teachers learn about the strategies that they use to analyze and solve various problems. Listening to informal conversations between students as they work cooperatively on problems provides further insight into their strategies. Through observation, teachers also learn about their students' attitude toward mathematics and their skill in cooperating with others. Observation is an excellent way to link assessment with instruction.

Adaptive Instruction

The goal of assessment in this curriculum is to adapt instruction to the skills and knowledge needed by a group of students. From reviewing student notebooks to simply observing, teachers learn which mathematical processes their students are able to effectively use and which ones they need to practice more. Adaptive assessment and instruction complete the link between assessment and instruction.

An Introduction to the Land and Its People, Geography, and Climate

Flying over the largely uninhabited expanse of southwest Alaska on a dark winter morning, one looks down at a white landscape interspersed with trees, winding rivers, rolling hills, and mountains. A handful of lights are sprinkled here, a handful there. Half of Alaska's 600,000-plus population lives in Anchorage. The other half is dispersed among smaller cities such as Fairbanks and Juneau and among the over 200 rural villages that are scattered across the state. Landing on the village airstrip, which is usually gravel and, in the winter, covered with smooth, hard-packed snow, one is taken to the village by either car or snowmachine. Hardly any villages or regional centers are connected to road systems. The major means of transportation between these communities is by small plane, boat, and snowmachine, depending on the season.

It is common for the school to be centrally located. Village roads are usually unpaved, and people drive cars, four-wheelers, and snowmachines. Houses are typically made from modern materials and have electricity and running water. Over the past twenty years, Alaska villages have undergone major changes, both technologically and culturally. Most now have television, full phone systems, modern water and sewage treatment facilities, an airport, and a small store. Some also have a restaurant, and a few even have a small hotel and taxicab service. Access to medical care and public safety are still sporadic, with the former usually provided by a local health care worker and a community health clinic or by health-care workers from larger cities or regional centers who visit on a regular basis. Serious medical emergencies require air evacuation to either Anchorage or Fairbanks.

Yup'ik has traditionally been used to reference the name of the people, singular or plural, as well as the name of their language. Steven Jacobson (personal communication, 2010), author of the Yup'ik Dictionary (1984), recognizes that some areas prefer the use of Yupiaq rather than Yup'ik, though the differences between areas and dialects are minimal, it is standard to use the established word, Yup'ik, in all instances. We follow this general rule throughout the module.

Yup'ik Values

For classroom purposes, perhaps the most important Yup'ik value is the notion of making everyday products from the natural resources existing in the environment. Although this is changing, the elders we work with continue to amaze us with their deep knowledge of the environment and their creative skills to fashion materials into products. This knowledge is being integrated into our latest work.

Many other important Yup'ik values are associated with each module. The elders counsel against waste. They value listening, learning, working hard, being cooperative, problem solving, and passing knowledge on to others. These values are expressed in the contents of the Yup'ik stories that accompany the modules, in the Cultural Notes, and in various activities. Similarly, Yup'ik people as well as other traditional people continue to produce, build, and make crafts from raw materials. Students who engage in these modules also learn how to make simple mathematical tools fashioned around such themes as Yup'ik border patterns and building model kayaks, fish racks, and smokehouses. This way, students learn to appreciate and value other cultures.

The Schools

Years of work have gone into making education as accessible as possible to rural communities. Almost every village has an elementary school and most have a high school. Some also have a higher education satellite facility, computer access to higher education courses, or options that enable students to earn college credits while in their respective home communities. Vocational education is taught in some of the high schools, and there are also special vocational education facilities in some villages. While English has become the dominant language throughout Alaska, many Yup'ik children in the villages still learn Yup'ik at home.

Yup'ik Village Life Today

Most villagers continue to participate in the seasonal rounds of hunting, fishing, and gathering. Although many modern conveniences are located within the village, when one steps outside of its narrow bounds, one is immediately aware of one's vulnerability in this immense and unforgiving land, where one misstep can lead to disaster. Depending upon their location (coastal community, riverine, or interior), villagers hunt and gather the surrounding resources. These include sea mammals, fish, caribou, and many types of berries. Knowledgeable elders know how to cross rivers and find their way through ice fields, navigating the seemingly featureless tundra by using directional indicators such as frozen grass and the constellations in the night sky. All of this can mean the difference between life and death. In the summer, when this largely treeless, moss-and grass-covered plain thaws into a large swamp dotted with small lakes, the consequences of ignorance, carelessness, and inexperience can be just as devastating. Underwater hazards in the river, such as submerged logs, can capsize a boat, dumping the occupants into the cold, swift current. Overland travel is much more difficult during the warm months due to the marshy ground and many waterways, and one can easily become disoriented and get lost. The sea is also integral to life in this region and requires its own set of skills and specialized knowledge to be safely navigated.

The Importance of the Land: Hunting and Gathering

Basic subsistence skills include knowing how to read the sky to determine the weather and make appropriate travel plans, being able to read the land to find one's way, knowing how to build an emergency shelter and, in the greater scheme, how to hunt and gather food and properly process and store it. In addition, the byproducts of subsistence activities, such as carved walrus tusks, pelts, and skins, are made into clothing or decorative items and a variety of other utilitarian arts and crafts products that provide an important source of cash for many rural residents.

Hunting and gathering are still of great importance in modern Yup'ik society. A young man's first seal hunt is celebrated, family members who normally live and work in one of the larger cities will often fly home to help when the salmon are running, and whole families still gather to go berry picking. The importance of hunting and gathering in daily life is further reflected in the legislative priorities expressed by rural residents in Alaska. These focus on such things as subsistence hunting regulations, fishing quotas, resource development, and environmental issues that affect the well-being of game animals and subsistence vegetation.

Conclusion

We developed this curriculum in a Yup'ik context and the results show that all students prosper from its content. The traditional subsistence and other skills of the Yup'ik people incorporate spatial, geometric, and proportional reasoning as well as other mathematical reasoning. We have attempted to offer you and your students a new way to approach and apply mathematics while also learning about Yup'ik culture. Our goal has been to present math as practical information that is inherent in everything we do. We hope your students will adopt and incorporate some of this knowledge and add it to their learning base.

We hope you and your students will benefit from the mathematics, culture, geography, and literature embedded in the *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders* series. The elders who guided this work emphasized that the next generation of children should be flexible thinkers and leaders. In a small way, we hope that this curriculum guides you and your students along this path.

Tua-ii ingrutuq [This is not the end].

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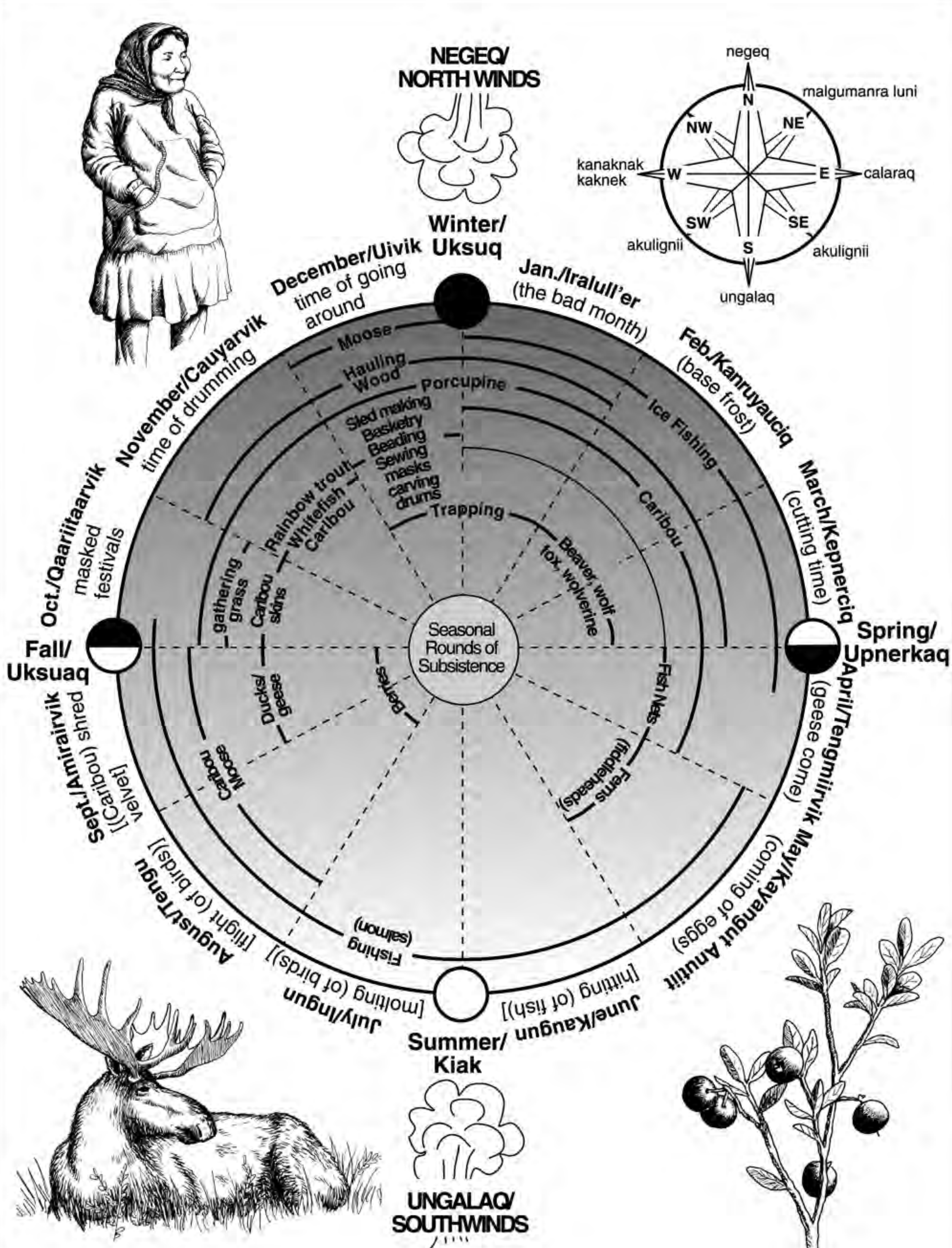
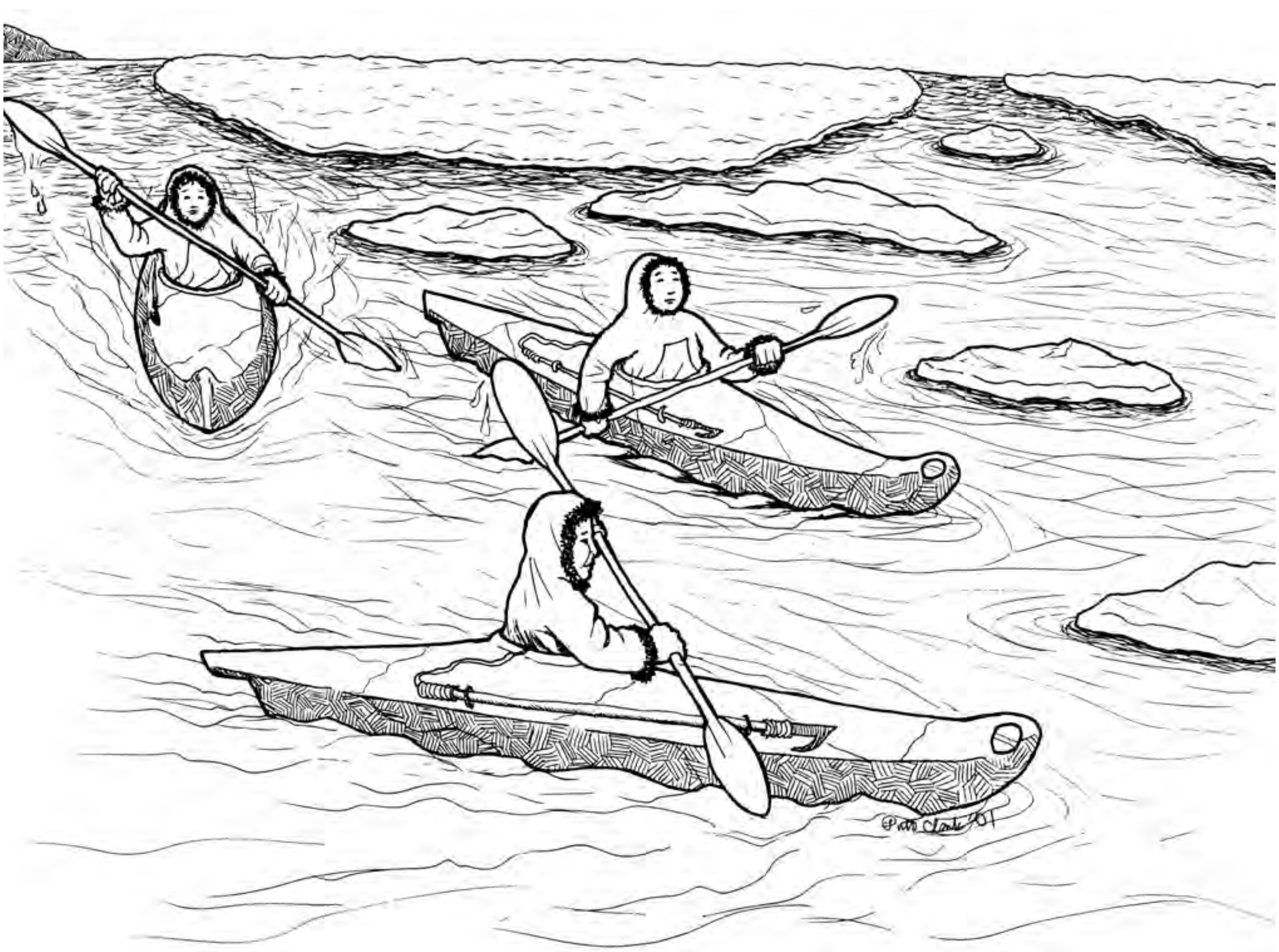


Fig. 1: Yearly subsistence calendar.

Introduction

Kayak Design:
Scientific Method and Statistical Analysis



Introduction to the Module

Why study the Yup'ik kayak in an elementary school mathematics classroom? Through creating simple clay kayak models, students are able to investigate the relationship between the kayak's shape and its function. By investigating these relationships, students are guided into a series of purposeful mathematical investigations, using the scientific method—controlling variables in a systematic way. They collect, organize, and analyze data by developing tables, converting tables into graphs (line, bar, and scatter plots), interpreting graphs, and using basic statistical techniques (mean, median, and mode) to determine if there is a relationship between form and function.

Thus, this module is organized so that students learn foundational concepts in math and scientific processes. The students learn and apply the scientific method as they test their model boats, changing one variable at a time. This skill set can be used throughout their educational careers. Likewise, as students collect and organize data there is a wonderful opportunity for them to learn to construct their own tables and graphs. These are also foundational skills that have applicability well beyond this module to many other subject areas. If the time is spent up front to help the students become more autonomous thinkers, time will be saved later because these students will be able to apply and use what was learned in this module. This module aims to have students become increasingly responsible for developing initial mastery of these concepts and for students to be resources to each other as they learn this knowledge.

Because students are responsible for designing their own boats, and the math of the module is directly integrated into this experience, students are highly motivated to not only test their boats but to perform the mathematics necessary to determine the results of their trials.

Students will establish a series of experiments to answer the question, "which boat will . . . ?" They will then conduct experiments, such as determining how much weight each boat is able to carry, take notes describing the experiment, collect and analyze their data, and draw conclusions based on the experiments. Throughout the module we intersperse a variety of notes: cultural, teacher, math, and science; these notes provide valuable information that can be integrated into the teaching of the module. As the students engage in these hands-on activities, they will learn about Yup'ik Eskimo culture of southwest Alaska through stories and myths as told by Yup'ik elders. These quotes make the module more personal and authentic. This module attempts to convey some of the Yup'ik people's ingenuity, creativity, and wisdom as demonstrated through their design and construction of the kayak.

The National Council of Teachers of Mathematics (NCTM, 2000) has encouraged the use of everyday knowledge and practices to be part of the mathematics classroom. In this module, we will analyze three Yup'ik watercraft designs. Students will build and test model boats based on the three traditional designs and collect data about their speed, load capacity, and stability. Through data collection, data organization, and analysis associated with hands-on explorations, students will also explore algebraic thinking and use dynamic measurement, partially based on the ways that elders measure. Other related measurement concepts that students will learn in this module include

- accurate estimating,
- body proportional measuring, and
- using body proportional ratios.

This integrated hands-on exploration of the relationship between design and function will demonstrate to students that mathematics is an integral part of everyday life. Other MCC modules in this series have used similar strategies with excellent results. We believe that by using this module, you can help your students both enjoy learning and retain more math vocabulary and concepts than they would through the use of standard math textbooks. We have found that students in both rural and urban Alaska have enjoyed the MCC series and have prospered from it. Students who used the *Kayak* module have made strong and consistent gains.

This module strives to hit several major mathematical and scientific processes, knowledge, and skills:

- Understanding and applying the scientific method
- Statistics, specifically measures of central tendencies
- Collecting, organizing, and analyzing data by constructing tables and graphs
- Ratios
- Form and function
- Expressing and solving simple algebraic problems

An Introduction to Traditional Yup'ik Watercraft

Over hundreds of years, a variety of boat designs have been developed by people across the Circumpolar North. The physical conditions where the boats were used (open sea to rivers to sea ice), the available material, the construction techniques, and the intended use all contribute to the development of their unique designs. The Yup'ik made a variety of craft to address these different uses. These include craft designs such as kayaks, round-bottomed boats, flat-bottomed rafts, and whaling vessels. Students may be familiar with some of these boats but may not know of their origins and their traditional uses.

Kayak

The kayak, a marvel of marine architecture and a formidable and seaworthy craft, is prominent today as more and more people use kayaks for recreation and sport. Kayaks were the "go everywhere, do everything" watercraft, the water version of what cars, pick-ups, and ATV's are for people today. The kayak needed to perform well for a variety of conditions. For example, not only would you need to be able to haul four (or even six people) in one kayak upriver, but sometimes, for carrying big or heavy loads in rough seas, two kayaks were lashed together with cross pieces to make a catamaran.

Some different types of kayaks are shown below.

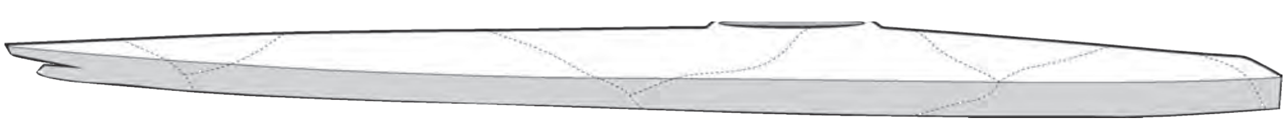


Fig. 2: The Aleut baidarka.

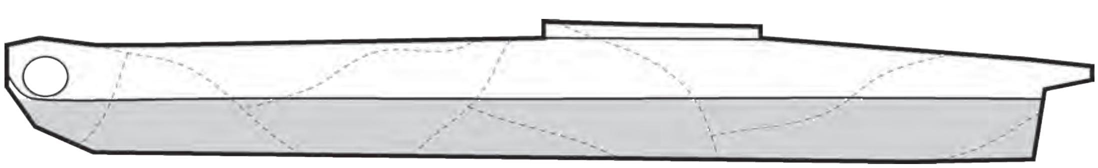


Fig. 3: Bering sea kayak.

These kayaks had driftwood frames, generally of spruce. Curved parts such as the prow pieces and deck beams were carved from specially selected spruce stump wood so that the curve of the piece would follow the natural grain of the wood (Figure 4). Boat building was traditionally a male occupation, except for preparing and sewing the skin kayak covers, which was done by women. The sewing required meticulous double seaming of the skins to provide a tight-fitting, waterproof cover. The skins were sewn together with braided sinew, using waterproof stitches and two layers of sewing: one on the outside, one on the inside. Each layer of stitches only went halfway through the skin layer, so the seam was waterproof. Kayaks were covered in sealskins, preferably bearded seal skins but sometimes seal in combination with other smaller and lighter skins. Rancid seal oil (which is thick and sticky) was used to waterproof the kayak skin by being rubbed in both inside and out. When the kayak skin began to absorb water, it was time to recoat it with oil. Kayaks could be simultaneously camouflaged and waterproofed by mixing a soft white powder scraped from rocks or the lake bottom with the rancid oil. The hunter would paint his kayak with this to make it look white, more like sea ice. The same mixture could be used to waterproof the stitches. If they lacked this material, they would burn grasses and mix the ash with the oil or melted tallow (personal communication Henry Alakayak, Fred George, Katie George, Sam Ivan, George Moses, John Pauk, Ferdinand Sharp, and Mike Toyukak at a workshop in Fairbanks, AK 2002).

"It seems to me that the Aleut baidarka [kayak] is so perfect in its way that a mathematician himself could hardly add anything to the perfection of its sea going qualities." (Veniaminov 1840:222)

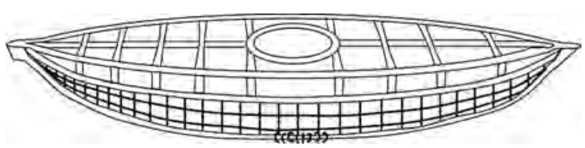


Fig. 4: Kayak frame.

Single-cockpit Yup'ik kayaks were relatively short (15 to 16 feet). They were designed to deal with the steep, closely spaced waves in the shallow waters of southwest Alaska. Speed and relative stability, both lightly and heavily loaded, were important design criteria. Not only did hunters have to deal with high winds and currents, but in precontact times, successful kayakers were sometimes those whose boats were faster than those of their enemies. But the more dangerous foes, day in and day out, were wind and wave, hence the importance of stability. Further, Yup'ik kayaks were designed to carry home butchered sea-mammals inside the hull, rather than dragging unbutchered seals alongside as was the practice in Greenland (and possibly in the Aleutians). This meant that the Yup'ik kayak had to work well either loaded with several hundred additional pounds or with just the weight of the paddler. This carrying capacity also allowed these kayaks to easily carry four people. In an emergency, two more people could be carried (Figure 5), one lying down on each deck. Hunters did tow sea-mammals, but generally only over to a shore or an ice pan where it could be hauled out and butchered. Henry Alakayak said that when you got a seal and were towing it, it was important not to tie the carcass of the seal to the kayak, in case it

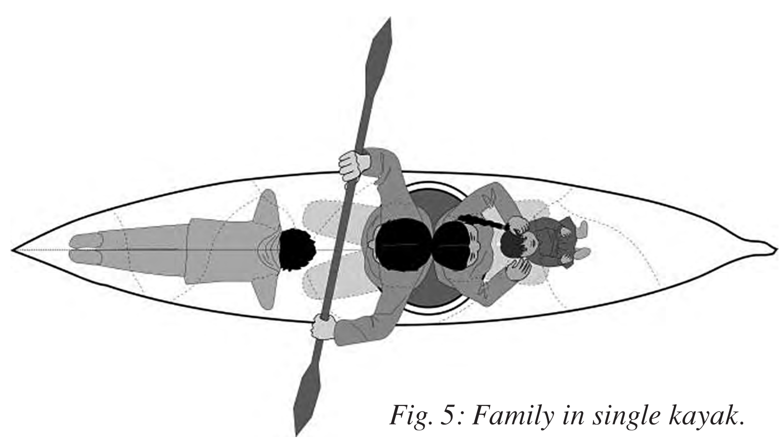


Fig. 5: Family in single kayak.

revives and “acts deadly.” He was taught to hold the rope holding the seal in his teeth, so it was not attached to the kayak.

Elders said that you could load a kayak up to the top fold line, where the top (or decks) start. If you went past that line it would tip over very easily. When families needed to move a lot of people or possessions, they would use poles to attach two kayaks together, like a pair of outriggers (Figure 6). First, they would tie two poles across in an ‘X’ shape in front of the cockpit holes and two behind. The bow poles would each run from the front of each cockpit rim to the bow on the opposite side. The stern poles would be the same, running from the rear of the cockpit rim to the stern on the opposite side. They would then tie cross bars so that it was like one raft in front and another behind the cockpit holes. When the paddlers got tired of paddling on the side they could reach, they would switch sides.

Fig. 6: Family traveling on a raft made from tying two kayaks together.

For several months of the year (when there was open water on the ocean) kayaks were the most important large tool that the Yup’ik made. There was much specialized technology that went along with kayaks, such as poles to shove things like pieces of meat far up into the bow and stern and specialized meat hooks to remove such pieces.

Women wove mats out of Alaska cotton grass (*iitaat*), which helped to keep the load from shifting back and forth inside the kayak. This was part of the balancing system. Fred George said that there were floorboards under the cockpit section, with the cotton grass mat on top.

When fish were loaded, they were put in head first, almost evenly distributed with slightly more weight in the stern. When a kayak is slightly stern heavy, it tracks normally, but when it is bow heavy it is very difficult to steer straight. The bow section was blocked off to keep the fish from shifting. With small fish such as herring, skins were used as liners to make getting the fish out of the far bow and stern easier. (When burlap sacks became available, these were used instead.)

After a man returned with fish, his wife would greet him and take the fish. Then they would remove the mats and take the kayak to the stream or haul water to wash it. They would put the kayak on its side, letting the water spill in, then rock it from side to side and dump the water out. They would repeat this to rinse it out. Then, the kayak would be stored on its side on the rack, so that the wind would dry it out.

If the kayak did not need to be dried out, they would just store it upside down. People were very careful of their kayaks, washing their feet before getting in, so they would not get sand and gravel inside the kayak, which would wear away the skin (personal communication Henry Alakayak, Fred George, Katie George, Sam Ivan, George Moses, John Pauk, Ferdinand Sharp, and Mike Toyukak at a workshop in Fairbanks, Alaska, 2002).

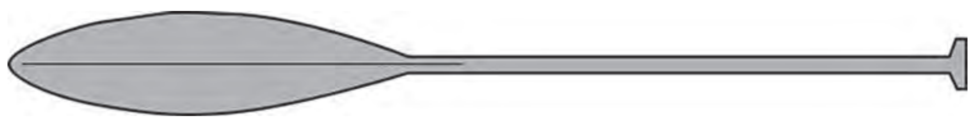


*Fig. 7: Kayak single-bladed paddle (*anguarun*).*

If someone was using the single-bladed paddle (*anguarun*), as people did most of the time, and needed to paddle harder, he/she shifted their lower hand closer to the paddle blade (Figure 7). This was the equivalent of shifting into a lower gear. To go faster, they dug their paddles deeper into the water. For top speed Yupiit used a two-bladed paddle (*paangrun*). This paddle was made by making the handle on a regular paddle longer and putting a blade on either end. There was a special paddle *(luqurun)* that was carved to be very thin, so it wouldn't make noise in the water. It was used with a sculling stroke to quietly move the kayak close to game (personal communication Henry Alakayak, John Pauk, and Mike Toyukak at a workshop in Fairbanks, Alaska, 2002).

The Yup'ik used two short poles for going upriver, pushing along the river bottom like a canoe on both sides of the boat. These poles were also used as they approached beaches with lots of rocks. They would lay the poles down parallel to the shoreline, and used them as rollers to pull the kayak up out of the water to protect the bottom from rocks.

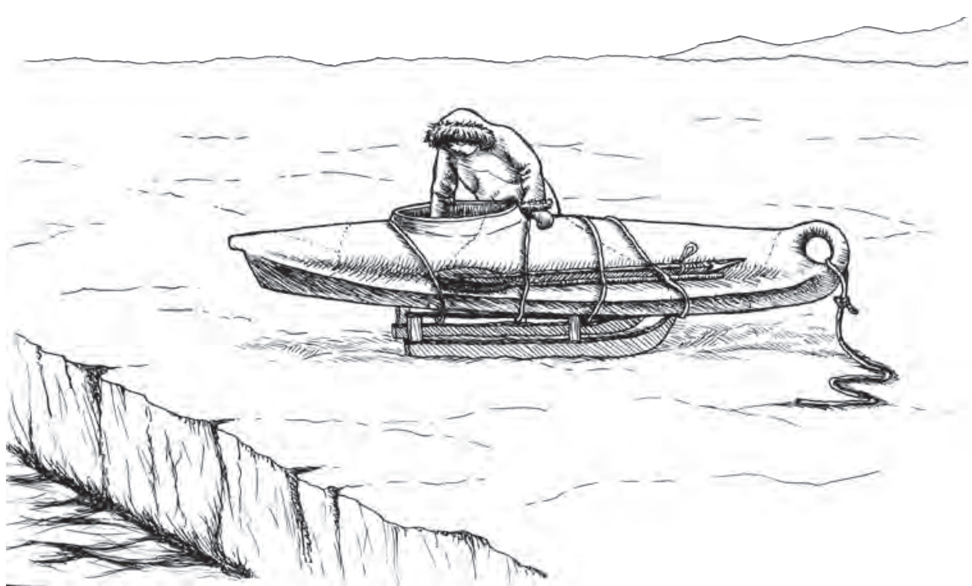


Fig. 8: Kayak sled.

Another brilliant adaptation was the kayak sled (Figure 8). In the spring, hunters often had to cross several sections of ice between open water sections. To do this, the hunters balanced their kayaks on small wooden sleds and pulled them across the ice. When they reached more water, they put the sled on the back deck of the kayak and paddled across. When they reached more ice, they could use their boat/ice hook (*negcikcuar*) to stabilize themselves while they got out of the kayak and onto the ice, then hook the kayak through the big hole in the bow to pull it up on the ice.

The kayak was used for travel. This travel included hunting trips, which meant travelling from the camp or village to open seas, bays, or rivers, depending on the time of year and purpose of the hunt. Another use was transporting the family from one camp to another. When a family traveled together in a kayak (*qayaq*), the man and woman would sit back to back; children could ride inside the kayak (Figures 9 and 10). These positions ensured that the passengers would not be injured if the paddler had to make a sudden move to maneuver the kayak. Children were lulled to sleep by the action of the water, the warm, dry, snug environment, and the close proximity of their parents. Because the oil-treated skins that covered the kayak were translucent, the interior of the kayak was almost luminous.

Fig. 9: Family in kayak.

Fig. 10: Children inside kayak.

When unloading, the paddler would get out first and hold the bow (grab the hole); next the woman or second person who was sitting back-to-back with the paddler would exit; then the person in back would get out, and finally, the person in the bow would exit the boat.

The two-hole kayak (*qayaq/malrugnek/pailuni/anguyaucessuun*) was used mostly for transporting either young boys or older men who were not capable of handling a solo kayak (Figure 11). Some pairs of men also hunted with the two-hole kayak. Two, two-holed kayaks (*pailun)* could also be connected, catamaran style, for stability when hauling large game (personal communication Henry Alakayak, Frederick George, Katie George, Sam Ivan, George Moses, Ferdinand Sharp, and Mike Toyukak at a workshop in Fairbanks, Alaska, 2002). Additional stories regarding the boats and the elders' experiences are interspersed throughout the module.

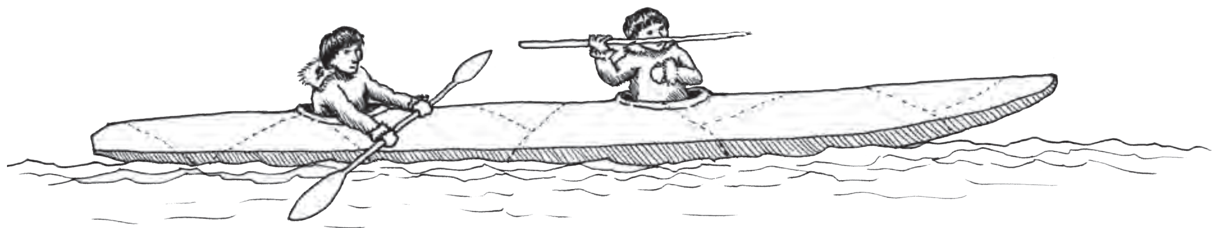


Fig. 11: Two-hole or two-hatch kayak.

There are many old pictures of three-hole kayaks, but none have survived, even in museums (Figure 12).

Fig. 12: Three-hole or two-hatch kayak.

Although the kayak (*qayaq*) is the most widely known northern watercraft, the Yup'ik developed other less-well-known boats. They are the skin raft (*angyaqatak)* considered a disposable craft; the rectangular raft (*angyarrluk*); and the whaling vessel (*aguun/egelrun*).

Skin Raft

The skin raft, *angyaqatak,* translated as big, old boat, is shown in Figure 13. It was used to move families downriver in late spring from hunting camp back to the village. This trip was often dangerous, requiring high levels of concentration and coordination among the men who paddled down the treacherous river.

The raft is put together with parts from the sled a family would have used to go upriver in the winter, along with green, freshly cut, pliable poplar saplings. This gave the frame an oval

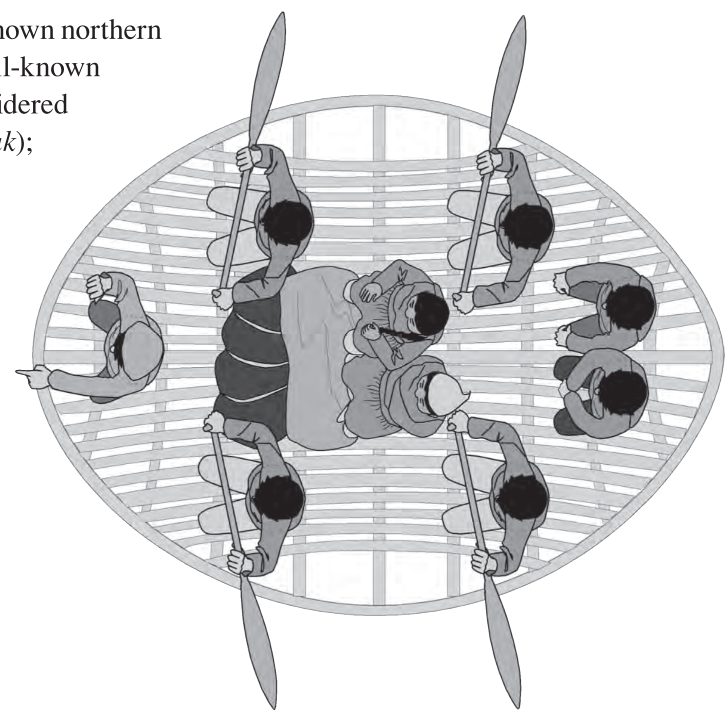


*Fig. 13: Skin raft or big old boat (*angyaqatak*) with passengers.*

shape with a round bottom. The tough hides of moose or bear were used for the slanted bow of the raft and thinner caribou skins were stitched together to complete the rest of the shell. Stitch holes were sealed either with thick grayish clay found along the river or with the ashes from bear grass that had been mixed with seal oil. Once the trip to the village was completed, the skin raft was dismantled. The wood was used for steam baths and the skin was used for boots. The "big old boats" could haul huge amounts, sometimes as many as three sleds or families.

Guides knelt in prescribed positions in the skin raft. The point man was located at 12 o'clock, paddlers were seated at 2:00 and 10:00, and backup paddlers were located at 5:00 and 7:00. The two men stationed in the back could take over paddling if the raft spun going through rapids. They guided the skin raft through difficult water conditions.

Every passenger faced the front and would not talk or otherwise distract the paddlers because rivers can be very dangerous. When the rapids were very challenging, everyone walked along the bank and the men would maneuver the boat through the rapids using ropes.

Sometimes the Yup'ik would build a separate skin raft for the dogs. They would not tie it to the larger skin raft and tow it because it would have been too dangerous for both the dogs and the humans (Figure 14). In that case, a small skin raft was constructed for a person and some dogs.

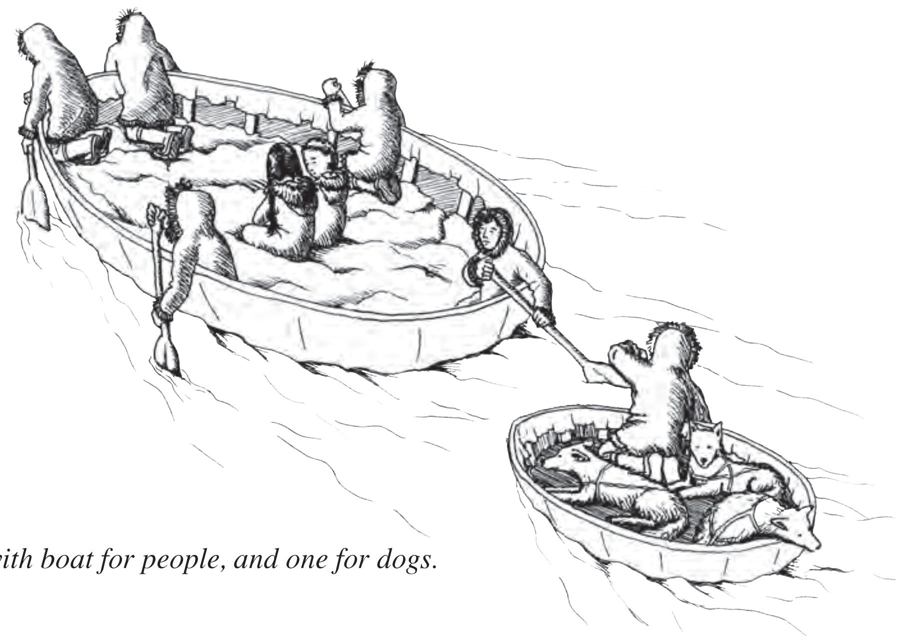


Fig. 14: Travel down river with boat for people, and one for dogs.

The Rectangular Raft

The rectangular raft (*angyarrluk*) could be narrow and long or fairly wide (Figure 15). These rafts could be used for hauling heavy loads. The Yup'ik chose dry spruce or cottonwood logs that could be as long as the original trees. They would lash them together with roots or use moose hide if it was available. The logs could also be pegged together. The Yup'ik would put on a second layer of logs going the opposite direction to form a criss-cross pattern. The logs would be notched so that they fit tightly or they would be nailed down. The Yup'ik used long poles to maneuver these boats down the river. Log rafts were primarily used on lakes, ocean bays, or the slower stretches of rivers. Those traveling could also wait until the wind was blowing the right direction, then raise a sail on the raft and sail across.

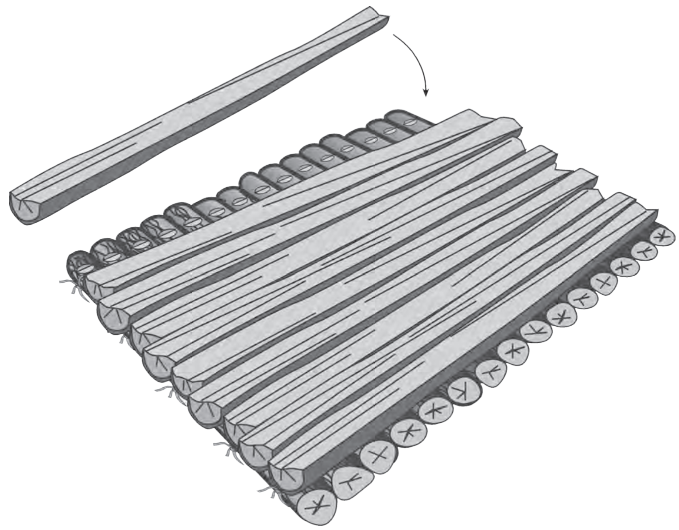


*Fig. 15: Log raft (*angyarrluk*).*

The Whaling Boat

The open whaling boat (*aguun*/*egelrun*) is another boat the Yup'ik used for transportation of people, equipment, and game (Figure 16). The whaling boat is closest in shape to the modern motorboats now in use.

*Fig. 16: Open whaling boat (*aguun/egelrun *or* angyaq/umiaq*).*

Today, the Lund is a modern all-purpose boat, made of aluminum, with a small motor (Figure 17). Most families use it for a variety of purposes such as transportation, cargo hauling, and fishing.

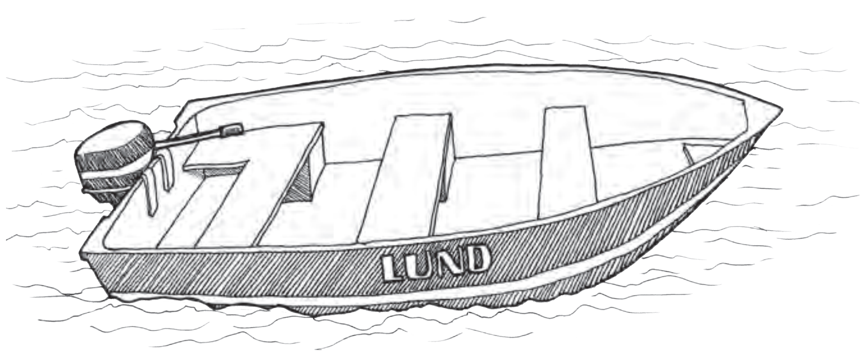


Fig. 17: Eighteen foot Lund.

(This section was translated from Yup'ik to English by Eliza Orr, Evelyn Yanez, and Dora Andrew-Ihrke.)

The Mathematics and the Pedagogy of the Module

The Mathematics

The functionality of the kayak and its ability to travel through challenging waters is based on its shape. In the previous section we explored many of the Yup'ik traditional watercraft. We now narrow our study to look at the effects of different bottom shapes for three of these crafts. In this module, students will investigate three of the main shapes used in Yup'ik boat design—v-shaped bottom (kayak), flat bottom (log raft), and round bottom (skin raft). Each of the designs causes the boats to perform differently with respect to weight capacity, speed, and stability. In this module, students will investigate each of these three designs for each of these performance criteria.

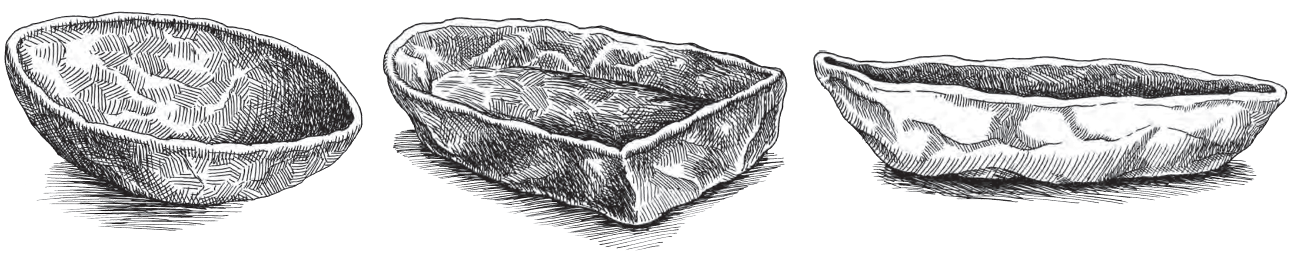


Fig. 18: Models of three types of boats, each made with 100g of clay: (left to right) round-bottom, flat-bottom, and v-bottom boats.

As part of analyzing their data, students will also learn about and apply statistical concepts (range, mean, median, and mode). They will construct scatter plots and bar graphs, and interpret the data within the cultural context of the Yup'ik boats. Students will learn which boats are most useful in different situations (for example, slow boats with large capacity would be good for transporting people, belongings, and supplies downriver, but not for moving just one person or for hunting). The key question that drives this module is "Does the design (or shape) affect the load capacity, speed, and stability of the model boat?" Students will work with data, which they will organize in tables and graphs to attempt to answer this question.

Does the design affect the load capacity, speed, and stability of a model boat?

As well as providing opportunities to learn about and apply statistical concepts, students will also work with their data to identify patterns as they relate to the influence that shape has on a boat's ability to go fast, carry a load, and remain stable. Because all explorations are hands-on and observable, and the math connects directly to the design of the students' kayak models, motivation to learn and apply statistical data and represent data graphically (e.g., scatter plots) and in numeric form (e.g., tables) should be high. In the final activity, students will then have an opportunity to redesign their kayaks to maximize performance, based on what they have learned.

Mathematics and Science Learning Goals

- Understand what a variable is, including what it means to control for a variable.
- Collect and organize data in tables and identify and interpret patterns in the data—understanding the shape or structure of the data.
- Understand the need for more than one measurement of a single variable.
- Analyze data, specifically using basic statistics—mean, median, mode, and range.
- Create and interpret graphic representations of data—scatter plots, line plots, and bar graphs.
- Familiarize students with traditional nonstandard units as well as metric and British system measures (standard units).
- Familiarize students with measurement conversion.
- Familiarize students with the use of a two-pan balance as an equation.
- Learn about degree measures of angles and using a protractor to measure angles.
- Learn prealgebra concepts and skills such as ratio, scale, and equivalent equations.
- Interpret and analyze data that consists of a single value (i.e., load capacity—weight) or two values (i.e., stability—weight and angle).
- Connect mathematics and statistical analysis to practical inferences about boat performance.
- Understand on an intuitive level what the basic statistical variables (mean, median, mode, range) mean and when they should be used.
- Understand and use the scientific method, including writing conjectures, organizing data, doing an experiment, collecting data, doing data analysis, and making conclusions.

National Council of Teachers of Mathematics Standards (2000)

Below is a list of the 6th grade math concepts addressed throughout this module and the individual activities that use or teach each of the standards.

Math Concept	Activity
Data Analysis and Probability	
Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population	2, 3, 5, 6, 7
Select, create, and use appropriate graphical representations of data, including scatterplots	3, 4, 5, 6, 7
Find, use, and interpret measures of center and spread, including mean and range	4, 5, 6, 7
Discuss and understand the correspondence between data sets and their graphical representations, especially stem-and-leaf plots and scatterplots	4, 5, 6, 7
Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken	2, 3, 4, 5, 6
Use conjectures to formulate new questions and plan new studies to answer them	3, 5, 6
Algebra	
Represent, analyze, and generalize a variety of patterns with tables, graphs, words	2, 3, 4, 5, 6, 7
Relate and compare different forms of representation for a relationship	2, 3, 4, 5, 6
Develop an initial conceptual understanding of different uses of variables	2, 3, 5, 6
Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships	2, 5, 6
Model and solve contextualized problems using various representations, such as graphs, tables, and equations	3, 4, 5, 6, 7
Use graphs to analyze the nature of change in quantities in linear relationships	5, 6
Process Standards	
Build new mathematical knowledge through problem solving	2, 4
Solve problems that arise in mathematics and in other contexts	2, 3, 4, 5, 6, 7
Recognize reasoning and proof as fundamental aspects of mathematics	2, 3, 5, 6, 7
Select and use various types of reasoning and methods of proof	2, 3, 5, 6, 7
Organize and consolidate their mathematical thinking through communications	1, 2, 3, 4, 5, 6, 7
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others	1, 2, 3, 4, 5, 6, 7
Use the language of mathematics to express mathematical ideas precisely	1, 2, 3, 4, 5, 6, 7
Recognize and use connections among mathematical ideas	1, 2, 3, 4, 5, 6, 7
Understand how mathematical ideas interconnect and build on one another to produce a coherent whole	1, 2, 3, 4, 5, 6, 7
Recognize and apply mathematics in contexts outside of mathematics	1, 2, 3, 4, 5, 6, 7
Create and use representations to organize, record, and communicate mathematical ideas	1, 2, 3, 4, 5, 6, 7
Select, apply, and translate among mathematical representations to solve problems	1, 2, 3, 4, 5, 6, 7
Use representations to model and interpret physical, social, and mathematical phenomena	1, 2, 3, 4, 5, 6, 7

The Pedagogy

This engaging module builds on students' curiosity and desire to explore. Thus, as students build a model kayak and test it for its capacity, stability, and speed, they collect data that becomes part of a question, hypothesize, observe/test, analyze, and design cycle.

As students go through this process, they learn about problem solving and the scientific method and how these processes can be cyclical as ideas and models are refined and improved. After their observations and measurements are complete, each student makes changes to their kayak's design. Through this repeating process, the students become involved in a "natural" cycle of testing hypotheses and using data to inform their decisions.

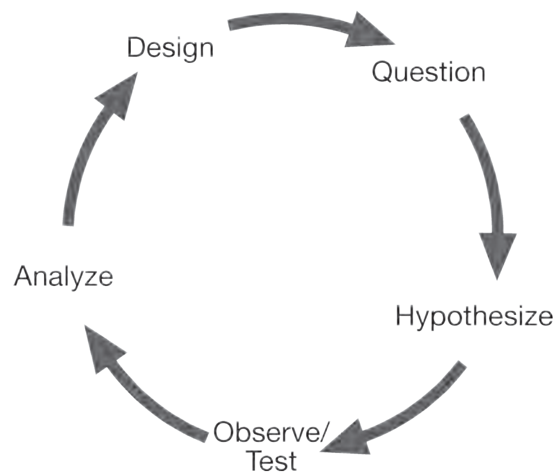


Fig. 19: The cyclical process used in experimental designs.

The key math content of the module is organized so that students should succeed. While the v-shaped boat might be faster, the round bottom boat holds more weight and the flat bottomed boat is the most stable. Thus, each student will do well in his/her own category of boat.

The pedagogical approach to this module recognizes that individual students have different ways of learning. Similarly, students have different cognitive strengths and weaknesses. The curricular design of the module aims to provide opportunities to use students' strengths and gain more practice as well as provide new insights for students in their weaker areas. More specifically, some students may excel at model building and other hands-on components of this module but may not be as skilled at paper-and-pencil math. Because the module is collaborative, it requires students to think and perform in a variety of ways and it provides interpersonal opportunities beyond traditional classroom communication. In this way, students who may not typically have a leadership role in the math classroom may find themselves being leaders.

As in all MCC modules, *Kayak Design: Scientific Method and Statistical Analysis* uses two culturally based approaches to teaching and learning that also incorporate key aspects of constructivist practice and reform mathematics. These pedagogical approaches, expert-apprentice modeling and joint productive activity, are traditional ways that Yup'ik elders teach and are well-suited to teaching and learning mathematics by engaging students as active learners. Expert-apprentice modeling and joint productive activity emphasizes collaboration and exploring concepts and processes by doing and showing and deemphasizes teachers lecturing while students passively listen. The activities in this module engage students in making or modifying a model, conducting experiments, and collecting and analyzing data. Therefore, students are engaged as active learners using expert-apprentice modeling and joint product activity.

In the classroom, expert-apprentice modeling typically begins with the teacher (expert) demonstrating a concept or process for the students (apprentices). This is often followed up by a student demonstration, with the teacher likely following up with an additional demonstration to refine the concept or process and provide further explanation for the students. When the teacher believes that the class is ready to work with the concept or process themselves, she begins a joint activity. In the joint activity, the teacher conducts her own work with the concept or process alongside her students, who are free to watch her and ask questions as they pursue their own work. In this way, the teacher and students work together on their own products and are able to learn in both a collaborative and independent way. Joint activity allows the teacher to continually model how they would approach or solve problems.

In this module, the teacher will typically begin an activity using expert-apprentice modeling. For example, in Activity 3: Scientific Method and Carrying Capacity, students make their model kayaks with 100g of clay. As outlined in the activity, the teacher should take some clay and demonstrate for the students how to form one of the three basic types of boats (i.e., v-shaped, flat, or round), then distribute clay. They could then invite students who are ready to make their own boats, with the rest of the class watching. As these students work, the teacher can show how to refine their work using their own model and encourage other students to begin making their model boats when they are ready. When all students and the teachers are working on their boats, the class is engaged in joint productive activity. The teacher finishes his or her own boat design as students work on theirs and, while they work, the students are free to ask each other and the teacher questions. In this way, all students produce a model boat but can take advantage of their peers' and the teacher's expertise. This pedagogical approach is consistent with traditional teaching by elders and is aligned with the Yup'ik view that "None of us knows more than all of us."

Further, these pedagogical approaches connect directly to students' relative cognitive strengths. A number of studies show that northern indigenous people have strong spatial abilities related to living off the environment (Berry, 1966; Berry et al., 2002). Allen, Adams, & Lipka (2009) also found this to be true for Yup'ik students in classroom settings. In addition to working with students' cognitive strengths, the pedagogy of the module relates well to Sternberg's triarchic theory of intelligence and Gardner's theory of intelligence. In fact, MCC was involved in joint studies with Sternberg et al. (2006) and Grigorenko et al. (2006), all leading cross-cultural psychologists in cognition and schooling. Their theory and the evidence indicate that students learn more thoroughly and retain more knowledge when they are taught in a way that taps into their analytic, creative, and practical knowledge. Sternberg viewed the MCC series as an exemplar of their theory in action (personal communication to Lipka, 2002).

The number of culture-based math curricula is very small. In fact, as far as we know, the MCC series is the only Alaska-based elementary school (supplemental) math curricula developed and tested in Alaska. Nationally, MCC is one of a few math curricula that studies consistently show performs well. Mary Ann Zehr (2008) wrote an article regarding culture-based teaching:

> The IES [U.S. Department of Education's Institute of Educational Science] staff could point to only one group of researchers that the institute has funded to study the effects of cultural accommodations other than language on student achievement. That study focuses on math, not literacy. The institute supported a team led by Jerry Lipka, a professor of education in the Geography Department at the University of Alaska Fairbanks, to examine how a math curriculum based on the Yup'ik culture of Alaska affected math achievement among students of all cultures and backgrounds in selected Alaska schools.

In summary, the design and pedagogy of the module encourages students to be independent thinkers. Students will have multiple opportunities to observe characteristics such as speed and stability of the boat, experiment with the design of their model boats, and, in the concluding activity of the module, make changes to their boat design to optimize its performance. Students will use math notebooks to record and organize their conjectures, data, findings, and conclusions.

Integrated Curriculum

Although the kayak module addresses statistics, data collection and interpretation, prealgebra, and geometry, it also includes physics, social studies, geography and cartography, cultural studies, historical myths, literacy, and problem solving. Students will follow the scientific method in testing their boats and learn proper data handling techniques. They will learn to control for variables in order to isolate the one they want to study. They will learn several ways to display data in order to determine its meaning. Non-Yup'ik students will also have an opportunity to learn about the culture of another group of people. Yup'ik students can learn more about their traditional culture and read about mythic heroes as well as listen to or read traditional stories told by Yup'ik elders.

The Science Connection

National Science Standards *(National Research Council, 1996)*

Below is a list of the 6th grade science concepts addressed throughout this module and the individual activities that use or teach each of the standards.

Science Concept	Activity
Unifying concepts and processes in science	
Systems, order, and organization	2, 3, 4, 5, 6
Evidence, models, and explanation	2, 3, 4, 5, 6, 7
Change, constancy, and measurement	3, 4, 5, 6
Form and function	1, 2, 3, 4, 5, 6, 7
Science as inquiry	
Understanding of scientific concepts	2, 3, 4, 5, 6, 7
Skills necessary to become independent inquirers about the natural world	1, 2, 4, 7
Physical science	
Motions and forces	3, 5, 6
Science in personal and social perspectives	
Populations, resources, and environments	1, 7
Natural hazards	1, 7
Risks and benefits	1, 7
History and nature of science	
Science as a human endeavor	1, 7

Although this is a mathematics module, it is highly integrated with science, and literacy. The above section provide tools for identifying the standards or grade level equivalents of the module.

The Literacy Connection

Strand 1: Developing Literacy through a Math Notebook

What is a Math Notebook?

The purpose of math notebooks is to help students build conceptual content and math skills, while at the same time the notebook serves as a context for developing literacy—reading, writing, and vocabulary development—as well as listening and speaking. The math notebook can be used as part of any math program to record students' mathematical findings, understandings of mathematical processes and vocabulary, and questions and reflections on their experiences with mathematical concepts. Math notebooks can also be a reference that students can use during large or small group discussions about math. Likewise, math notebooks can provide a space for students to extend their individual learning by recording additional insights that may be developed through large- and small-group discussions.

The use of a math notebook and journaling has been found to increase students' learning. Therefore, on steps that require students to use their notebooks, we add a book icon to the step and ask teachers to make a point of having their students discuss the subject among themselves and write in their journals about their thoughts and the data.

How is a Math Notebook Different from a Math Journal?

In recent years, math journals have been incorporated into math instruction as a response to NCTM standards and testing requirements that focus on students' written explanations of their mathematical processes. However, math journals are often employed after a math activity, as a way for students to record their reflections about their learning. Typically, students respond to prompts that may be something like, "In math today, I learned …" In this context, math journals are not necessarily records of data gathered at the time of their mathematical explorations.

In contrast, math notebooks are tools your students can use

- to record what they already know before the hands-on mathematical problem-solving activity
- to record data as they are involved in the math activity
- to pose questions throughout the process
- for vocabulary development
- to reflect on their thinking and learning after the inquiry
- as a resource for large or small group discussions

What Kinds of Information Should I Ask Students to Record in Math Notebooks?

A teacher must first consider what information will be recorded by the students during each math activity. You may decide on a basic template and then add additional entry requirements, depending on the goals of your math lesson. For example, possible elements to include as part of the basic template might be:

- date
- materials (needed or used)
- topic of lesson
- goal for lesson
- how to design and construct tables and graphs

- how to interpret and analyze graphs
- how to apply basic statistics to understand data

A basic template with this information can provide your students with a point of reference to locate information for discussion groups and to review previous lessons.

There are several methods available for students to represent information presented in the math module activities. The processes (observing, collecting and displaying data, comparing, etc.) and specific tasks (testing the load, speed, and stability of different boats) suggest different ways information can be represented. Some of the methods your students can use with the MCC activities are:

- taking notes about the topic
- making lists
- creating charts, tables, and graphs
- writing down observations as they are doing the activities
- making technical drawings
- writing definitions of vocabulary words
- writing equations, proofs, and conjectures

It is important to provide instruction and modeling of each of these methods *before* you ask students to record entries in their math notebooks. Through modeling, students can become comfortable with each method and feel more confident to begin recording on their own.

Model and Practice

Before asking students to record any entries using these methods, model and practice each one as a large group as well as in small-group settings. Use expert-apprentice modeling and cognitive apprenticeship to show your students how you keep notes and how you will use them. Share your thinking processes, modeling cognitive behavior.

In MCC modules, there are learning goals for math lessons. As you plan the lesson from the module, take time to note the different kinds of explorations students will be involved in during the lesson. For example, students may be asked to draw different graphs or write an equation. These are data that can be recorded in the math notebook.

Strand 2: Developing Literacy Through Stories

In an effort to address literacy development and reading comprehension, which are important to all disciplines, MCC included in the module traditional Yup'ik stories and personal stories shared by elders. Anecdotal data gathered from related research projects about the implementation of MCC between 2000 and 2004 suggests that stories associated with traditional Yup'ik cultural and subsistence activities such as egging, included in *Going to Egg Island* (2003), and berry picking, included in *Picking Berries and Gathering Data* (2004), play an important role in students' engagement in the traditional storytelling genre as well as help students connect to the math activities and concepts presented in the modules.

It is important to recognize that stories in the oral tradition represent a well-developed literary genre that contains unique stylistic and rhetorical features and structures. These features and structures are comparable to the rhetorical devices found in Western literary genre (e.g., foreshadowing and repetition) and at the same time reflect devices like voice inflections, pacing, and gestures that are only available to the oral storyteller. Thus, because traditional oral storytelling is a literary genre, it seems important for children, regardless of linguistic, ethnic or cultural backgrounds, to experience this genre as part of the curriculum of school (Webster and Yanez, 2007).

The personal stories throughout the module and the optional reader, *Kukugyarpak,* have been created from transcriptions and translations of live storytelling events and represents a close adaptation to written text. We suggest you first read the stories aloud in their entirety. This will provide students with an experience that is similar to listening to a traditional oral story being told at a storytelling event. It will also give students a preview to the story, so that when they are asked to read it on their own or with a buddy, they will have had experience with the style and vocabulary of the story.

Kukugyarpak*: The Reader* (optional)

Accompanying the *Kayak Design: Scientific Method and Statistical Analysis* module is the traditional story *Kukugyarpak*. The story is told by Annie Blue, a renowned storyteller from Togiak, Alaska. The epic adventures of Kukugyarpk provide students and adults alike a rare glimpse into Yup'ik values: rules for living in a harsh environment by people who have uniquely adapted to the north.

The epic story *Kukugyarpak* can be read aloud to students throughout the module, either as a class or by using the accompanying CD which has both the Yup'ik and English versions of the story. This story provides students with a fantastic and epic journey full of fantasy, adventure, and strange people. Kukugyarpak, separated from his family and village while hunting, goes on an amazing voyage. His travels take him to places where he meets people with very big mouths, scary women, mouthless people, and giants. He encounters natural hazards, monsters, and magic. He helps the very strange people he meets and in turn is helped along his journey. Kukugyarpak is curious about the world he travels through and, although warned against going certain places, he goes there anyway, sometimes only narrowly escaping death. His adventurous journey is filled with peril.

Storytelling is a historical tradition in the Yup'ik cultural and is a natural way to pass cultural knowledge and values to the learner. This story and the personal kayak stories included in the module provide literacy, historical, and cultural connections about the kayak. Below is a suggested scheme for using *Kukugyarpak* throughout the module.

Kukugyarpak

Section Title	*Summary*	*Cultural Notes*	*Kayak Module*
Introduction Page 1–3	The young Yupik hunter Kukugyarpak goes hunting and starts his adventures.	Young men kayak together for safety; provide for and take care of your family	**Activity 1** Travel conditions. *Ingenuity of design; kayak is adaptable to environment*
The Grieving of Kukugyarpak Page 4–10	Kukugyarpak arrives at a village and is offered food with different weapons, which, if he chooses, is how he will die.	Respect the wisdom and knowledge of elders; respect spirituality about grieving; be helpful to others	
Teaching about Natural Childbirth Page 11–12	In the same village, Kukugyarpak teaches them about natural childbirth.	Be helpful to one another	

Kukugyarpak

Section Title	Summary	Cultural Notes	Kayak Module
The People with the Big Mouths Page 12–15	Kukugyarpak meets a woman and her son who have big mouths.	Be helpful to one another; have respect for the land and its resources.	**Activity 3** Storage of kayaks, seal oil, and tree sap for waterproofing; load of the seal: *ingenuity of design, stability with a load, using resources*
The Compassionate Woman Page 16–17	Kukugyarpak meets a compassionate woman	Accept help	
The Scary Women Page 17–20	Kukugyarpak, being very curious, peeks into the houses and sees women sleeping.	Have respect for the land	**Activity 5** Kayak gear and double-bladed paddles*: ingenuity of design*
The Island Page 21	Kukugyarpak travels on the ocean	Have respect for the land	Kayak travel and gear: *ingenuity of design, stability, load, speed*
Mouthless People Page 22–25	Kukugyarpak visits a village where people have no mouths.	Teach and learn outdoor survival skills and hunting, be helpful to one another	
The Big People Page 25–30	Traveling in his kayak, Kukugyarpak meets the Big People.	Teach and learn outdoor survival skills; be helpful to one another; share knowledge; accept help; respect spirituality; always be ready	**Activity 6** Kayak, round boat, and gear: *ingenuity of design, stability, using resources*
The Portal Page 31–32	The Big People tell Kukugyarpak that the round boat will save his life when he enters the Portal.	Be helpful to one another; accept help and always be ready	Round boat: *ingenuity of design, stability, load, using resources*
People (Creatures) with Tails Page 33–34	Kukugyarpak travels to a village where there are people with tails.	Respect the world of nature	
The Bearded Women Page 34–41	Kukugyarpak visits a big house where there are women with beards that kill men.	Provide for and take care of your family; be helpful to others	
Two Who Were Watching His Village Page 41–46	Kukugyarpak continues to travel toward the light. He visits a community house. Kukugyarpak continues his journey home.	Never give up; respect elders; respect spirituality; always be ready	**Activity 7** Versatility of kayak throughout journey, use of seal oil and tree sap for waterproofing: *ingenuity of design, stability, load, speed, using resources*
Going Home Page 47–49	Kukugyarpak finally returns home	Never give up	

Personal Kayak Stories

Personal and cultural stories have been interspersed throughout the module. For example, in the first activity we suggest having your students visualize what it might feel and sound like while traveling inside a kayak. Then by retelling elders' personal experiences of what it was like to ride lying down inside a kayak, students may gain further insights into those earlier times. Further, in the last activity we have a story told by Henry Alakayak, an elder from Manakotak, that connects all of the important features of the kayak that students have been studying: load capacity, speed, and stability, as well as versatility. Henry's and other elders' stories show the extraordinary ways in which elders used a variety of watercraft during their younger days. Personal experiences such as these throughout the module enrich students' learning by placing the modules activities in a larger cultural, historical, and personal context.

The stories are presented in the elders' voices to preserve their words, phrases, and ways of telling stories. These stories establish the context in which Yup'ik used different watercraft at different times of the year under widely different circumstances. They considered the environmental conditions, including the time of year, the purpose of travel, available resources, and personal knowledge about the duration of the trip and the number of people, animals, and cargo involved to decide which type of boat to use. As in the module, these factors were considered and the boat design corresponded to these conditions. In the classroom, you and your students will be given the same challenges faced by the elders—the need for stability, the need to carry goods and people, and the need for speed. Elders solved these problems and optimized their watercraft over the generations. Your students, in a "laboratory" setting, will be controlling these factors so that they can study one factor at a time. Each story will reveal the relationship of the form and function of the boat to the specific environmental conditions, Yup'ik ingenuity in using the available resources so there was no waste, the importance of balance, personal knowledge required to travel by boat, history and culture, and, most of all, the amazing versatility of some Yup'ik kayak that were able to travel across rivers, bays, and Cook Inlet.

Module CDs

The *Kayak Design: Scientific Method and Statistical Analysis* module includes two CDs that contain teacher resources, including:

- blackline masters found in the module by activity
- short instructional video on using a balance to discover math properties
- audio recordings of Evelyn Yanez and Dora Andrew-Ihrke reading the short stories found throughout the module
- Yup'ik interactive glossary

The optional reader, *Kukugyarpak*, also comes with a CD with audio recordings of Evelyn Yanez and Dora Andrew-Ihrke reading the *Kukugyarpak* story, both in Yup'ik and English respectively. These resources are for the teacher to use as they see fit in the classroom to provide depth and a deeper level of engagement for their students.

Developing Vocabulary

Throughout this module, an emphasis is placed on vocabulary development. Vocabulary development is a key factor in reading comprehension and in understanding math problems. In Thorndike's (1917) seminal study on reading comprehension, correct reading was described as occurring when (a) the correct meaning of each word was known, (b) the importance of the word and contextual meaning within the sentence was understood, and (c) the purpose and comprehension of the passage was examined and validated for understanding and adjustments were made when there was a breakdown in understanding. Biemiller (2001) suggested that when readers understand less than 95% of the words in a text, they will probably have difficulty comprehending the meaning of the text.

There are various approaches to vocabulary instruction. These include (a) incidental, developed through wide reading by the reader; (b) implicit, developing vocabulary through use of contextual clues within text by the reader; and (c) explicit, direct instruction. We found online mathematics dictionaries to be an excellent resource to ensure that the mathematical definitions used were accurate and precise (Eather, 2010).

Master Vocabulary List

Algorithm—mathematical process or formula used for calculation or problem-solving

Angle—the amount of rotation needed to bring one line into coincidence with another, generally measured in radians or in degrees

Area—the number of units that cover a surface

Bar graph—chart that uses rectangular bars to link a number to a category, such as a load value to the category of the boat that carried it

Body proportional measures—units of measure within body lengths that are generally the same for all people

Carrying capacity—the amount of weight placed in a boat that can be supported by that craft

Central tendencies—a number that in some way conveys the "center" or "middle" of a set of data; examples include the mean, median, and mode

Controlling for variables—holding one variable constant so that another variable can be measured

Conversion—changing one thing into another. Converting units from one measurement system (such as imperial units, body units, metric units) to another is done using the ratio between two measurement units

Data analysis—process of making sense of and drawing conclusions from data

Degree—unit used to measure angles; 360 degrees make a full circle

Equivalent—two values that are the same

Equation—mathematical statement containing an equals sign, to show that two expressions are equal

Inversely related variables—when the value of one variable goes up and the other goes down

Kayak—one of the traditional Yup'ik watercraft with a skin cover on a light framework, made watertight by flexible closure around the waist of the occupant and propelled with a paddle

Line plot—a line plot shows data on a number line with an X or other marks to show frequency

Load—weight placed in a vessel

Mean—sum of all data points divided by the number of points

Median—the value that is in the middle of an ordered data set

Mode—the most commonly occurring value or values in a set of data

Nonstandard measurement units—a customized unit of measure, e.g., traditional Yup'ik body measures or pattern blocks

Ordered pair—a pair of numbers used to locate a point on a coordinate plane, written in the form (x, y) where x is the x-coordinate and y is the y-coordinate

Optimization—to make as effective as possible

Outlier—a data point that is extreme compared to the rest of the data

Protractor—device used to measure angles in degrees

Range—difference between the smallest and largest data values

Ratio—the relationship between two units, expressed in one of three ways: a fraction (1/2), "1 to 2" or "1:2"

Relative speed performance—how fast or slow a boat moves in comparison to other boats

Reliable—if an experiment yields the same or very similar data when repeated

Scatter plot—a graph that displays how two variables in a data set are related

Scientific method—a series of steps for scientific research: (1) identify a problem you would like to solve, (2) formulate a hypothesis, (3) test the hypothesis, (4) collect and analyze the data, (5) make conclusions

Second—unit used to measure time

Speed—distance per unit time

Stability—tendency of a boat to return to level or neutral position after a roll or tilt

Standard measurement units—units of measure that are well known, standardized, and agreed upon, e.g., inches, centimeters, meters

Stem and leaf plot—a plot where each data value is split into a "leaf" (usually the last digit) and a "stem" (the other digits); for example "32" would be split into "3" (stem) and "2" (leaf)

Valid—results in an experiment that accurately represents the variable tested

Variable—something that can be changed or controlled in an experiment; also a letter used to represent an unknown quantity in a mathematical expression

Δ Notation—the Δ symbol is the Greek letter "delta" and is used to indicate change. The Δ symbol is often used by scientists and mathematicians to indicate the change of a measurement, variable, or other quantity

Master Vocabulary of Yup'ik Words

Aguun/Egelrun—open whaling boat

Angyaqatak—skin raft (translated as big, old boat, round bottom)

Anguarun—single-bladed paddle

Angyarrluk—log raft (flat bottom)

Iitaat—Alaska cotton grass

Ikuyegarneq—length from elbow to the end of your fist

Ikuyegarnerek malruk—the length between your elbows, when the fists meet in the middle of the chest

Kakaanaq—Yup'ik game

Kukugyarpak—name of the main character in the epic story, *Kukugyarpak*

Luqurun—special paddle, carved to be very thin so it wouldn't make noise in the water

Negcikcuar—boat or ice hook

Paangrun—two-bladed paddle

Qayaq—kayak (V-SHAPED bottom)

Qayaq/Malrugnek/Pailuni/Anguyacessuun—two-hole kayak

Tallim cuqii—length between fist and armpit

Tallinin—length between fist and armpit (same as *Tallim cuqii*)

Taluyaneq—length from the middle of the body to end of outstretched fingertips

Yagneq—length between the tips of your fingers on outstretched arms

Qaspeq—a one piece pull-over, lightweight clothing

Master Materials List

Package Includes

CD-ROM: Kayak Reference: blackline master, instructional video and short story audio recordings
CD-ROM: Yup'ik Glossary
Poster, Body Measurements
Poster, Kayak Measurements

Blackline Masters

Handout, Boat Type and Load Table
Handout, Circle for Protractor Construction
Handout, Kayak Body Measurement Table
Handout, Target for Central Tendency Game
Transparency, Boys in Kayak
Transparency, Building Kayak in Shop
Transparency, Hooper Bay Kayak and Sled
Transparency, Kayak (*Qayaq*)
Transparency, Kayak Hunter
Transparency, Log Raft (*Angyarrluk*)
Transparency, Map of Southwest Alaska
Transparency, Skin Raft (*Angyaqatak*)
Transparency, Whaling Boat (*Aguun/egelrun*)

Reader (optional)

Kukugyarpak
CD-ROM: *Kukugyarpak*

Teacher Provides

Blank paper
Butcher paper
Calculators
Clay
Construction paper
Fishing sinkers (1/8 oz.)
Graph paper
Heavy fishing sinker
Index cards
Markers/pens
Math notebooks
Overhead projector
Paper strips (uniform size)
Paperclips
Pattern blocks
Post-It easel and/or large/butcher paper
Protractors
Rulers (plastic is best)
Safety pins
Scissors
Small container for floating boats
Sponges or paper towels
Standardized weights
Stickers or Post-Its (multiple colors)
Stopwatches
Straws or popsicle sticks
String
Tape
Three-foot long container for boat speed test
Transparencies
Two-pan balance
Washers (two to three different weights)
Water
Waxed paper
Yard or meter stick or tape measure
Optional: cardboard, hole punch

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Zehr, M. A. (2008). Evidence on effect of culture-based teaching called thin. *Education Week, 27*(17), 8.

Photography Credits

Alaska State Library:
Hooper Bay School Shop 3/1942; Butler/Dale Photograph Collection, Alaska State Library, P306-0227.

Anchorage Museum of History and Art:
Eskimos in their kiak [sic], Alaska; O. D. Goetze, Goetze Collection, Anchorage Museum, B01.41.75.
Eskimo hunter with kayak and a seal; Lomen Brothers, Eide Collection, Anchorage Museum, B70.28.17.

Silas Tomaganak Hooper Bay. Personal photograph by David Zimmerly. 16 April 1978.

Resources for Teachers

Alaska's Digital Archives http://vilda.alaska.edu/
An online resource for finding images from Alaska.

Anchorage Museum at Rasmuson Center: http://www.anchoragemuseum.org/
The Anchorage Museum website can be used for online ordering for books and videos about kayaks.

Blue, A. (2009). *Kukugyarpak*. Calgary, Alberta, Canada: Detselig Enterprises, LTD.
An optional reader telling the traditional story of a young man on a journey, and is tied into the Kayak module.

National Council of Teachers of Mathematics (NCTM) website: http://nctm.org/
The NCTM website has resources for classrooms and teachers for all areas of math.

Qayaqs and Canoes: Native Ways of Knowing. VHS, Alaska Native Heritage Center.
This 56 minute video shares the construction of and launching of kayaks made by five major Alaska Native cultures.

Steinbright, J. (2001). *Qayaqs & Canoes Native Ways of Knowing* Alaska Native Heritage Center.
A book discussing kayaks and other watercraft from native groups.

Vocabulary dictionary: http://www.amathsdictionaryforkids.com/
A user-friendly online dictionary for math terms.

Yuungnaqpiallerput/The Way We Genuinely Live: Masterworks of Yup'ik Science and Survival. DVD, Anchorage Museum at Rasmuson Center, 2008.
This 33 minute video shares how kayaks were made, along with other traditional watercraft.

Activities

Kayak Design

Activity 1: Kayak Design and Use in Southwest Alaska

The kayak represents the ingenuity and creativity of indigenous people. This watercraft is capable of navigating everything from rough ocean swells to swift-moving rivers. Its ingenuity is recognized by people all over the world who use kayaks for recreational purposes. The marvelous kayak design reflects balance and stability because of the relationship between its form and function. Building model kayaks offers students wonderful opportunities to test and record its performance under varying conditions.

This activity introduces students to the kayak that was used by the Yup'ik and other coastal indigenous groups for hunting and travelling. Kayak builders had to keep the craft's many functions in mind as they constructed a well-balanced kayak. The kayak needed to be sturdy for rough waters and light enough for portaging, it had to carry a lot of weight, be stable and relatively fast when both lightly and heavily loaded, and it needed to be made and maintained out of locally available materials. In order for a boat to perform safely and adequately, you must consider its purpose and the environment it is being used in.

The practical issues of designing and building a kayak (*qayaq*) or skin raft (*angyaqatak*) connect students quite directly to aspects of the Yup'ik culture and many mathematical concepts. They will learn about the concepts of design, environment, and body proportional measures. What size kayak is right for the user? Yup'ik people solve this problem by using an agreed-upon set of body proportional measures for the length and width of the kayak. These measurements, compared to the paddler's height, form ratios that produce a well-balanced, custom-made kayak. In this activity students will consider the form and function of Yup'ik kayaks and other boat forms. Students will learn to appreciate the complexity of a kayak's construction to create balanced kayaks by using body measurements and ratios. Make sure that students understand that although each of them has different body length measures, the relationship between kayak length and kayak width results in a ratio.

Teacher Note

Students may use calculators throughout this module.

Goals

- To increase cultural knowledge about kayaks and the design criteria used in developing kayaks
- To understand that ratio is a relationship between two quantities
- To use ratios to find unknown values

- To understand the value of and use of Yup'ik body proportional measurements

Materials

- CD: Kayak Reference (can be used instead of making transparencies of Blackline Masters)
- Math notebooks
- Poster, Body Measurements
- Poster, Kayak Measurements
- Scissors
- Storybook, *Kukugyarpak* (optional)
- Storybook, *Big Henry Little Joh*n (optional)
- String (a nonstretchy string works best)
- Transparency, Boys in Kayak
- Transparency, Building Kayak in Shop
- Transparency, Hooper Bay Kayak and Sled
- Transparency, Kayak (*Qayaq*)
- Transparency, Kayak Hunter
- Transparency, Log Raft *(Angyarrluk*)
- Transparency, Map of Southwest Alaska
- Transparency, Skin Boat (*Angyaqatak*)
- Transparency, Whaling Boat (*Aguun*/*Egelrun)*
- Worksheet, Kayak Body Measurement Table
- Yardsticks or measuring tape

Resources

- A story about friends building kayaks for each other.
  Myers, Seth. (2004). *Big Henry Little John*. Calgary, Canada: Detselig Enterprises Ltd.
- Information about making sea kayaks and other boats, including pictures. http://www.guillemot-kayaks.com/Building/Building.html
- A website dedicated to the preservation of Yupik culture, and specific information about the kayak. http://qayanek.com/kwig/
- An index of images of kayaks and associated watercraft. http://www.arctickayaks.com/images/
- The optional reader, *Kukugyarpak*, about the epic journey of a Yup'ik man
  Blue, A. (2009). *Kukugyarpak*. Calgary, Alberta, Canada: Detselig Enterprises, LTD.

Vocabulary

Body proportional measures—relationships between specific body lengths that are generally the same for all people

Conversion factor—ratio between two standard measurement units

Kayak—traditional Yup'ik watercraft with a watertight skin cover on a light wooden framework

Nonstandard measurement units—units that are not nationally or internationally recognized such as traditional Yup'ik body measures

Optimization—to make as effective as possible

Ratio—the relationship between two units, expressed in one of three ways: a fraction (1/2), "1 to 2" or "1:2"

Aguun/Egelrun—open whaling boat

Angyaqatak—skin raft (translated as big, old boat, round bottom)

Angyarrluk—log raft (flat bottom)

Ikuyegarnerek malruk—the length between your elbows, when the fists meet in the middle of the chest

Qaspeq—*a one piece, pull-over, lightweight clothing*

Qayaq—kayak, see definition above (V-shaped bottom)

Tallim cuqii—length between fist and armpit

Taluyaneq—length from the middle of the body to end of outstretched fingertips

Yagneq—length between the tips of your fingers on outstretched arms

This symbol means students need their math notebooks.

Preparation

- Read the introduction to the module to provide cultural background on the boat types and functions used by the Yup'ik people.
- Make transparencies of black line masters or use a projector to show the images from the Kayak Reference CD.
- Decide if you will use the optional activity at the end of Activity 1.
- If you read *Kukugyarpak*, you can read pages 1 through 3 which have a close tie to Activity 1, or listen to the audio recording on the CD.

An audio recording of the optional reader can be found on the *Kukugyarpak* CD.

Duration

Three to four class periods.

Instructions

1. **Overview of Module:** Explain to the students that during the next few weeks they will each have a chance to build and test model boats using the scientific method. Share with them that they will be learning how to

perform experiments with the boats to test them for different functions, and at the end of the module they will be given the chance to modify their boats to make them out-perform their initial boats.

Teacher Note

This is a good place to include the Cultural Writing activity listed on the next page.

2. **Introduction:** Ask students to share what they know about boats and boat design. Explain that the Yup'ik people used several different types of boats for travel and hunting. Introduce the whole class to the log rafts (*angyarrluk*), the skin boats (*angyaqatak*), whaling boats (*aguun/ egelrun*), and kayaks (*qayaq)* using the blackline masters at the end of the activity.

3. Introduce students to the concept of a math notebook. Explain that they will be using their notebooks to record information they learn and how to organize data on boats, and eventually on how their own boats perform and ways to improve their boats.

4. The Yup'ik people used kayaks in rivers and up and down the coast of southwest Alaska. Show the transparency, Map of Southwest Alaska. While looking at the transparency of the map, read the following scenario to students while they visualize it in their minds:

> You're on the edge of the Bering Sea. It's April, cold and windy, and there is lots of ice in the water, but there are also lots of fat seals. These seals include bearded seals, which can weigh 600 pounds, but they are too far out to sea to reach them from the shore. You are hungry and there isn't other game or fish. To feed yourself and your family—to survive—you need to be able to go out on the ocean where the seals are, harpoon one without tipping over, and bring it back without sinking.

5. **Discussion:** Start a discussion with the class, asking students to name one thing their boat must be able to do based on the scenario you have read. Have a few students share ideas on the characteristics a boat might need to accomplish this. Draw on the previous discussion about boats and their functions as well as students' knowledge of boats and environmental factors that affect the types of boats that can be used. Guide the discussion to the idea of form and function by referring to information in the Introduction to deepen the discussion.

Teacher Note

Students might say that the boat has to be stable and not tip over when you thrust the harpoon into the seal. If possible, push the discussion further so that students share characteristics of boats that are stable, such as a flat or round bottom, based on experience they've had with boats. See figure 1.2 and 1.3.

Science Note

This discussion offers a great opportunity to tie in other **science concepts**: friction, ballast, stability, aerodynamics, center of gravity, lines of symmetry, etc.

Cultural Connections Through Writing (optional activity)

Below is a passage explaining how elders used to travel in kayaks. Read it aloud to your students while they close their eyes and visualize the experience. Have them imagine that they are living in a different time. It is a time without motorboats or airplanes, and their family is traveling to the next village. Here is how Annie Blue, an elder for Togiak, described the scene:

> When traveling they would have two small children go in the back side of the kayak and another two small children to the front. But the parents sat back to back at the opening of the kayak. While they rowed you could see the current from the inside of the kayak and when we got to wherever we were going, we'd get out of the kayak. (Personal communication with Annie Blue, in Manokotak, Alaska 2003)

Have your students do a descriptive writing sample in which they imagine what the journey would be like if they traveled by kayak. What would it feel like to be inside a kayak? What does it sound like? What does it look like? What might it smell like?

Fig. 1.1: How children traveled inside the kayak.

After students write their ideas, have them share, and then read the *Personal Kayak Accounts* of how the elders felt about being inside a kayak as kids. Ask students to compare their imaginations to how Yup'ik elders felt about their experiences. Note that the elders were not afraid of traveling in a kayak despite not knowing how to swim because they placed their trust in the construction of the kayak.

Personal Kayak Accounts

In May 2003, Mike Toyukak, a consultant, and elders George Moses, Mary Active, Annie Blue, and Jerry Lipka (author) met in Manokotak, Alaska, to discuss experience with traditional Yup'ik watercraft. Kayaks (*qayaq*) were used for travel and hunting. All those who experienced being inside the kayak as kids said it was not scary but that it was enjoyable. They could see the water along the bottom of the kayak and hear the water.

- Mike said it was like music to the ears and helped him go to sleep.
- Annie said that the kayaks were smeared with oil, which made it hard to see through from the inside.
- Mary said that she could still see the water from the inside and she also said that it made her very sleepy.
- Annie agreed that she could still see the water and the current.
- Mike said that they didn't keep the kayaks too long. The ribs of the kayak had to be changed before they became too weak.
- Mary said that the skin of the kayak began to darken after a year.

6. Divide the class into small groups and have them draw a picture of a boat that would work for the scenario you read. Their entry should include an explanation of the boat's features and how they serve the function of the boat. See student examples in figures 1.2 and 1.3.

7. **Share:** Have volunteers share their boat designs and ask students to explain why their features would help in the situation described. Ask them how they resolved the problem when there was a conflict between the different needs of the boat (e.g., they wanted a boat that is fast enough to sneak up on a seal while at the same time wide enough to hold the seal meat).

Math Note

These resolutions are the beginning of the mathematical process of optimization, in which different variables are balanced to find the optimal solution.

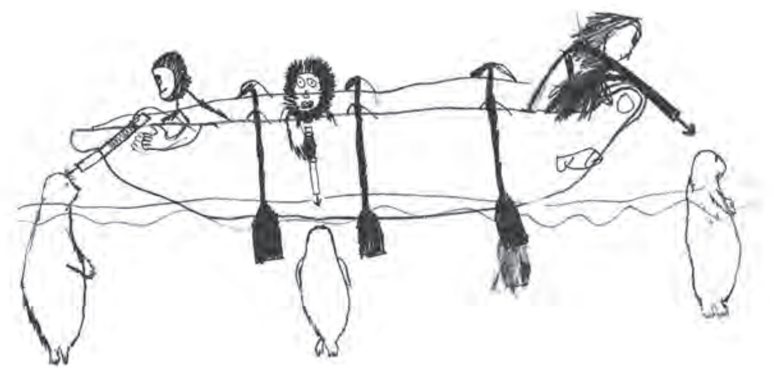


Fig. 1.2: A student diagram representating the boat in the scenario. Togiak, Alaska.

8. Show the pictures of the different kayaks from the blackline masters (Kayak Hunter, Boys in Kayak, Building Kayak in Shop, Hooper Bay Kayak and Sled). Explain that each kayak was designed (optimized) to fit particular conditions. Ask students to compare these kayaks to the boats they designed and to make conjectures about what conditions each boat might do well in.

This would be a good place to stop for the day.

9. Connect back to yesterday's activity when they looked at the characteristics and designs of boats, and as a group recall the three main boat types (kayak, skin boat, and log raft). Have students draw an example of each type of boat and record the form and function of each boat type. This could include boat characteristics, the environmental factors that affect their use, and what they are used for.

10. Share *Annie Blue's Kayak Story,* describing how kayaks were made, who was involved, and how important they were to the Yup'ik way of life.

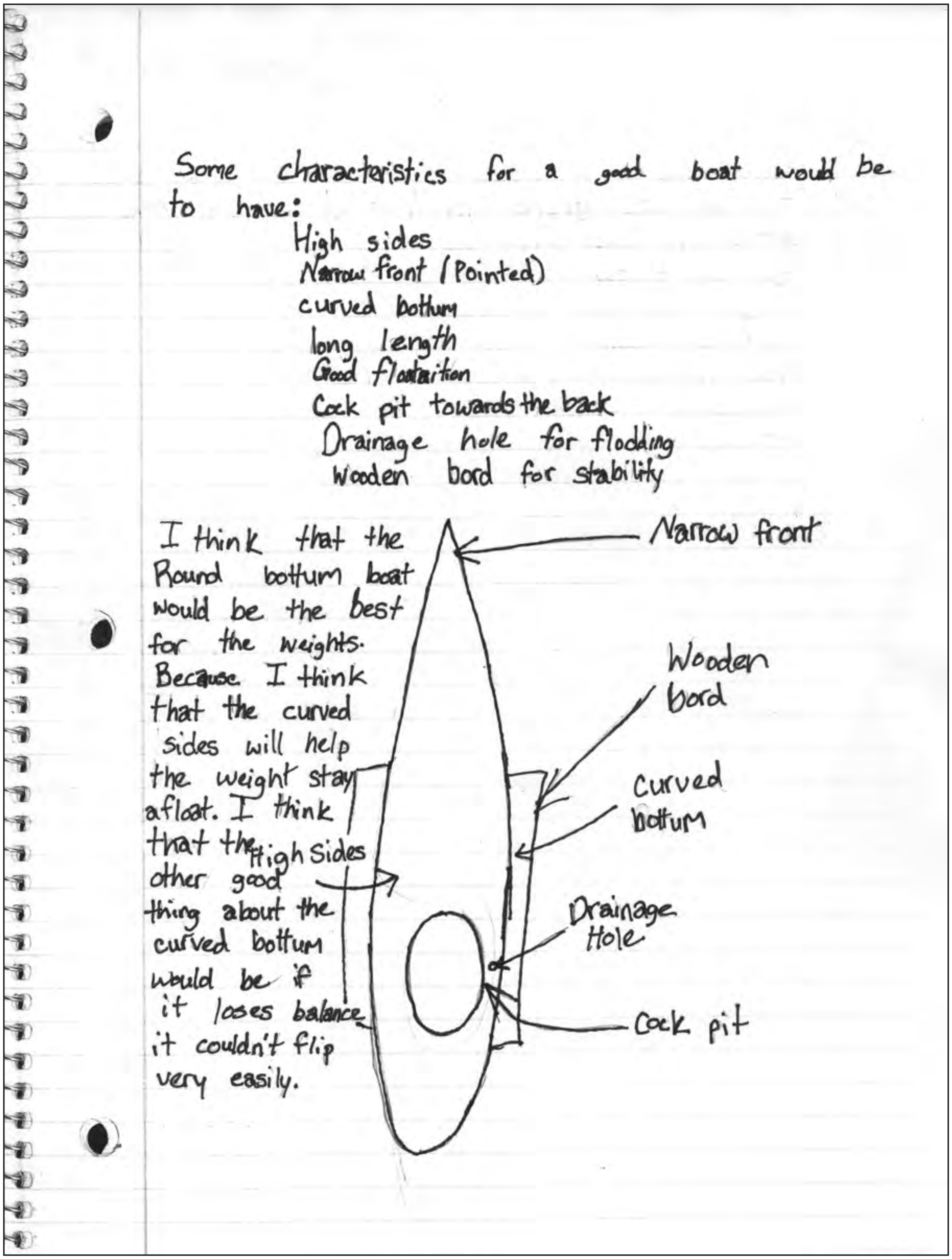


Fig. 1.3: A student notebook entry representing a boat of their own design. Fairbanks, Alaska.

Annie Blue's Kayak Story

Translated by Eliza Orr and Evelyn Yanez

People couldn't be without their kayaks back then. When they made kayaks, with skin like this, they would bring the skin into the house and I would be among the people making the kayak skin. To make thread, the women and the old women would make their thread by braiding caribou sinew. When they finish braiding it, the thread for the kayak, they would stretch it and not cut it into pieces. When it is done, it is ready to be used on a kayak. Although the kayaks are long they were able to sew for a long time because of the way they stretched the sinew.

When they finish getting the skin ready for the kayak, they would bring it to the community house. And since I would not allow myself to be left behind, we, the girls, usually went along to watch when they would then slip it onto the kayak. They would tie the end, tote-hole of the kayak, when they were about to slip the skin on. Finally they would take a hold of the end of the kayak and somehow slide the skin on, pulling with their hands. And when that is done they would then start sewing it until it was done.

When the sewing was done they would then bring the kayak down to where they had set up two sets of kayak posts to hang the kayak on and set it on top of that and make thick pitch to cover the stitches with. They probably had big stitches as they sewed, but they finished them in good condition. When it's finally done and ready, they take it onto the water and use it to go upriver and to move camp. At springtime after being in a camp for a long time, they would get ready to go home by putting two kayaks side by side, put something on top of those kayaks so the people and their belongings could be on top of it as they move camp, and they got to their destination in that way.

An audio clip of this story can be found on the *Kayak* CD.

That's what they did with the kayaks. They were needed very much, and couldn't be without them. Those without kayaks couldn't do anything. Even if they wanted to cross over to the other side of the river, they couldn't. That is the end about kayaks. [This is how Annie ended the story.]

Body Proportional Measuring and Ratios

11. Explain how successful the Yup'ik people were in making kayaks: people all over the world use Yup'ik designs. Ask students how they think Yup'ik people designed kayaks. How wide were they? How long were they? What tools did they use to measure their kayaks? What would happen if the kayak was too big for you? What if it was too small? Lead the discussion to the idea that kayaks needed to fit the size of a person, otherwise they wouldn't be able to use them effectively. The Yup'ik people use a specific relationship (ratio) between the paddler's height and the kayak length and width to make it fit perfectly. Share that

the Yup'ik people used body measurements to make kayaks. Explain that today the students will be using their own bodies to determine the size of a kayak that would fit their bodies. Differentiate between body measurements and ratios: the former is a length, and the latter is a comparison of lengths, in this situation.

12. **Introduce Body Measurements:** Ask students if they have ever worked with body measures. Note their ideas on the board (e.g., in soccer when players walk to measure where a penalty kick would occur, measuring rope using an arm span as a unit of measure, seeing their mother sew, using her arm span to measure cloth for a lightweight pull-over top (*qaspeq*). Tell students that these are called nonstandard units and then ask students why nonstandard units are useful (e.g., portable, quick, customized, accurate etc.).

13. Hang the Body Measurements poster for reference. Explain that in the Yup'ik culture, there are many different body lengths used for measuring.

14. Demonstrate the "outstretched arms from fingertips" (*yagneq*). Have a volunteer stand up and make this same measurement. Ask students to observe, compare, and briefly discuss. Using string, have students work in pairs to find the following measurements on their own bodies: the length of outstretched arms from fingertips (*yagneq)*, the length from fist to armpit (*tallim cuqii)*, the length from middle of body to end of fingertips (*taluyaneq)*, the distance of two elbow lengths (*ikuyegarnerek malruk)* made by putting the fists together, and their height. Students should label the strings for future reference with their name and the body unit measured. **(An optional activity, found at the end of this activity, can be inserted here.)**

15. Ask students to find relationships between these five body measurements using their strings. Have a brief discussion about the relationships, explaining that these represent ratios: a comparison of two values. Have students record the ratios they found in their notebooks (e.g., 2 *taluyaneq* to 1 *yagneq)* and on a poster for reference.

16. Share and post the Kayak Measurements poster explaining how these specific body measurements were used in determining the dimensions of a kayak (Figure 1.4).

17. Hand out the Kayak Body Measurement Table to each student and explain that they will now measure their strings in inches and record their values. Then they will find the kayak length (L) by adding the four measurements. See Figure 1.5 for an example of a table with data. Note there are two *yagneq* values on the table to account for the two required in the total kayak length.

Teacher Note

Traditional Yup'ik kayaks are made for a person using her or his own body measures. This ensures that the kayak will "fit" that person and that he or she can effectively pilot and use it. If the kayak is too small for a person, they won't be able to comfortably fit into it, and if it is too big they won't be able to pilot it well. The story Big Henry Little John, by Seth Myers (2004), depicts this very problem.

Kayak Measurements

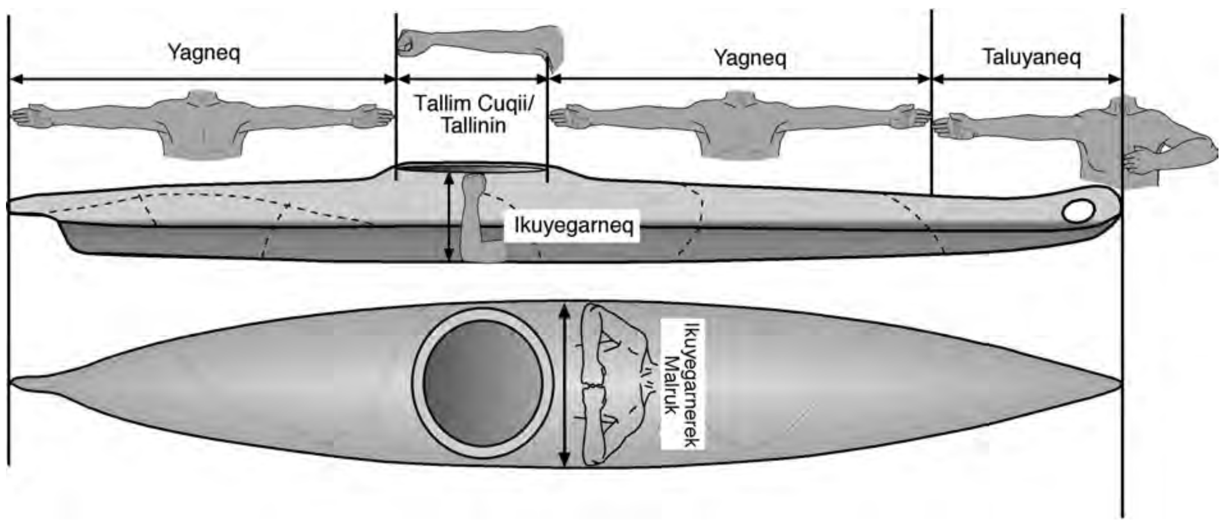


Fig. 1.4: Top view of body measures used to construct a Yup'ik kayak (right end is front of kayak).

- The width of kayak is "two elbow lengths" (*ikuyegarnerek malruk)* made by putting the fists together at the center of the chest.
- The diameter of the cockpit opening is "from fist to armpit" (*tallim cuqii*/*tallinin)*, made by making a fist and extending the arm.
- The "length of outstretched arms from fingertips" (*yagneq*) is made by extending the arms from the body. Note that a *yagneq* is about the same length as a person's height.
- The last measurement for the kayak length is from finger tip to chest center (*taluyaneq*).

As the above figure shows, two *yagneq*, one *tallim cuqii*/*tallinin*, and one *taluyaneq* are used to make the total length of the kayak. These measures also identify the position of the cockpit within the kayak.

Body Measurement - Length (in.) *Example*			
	Nicolle	**Jerry**	**Karen**
yagneq	67 in.	74.5 in.	65.5 in.
yagneq	67 in.	74.5 in.	65.5 in.
tallinin	24 in.	25 in.	23 in.
taluyaneq	33.5 in.	37.25 in.	32.75 in.
Total Kayak Length (L)	**191.5 in.**	**211.25 in.**	**186.75 in.**
Height of Paddler (H)	68 in.	75 in.	68 in.
Ratio (L/H)	191.5/68 = **2.816**	211.25/75 = **2.816**	186.75 = **2.75**

Body Measurement - Length (in.) *Example*			
	Nicolle	**Jerry**	**Karen**
Total Kayak Length (L)	**191.5 in.**	**211.25 in.**	**186.75 in.**
***ikugarnek malruk* Kayak Width (W)**	27.5 in.	30.35 in.	26.83 in.
Ratio (L/H)	191.5/27.5 = **6.96**	211.25/30.35 = **6.96**	186.75/26.83 = **6.96**

Fig. 1.5: Example of body measurement tables with data.

18. **Explore Ratios:** Now that students know the length of their own kayak, have each student cut a new string to represent the length of their kayak (L). Ask the students, "Do you think the smallest student and largest student will have the same ratios?" Now have students find ratios between their kayak length (L) and the body measurement strings they have.

19. **Ratios of Kayak Measurements:** Have students share ratios they found and see if students find any patterns between their answers. Make sure students are using the same two measurements when comparing their ratios. Ask students what having the same ratio means. Why is this important? Explain that the Yup'ik people knew that to have a balanced kayak for the paddler, the kayak needed to be proportional to the paddler's body so that it was easily handled.

20. **Calculate Ratio:** Ask students if they can figure out how close to 3 the ratio between paddler height and kayak length is. Ask them if they can get more accurate. Guide students to calculate their ratio by dividing the kayak length by paddler height (L/H) with or without a calculator, rounding their answers to tenths. Students should record their value on the Kayak Body Measurement Table handout. Have all students put

Ratios are generalizable

Math Note

A **ratio** is the relationship between two quantities and is only meaningful if the two referents are known. In this case a kayak length and the paddler's height are the units. The ratio between paddler height and kayak length can help us find the best length of this kayak style in a mathematical way by just knowing the height of the paddler. This is because the approximate kayak length is found by multiplying the paddler height by 2.8. Now students can find the dimension of a kayak mathematically. This is because while Yup'ik body measures will have different measures in standard units, the ratio of those body measurements is the same.

Building a foundation for algebraic thinking.

Teacher Note

Students can apply the concept of **ratios to everyday items**. An example of a ratio people are familiar with includes the cost of gas by the gallon ($/gallon). Jack pays $25 for 5 gallons of gas. How much did he pay for a gallon? Allison pays $20 for 4 gallons of gas. How much did she pay for a gallon? Each paid $5 per gallon. But they each had a different price and a different amount of gas. $5 per gallon is the ratio between price and gallon regardless of the different amount spent and the quantities purchased.

their answers on the board. (Depending on the level of accuracy in your students' measurements with the strings, you should find that most of the ratios are close to 2.8 or 2.9.) See Figure 1.6 for a visual reference of the value.

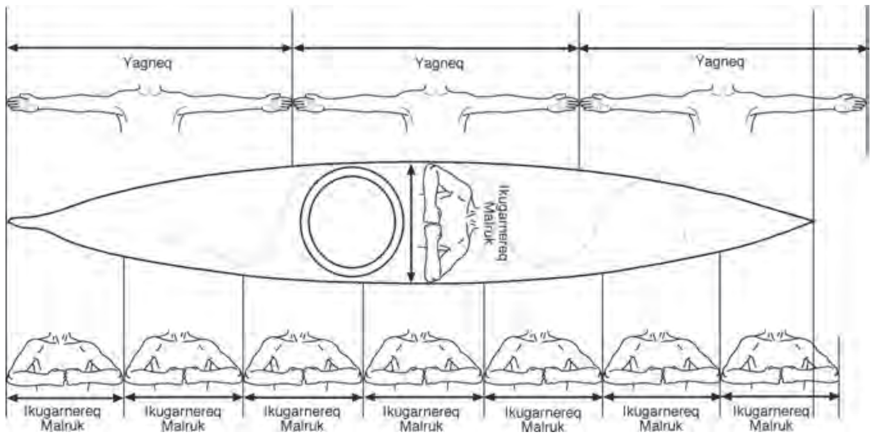


Fig. 1.6: Visual representation of the ratio values.

21. **Discuss Ratio:** Looking at all of the data, ask students if they notice any patterns. Have students explain their understanding of what 2.8 means and how it can be used (they could say that the ratio shows that their kayak length equals 2.8 times their height). Have students explain this value in a sentence. For example: "My kayak length equals 2.8 times my height." Ask students how they might convert this to a math sentence. Guide the students toward writing the equation: $L = 2.8 \times H$.

22. Now, use the same process with the kayak length and kayak width (L/W). Have students record their answers on the board and in their tables. Repeat the process by having students discuss any patterns they see in their results and how to write the relationship in a mathematical form: $L = 6.8 \times H$. (Depending on the accuracy of measurements, the ratio should be roughly 6.8 or 6.9). See Figure 1.6 for a visual reference of the value.

23. Now challenge students to find the length of a kayak for a paddler who is 72 inches tall. Facilitate a discussion using methods the students come up with. (The answer will be 201.6 inches.) Continue with more examples if your students would benefit from the practice.

24. To summarize the activity and discussion, ask students to explain in their notebooks why kayak measurements were based on a person's body measurements. Have them include how they could determine the length and width of a kayak for someone they know. (For example: You would find the length of someone's kayak by multiplying their height by 2.8.)

Optional Activity: Building a Model Kayak Outline

In this optional activity, students have a chance to physically model the size of a kayak that would be proportional to their own bodies.

Goals

- To use the ratio from the previous activity to estimate the length of butcher paper needed for their kayak outline
- To visualize how the height of a person affects the length of a kayak and make comparisons between kayak outlines

Materials

- Butcher paper for each student or group
- Handout, Kayak Body Measurement Table from Activity 1
- Markers
- String lengths from Activity 1

Preparation

- Butcher paper long enough for a kayak outline for each student in the class.
- For making the outline of the kayak, students will need a large open space, such as the gym or cafeteria floor.

Duration

One class period.

Instructions

1. **Model:** Model for the class how to use your measurement strings and the picture on the Kayak Body Measurement Table to mark an outline of a kayak. This could be a scaled version (scaling is an application of a ratio) or the full model.

2. Have students estimate how long their butcher paper needs to be. Ask your class, whose kayak is going to be the largest? Whose will be the smallest? How do they know? A reminder about the previous activity with ratios may help in their problem solving. Help your students get the butcher paper for their outlines based on their estimates.

Teacher Note

Depending on time and space, allow each student to draw their own kayak outline, or have students work in small groups by picking one group member's kayak to outline.

Cultural Note

The following information from George Moses, an elder from Akiachak, emphasizes the importance of Yup'ik body proportionality, balance, and ratios.

George Moses' father bought two kayaks from someone who lived down by the ocean, one for him [George] and one for his brother. His brother wanted the better-made kayak, but it was too small for him; it didn't fit. The other kayak was too big for George. The better one was only a little too big for him. So George got the better-made kayak, and his brother took the one that fit him. Because George's kayak was still too big for him, they rolled up a sleeping bag to put in with him. Paddling was harder for him than for his brother because the kayak was not balanced with his body.

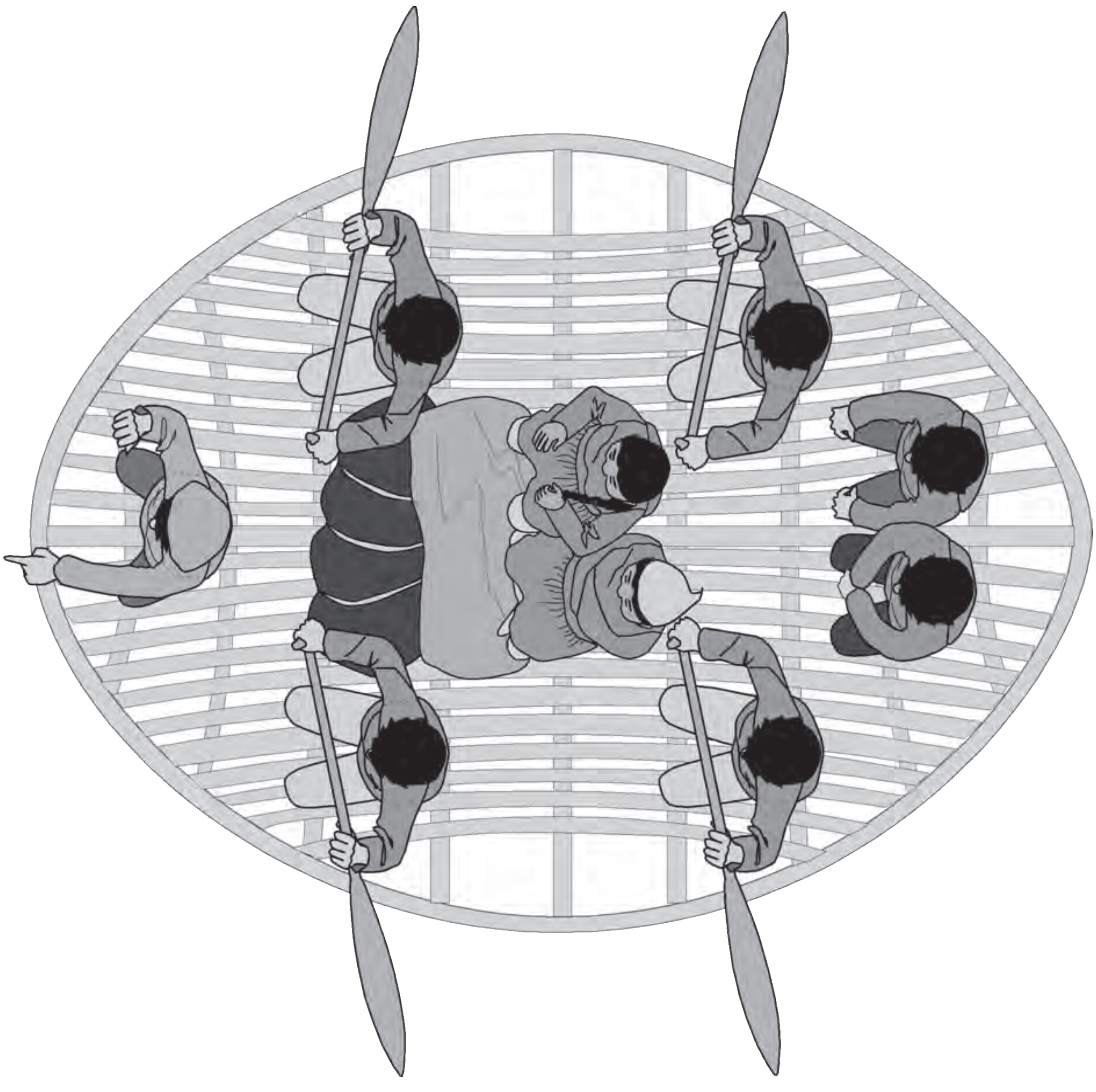
3. Students will need to collect their strings, a marker, Kayak Body Measurement Table, and their butcher paper to begin working on the outline of the kayaks. Circulate among the pairs of students to make sure students are using the correct body measures and build student autonomy by having students help each other complete the task.

4. Once the kayak outlines are complete, have students circulate around the room and see others' actual-sized kayak outlines. Discuss differences and similarities they see and why these differences exist. Remind them to consider the ratio they worked with previously.

5. Display the kayaks for the class and others to see (Figure 1.7).

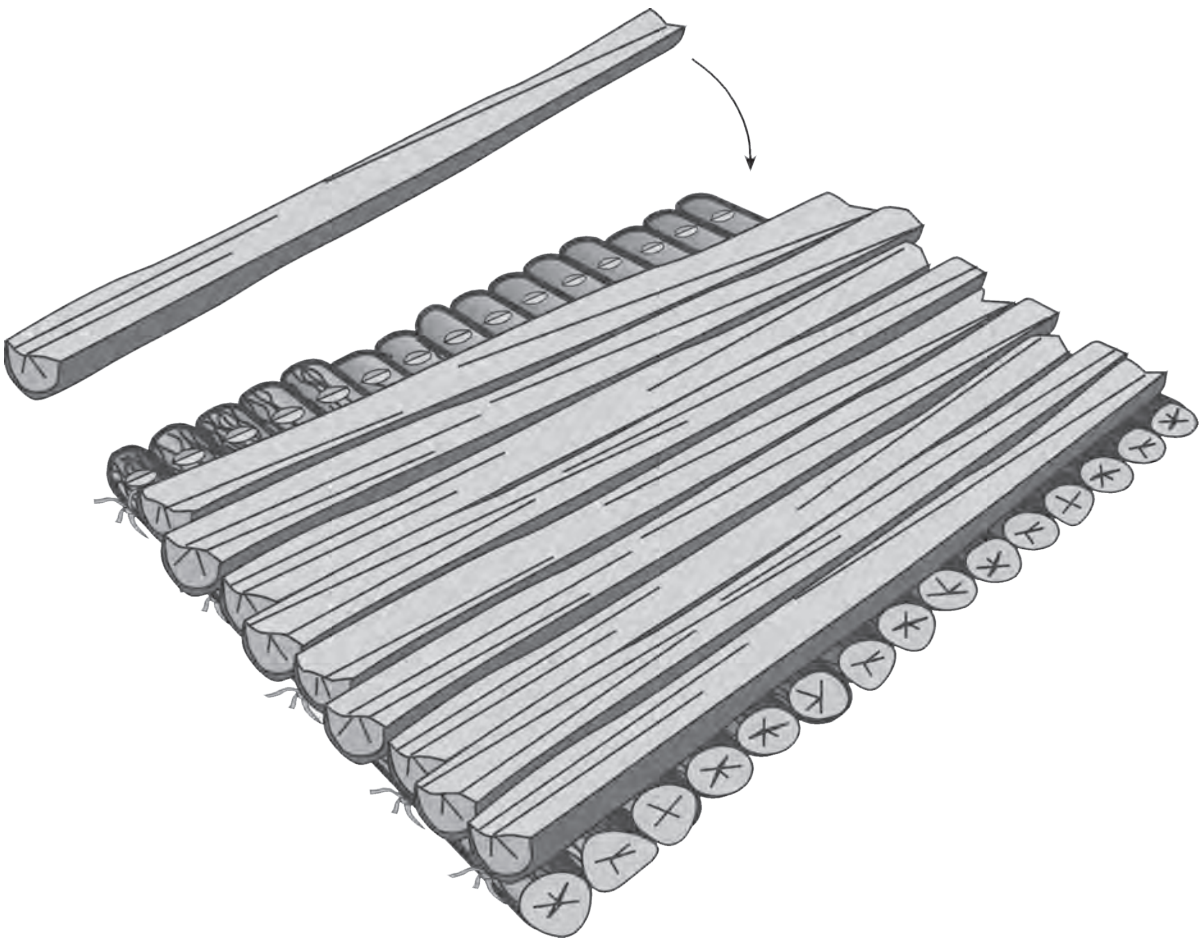
Fig. 1.7: Examples of kayak outlines made by students in Togiak, Alaska.

Skin Raft (*Angyaqatak*)



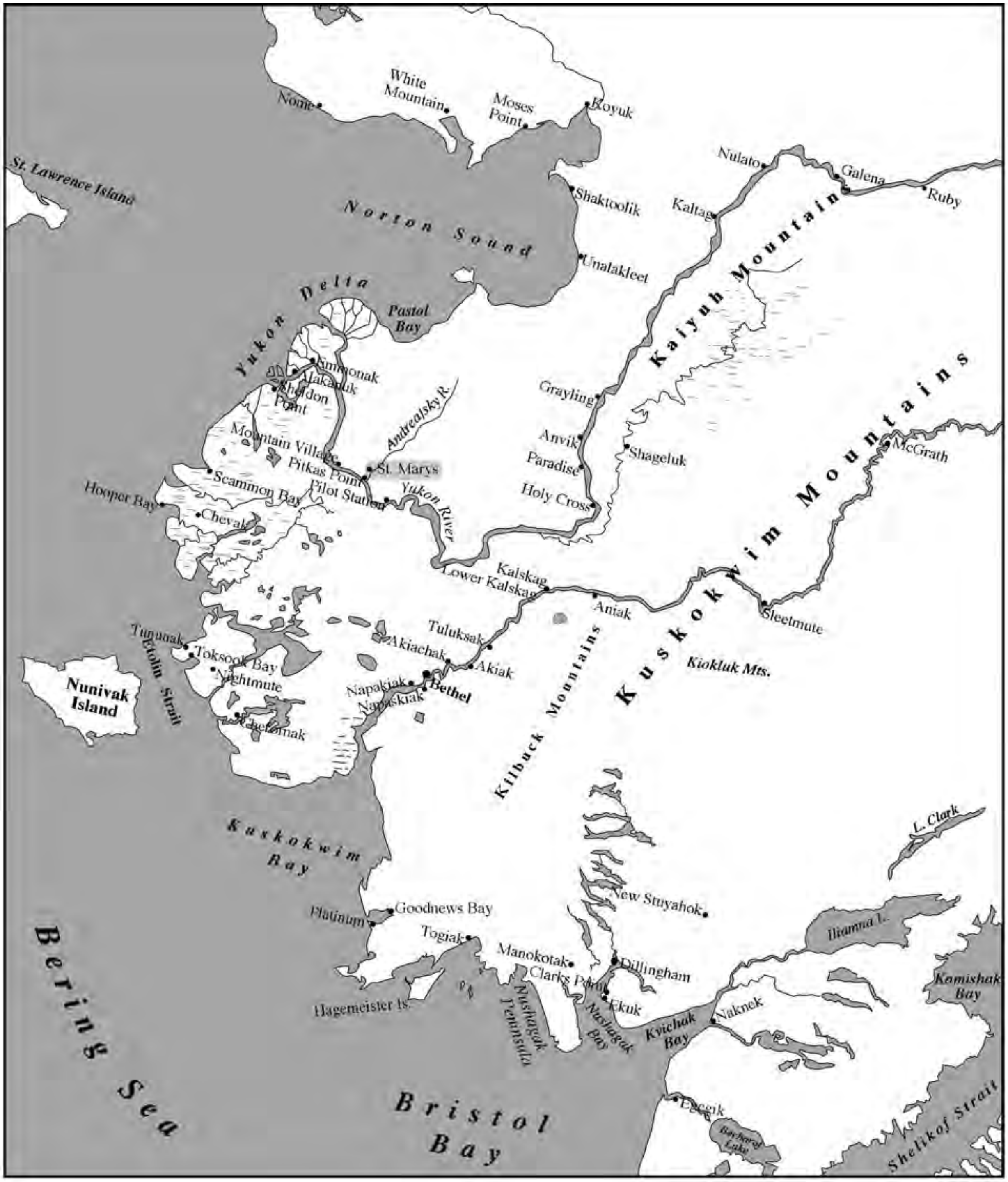
Whaling Boat (*Aguun/Egelrun*)

Log Raft (*Angyarrluk*)



Kayak (*Qayaq*)

Map of Southwest Alaska



Kayak Hunter

Eskimo hunter with kayak and a seal.
Lomen Brothers, Eide Collection; Anchorage Museum, B70.28.17.

Boys in Kayak

Eskimos in their kiak [sic], Alaska.
O. D. Goetze, Goetze Collection; Anchorage Museum, B01.41.75.

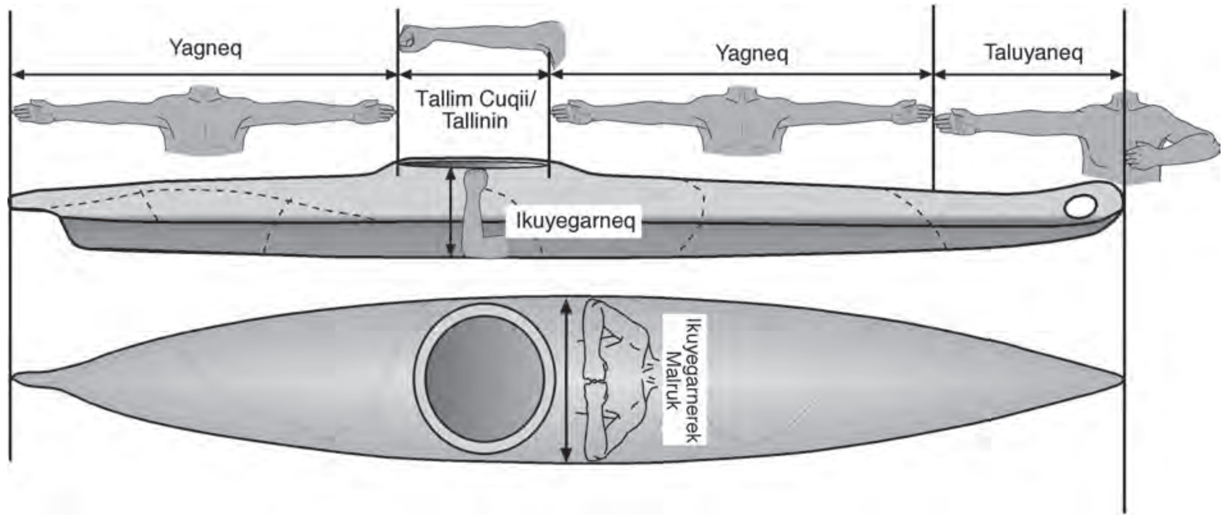
Hooper Bay Kayak and Sled

Silas Tomaganak Hooper Bay.
Personal photograph by David Zimmerly. 16 April 1978

Building Kayak in Shop

Hooper Bay School Shop 3/1942.
Alaska State Library, Butler/Dale Photograph Collection, P306-0227.

Kayak Body Measurement Table



Body Measurement - Length (in.)	
Name	
yagneq	
yagneq	
tallinin	
taluyaneq	
Total Kayak Length (L)	
Height of Paddler (H)	
Ratio (L/H)	

Body Measurement - Length (in.)	
Name	
Total Kayak Length (L)	
ikugarnek malruk **Kayak Width (W)**	
Ratio (L/H)	

Activity 2: Making a Weight Set

In this lesson and throughout the module, students will investigate boat performance through hands-on explorations, using the scientific method coupled with data collection and analysis. They explore through a series of tests how their boat performs regarding load, speed, and stability. Students will conduct these tests in three activities, but in order to explore in a systematic way, they will need to have tools to accurately measure differences in performance. One tool that they need is a weight set. Because classroom sets of standardized weights are not always available, students will use the concept of ratios and algebraic reasoning to create their own nonstandard weight sets using pattern blocks.

Students will begin by identifying the area relationships between the different pattern blocks. These area relationships can be translated into words and then into mathematical equations using variables. From there, students identify the weight (in grams) of a single pattern piece, and by using the area relationships and equations, are able to determine the standard weight of each pattern piece. Students use algebraic reasoning to identify equivalent values in area and weight of their pattern pieces and to translate these relationships into algebraic equations. Gaining confidence in this process, students practice writing algebraic equations with more complicated groups of pattern blocks and use the two-pan balance to verify their algebraic reasoning.

This lesson connects the math standards of geometry (area relationships and part/whole relationships) with algebraic thinking, providing students a better chance to develop deeper mathematical understanding. In Activity 3, they will use this nonstandard weight set to sink their boats.

Goals

- To use standardized weights to define and assemble a set of nonstandard weights
- To use conversions to change the unit being used
- To measure unknown weights using nonstandard units and convert to standard units
- To use a two-pan balance to develop algebraic thinking skills, specifically equation balancing
- To write a mathematical expression using variables
- To use fractional understanding to find weights of unknown geometric pieces

Materials

- Clay boats (two per class) and water tanks
- Construction paper
- Heavy sinker
- Index cards
- Markers
- Math Notebooks
- MCC DVD: video
- Miscellaneous items from the classroom (e.g., chalk, scissors, markers) to be used as unknown weights
- Pattern blocks (not hollow: hexagons, trapezoids, blue rhombi, squares, and triangles)
- Set of standardized weights
- Storybook, *Kukugyarpak* (optional)
- Two-pan balance

Vocabulary

Area—the number of units that cover a surface

Conversion—changing one thing into another. Converting units from one measurement system (such as metric units or pattern blocks) to another is done using the ratio between two measurement units

Equation—mathematical statement containing an equals sign, to show that two expressions are equal

Equivalent—two values that are the same

Load—weight placed in a vessel

Ratio—the relationship between two units, expressed in one of three ways: a fraction (1/2), "1 to 2" or "1:2"

Nonstandard measurement units—a customized unit of measure, e.g., traditional Yup'ik body measures or pattern blocks

Standard measurement units—units of measure that are well known, standardized, and agreed upon, e.g., inches, centimeters, meters

Variable—something that can be changed or controlled in an experiment; also a letter used to represent an unknown quantity in a mathematical expression

Preparation

- Gather the standardized weights (grams), solid pattern blocks, and two-pan balances for groups of two, three, or four students.

Teacher Note

Be sure to **use solid pattern blocks** for this activity and those to follow. Solid blocks, wood or plastic, are proportional in their area and weight values, unlike hollow pattern blocks. Ideally, solid pattern blocks having the same shape will have the same weight (a solid wood hexagon is about 9 grams, and the solid plastic hexagon is about 18 grams). As your class works through this activity, you might find that this is not the case. Have your students work through a solution to this problem. This might include finding a set of blocks that has equal weights, using another nonstandard unit (marbles, washers, nuts, bolts) or finding an average weight of the block pieces and using that value.

This symbol means students need their math notebooks.

- A note card with an equals sign (=) should be taped to each balance between the pans.
- Make two clay boats out of 100 grams of clay each for use in demonstration. Have two water tanks and weights handy. Make sure one weight is a large and heavy sinker and the others are solid pattern blocks.
- If you choose to use *Kukugyarpuk*, you may want to read pages 4 through 12 during Activity 2.

An audio recording of the optional reader can be found on the *Kukugyarpak* CD.

Duration

One to two class periods.

Instructions

1. **Introduction:** Review with the students the form and functions of different boats. Explain that after they build their boats, they will test them to see how much weight they can carry before sinking. Demonstrate loading one of your prepared boats with one large standard weight and the other with multiple pattern blocks. Conclude that the boat with multiple pattern blocks holds more weight because there are more pieces. Ask students if they agree with your conclusion. Have the students explain their reasoning and justify their responses.

Teacher Note

In the above example, the multiple pattern blocks weighed less than the one larger weight. Students might conclude the boat with many pieces had more weight because of the greater number of pieces.

Establishing the principle of evidence and scientific thinking

2. **Discussion:** Have a conversation about the difference between standardized weights (grams) and nonstandard weights (marbles, pattern blocks, bolts etc.). Include in your discussion why having uniform weights might be important in their boat experiment. For example, there may not be enough standard weights for everyone to use, so they may need to use other objects that are readily available. Explain they will need to track how much weight a boat holds, using the pattern blocks as their nonstandard weights. This is a good point to stress how important their mathematical reasoning will be today, because it will help them solve the problem of finding the weights of different objects.

3. **Area Equivalence and Ratios:** Group students depending on class size and the number of two-pan balances available. Give each group a set of

Developing Students' Algebraic Thinking—Equality and Variables

solid pattern blocks (hexagons, trapezoids, blue rhombi, and triangles). In their groups, have students determine the geometric (area) relationships between the hexagon, trapezoid, blue rhombus, and triangle. For example, two trapezoids are the same as one hexagon, or 2:1, etc. These are more examples of ratios.

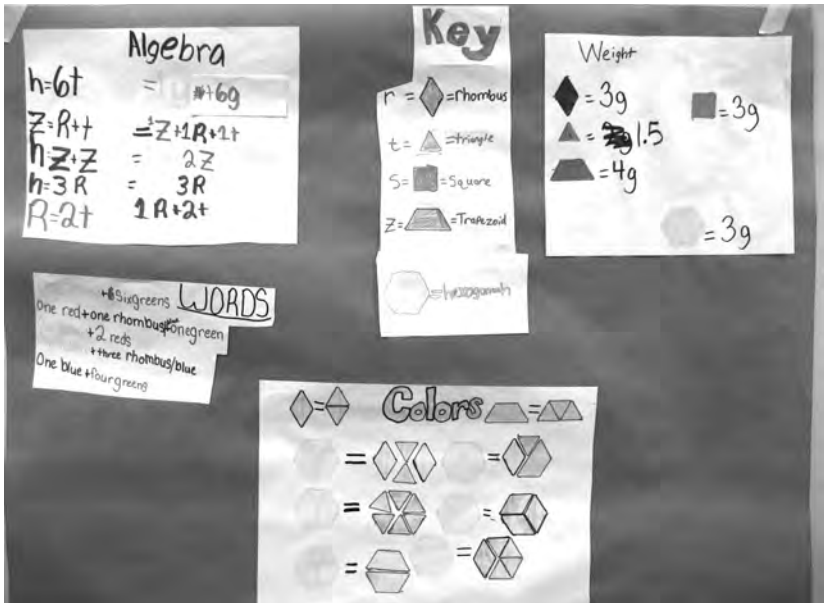


Fig. 2.1: A complete reference poster by a student group in Togiak, Alaska.

4. **Posters**: Have students begin creating a reference poster. The first section will show the area equivalencies of the pattern blocks by drawing the area relationships of the shapes they just discovered (see a student example in Figure 2.2).

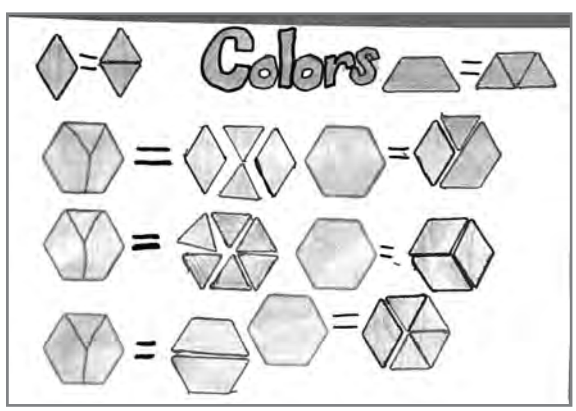
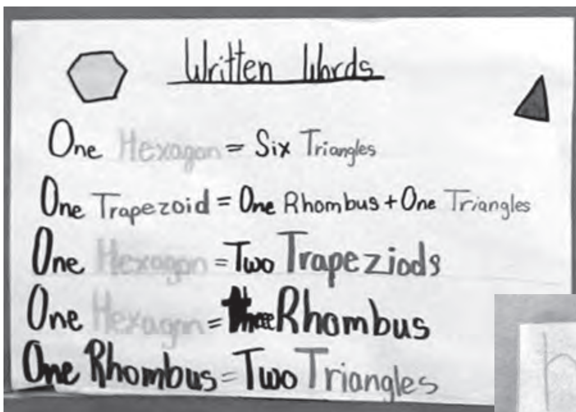


Fig. 2.2: Area equivalence portion of the reference poster.

5. **Write Equations from Area Model:** Discuss what "equals" or "equivalent" means with the students. Have them write simple statements using an equal sign to show the area relationships they just discovered with either pictures or words (e.g., 2 trapezoids = 1 hexagon). As the students explore and come up with equivalent statements for combinations of shapes (trapezoids and rhombi, trapezoids and triangles, etc.), engage in joint product activity and create your own set of equations to share. You may want to introduce the idea of variables to make writing the equations easier. See the Math Note about the use of variables. Students should record their equations in their Math Notebooks and on their reference poster. See Figure 2.3 for student examples.

Math Note

Introducing **variables**, or abbreviations, for the shapes will prove useful (the authors use H = hexagon, P = trapezoid, R = rhombus, T = triangle). Using variables, the relation between the pattern pieces could be written like this: 2P = 1H. Students may use the phrase "is the same as" and "equals" interchangeably.



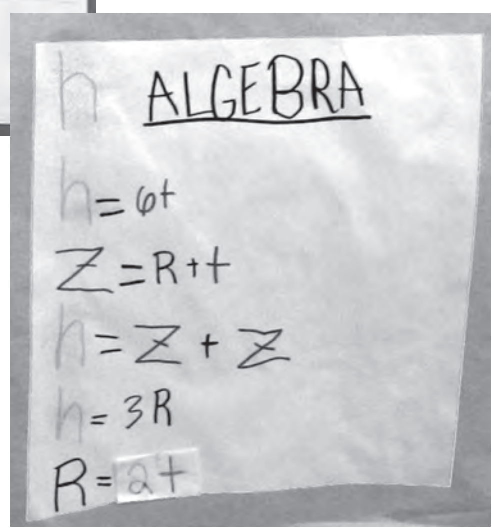


Fig. 2.3: Sentences and algebraic equations for pattern blocks from student posters in Togiak, Alaska.

6. **Demonstrate:** Explain to the class that they will now go from nonstandard units to standard units of weight, using grams. Write "1H = ?" on the board. Using a two-pan balance and gram weights, ask a student to join you at the front of the room to help determine how much a hexagon weighs by placing gram weights on one side of your balance until it is equal to a hexagon piece in the other pan (the authors found 9 g; yours may vary).

7. **Find Weights:** Knowing the value of the hexagon, now have students return to their groups to use the weight of a hexagon to determine the weights of the other pattern blocks without using a balance. Have students record their results in their notebooks and share their values with the class, explaining how they got their answers.

8. **Verify Weights:** Now distribute the two-pan balances to the small groups and have the students weigh the three other pattern blocks (trapezoid, rhombi, triangle) with gram weights to check their calculations from the last step. Have students practice reading the pans like an equation: listing what is in one tray, then saying either "equals" or "is the same as" and then listing what is in the other tray. For example "one rhombus is the same as three grams." Notice how they handle the trapezoid and triangle, which have weights that are not whole grams. (The authors found that the trapezoid equaled 4.5g and the triangle was about 1.5 g.) Students should write the weights they find for each piece in their notebooks.

Teacher Note

As you walk among the working students, interact with each group and ask questions to assess their understanding. **Encourage students to ask questions and to help each other understand.** What does "equals" mean? Does it matter which pan is used for the different groups of blocks? What are the similarities and differences of the two-pan balance and the equals sign?

9. **Discussion:** Ask students, "How do these values compare to your previous results?" Discuss why there may be differences. As a class, come to a consensus about the weight of each piece and have them record these confirmed values in a table in their Math Notebooks for use in the next activity. Have them add this information to their reference poster too. (This is the only step where gram weights are used.) See Figure 2.4 for a student example.

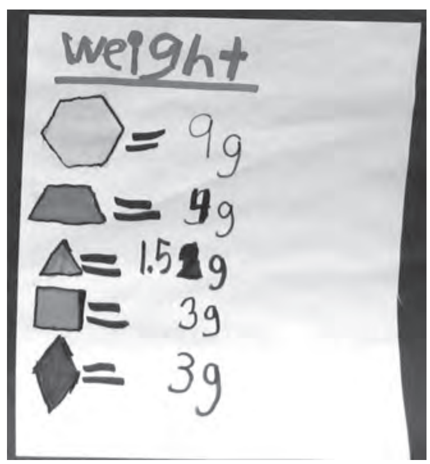


Fig. 2.4: Weight values of pattern blocks.

10. **Complex Equations:** Now have the students, in their groups, use the balance to explore the relationships between the four different pattern blocks by putting more than one type of pattern block on one side of the scale and using a different combination of pattern blocks on the other, adjusting it until it balances. Create your own list of equations simultaneously. Have students write the mathematical equation in their Math Notebooks (e.g., 3P + 2R = 1H + 7T).

11. Have a few students share some of their equations and contribute your own to the list. Have students brainstorm a way to check if their equations are balanced without using the actual balance. For example,

students could use the geometric relationships or the weight values (in grams) they previously calculated. Allow students to verify their equations by working in their groups. Then have students share their equations with the class and discuss how they checked if they had balanced equations.

12. **Unknown Block:** Introduce the square pattern block. Ask them how they could find its weight, knowing what they know about the other pieces. Discuss possible methods as a class and how they might check their work. Have them work in small groups to determine its geometric relationship to the pattern blocks and its weight in standard gram units. Ask volunteers to share their findings and how they figured it out. This information should be added to the reference poster and the table in their Math Notebooks.

Math Note

Consider the illustration below, which shows how students might determine the weight of three identical, unmarked, nonstandard weights; since the three equal weights have a combined weight of 24 grams, each must weigh 8 grams (the gram weights are standard weights/units).

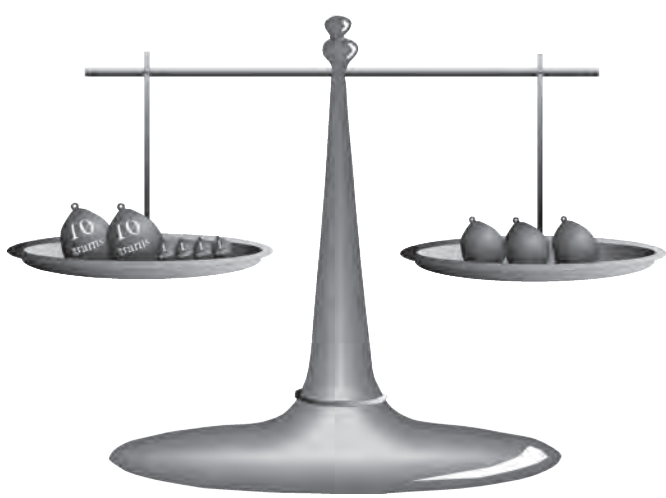


Fig. 2.5: A two-pan balance with known and unknown weights used to practice prealgebraic thinking.

This scale illustrates the algebraic equation 3x = 24, where the weight of each of the three equal weights is "x." Solving the equation confirms that each unmarked weight on the right side weighs 8 g. Note that this establishes a relationship between the nonstandard weights and the standard weights, i.e., each nonstandard weight weighs 8 g. This is an example of algebraic thinking.

Teacher Note

The accompanying Kayak Reference CD has a short instructional video showing how to use the balance to demonstrate several **mathematical properties**: commutative, transitive, and principles of equality. This may be a good place to bring those ideas in.

Science Note

The word "weight" is used throughout the module, even though the students are actually **finding and using mass**, because the distinction is not important in the execution of the activities. A curious student, however, may notice that things (boats, fish, etc.) weigh less in water than they do on dry land and wonder why. The crux of the matter is that mass is a measure of how much "stuff" is in an object, and weight is the downward force, due to gravity, exerted by that "stuff". When an object is in the water, the opposing force, buoyancy, reduces the weight—but the mass stays the same because the object has not lost any material. This is why people use boats to transport heavy objects.

13. **Unknown Objects:** Have each group pick a classroom item (chalk, marker, scissors, etc.—nothing too heavy) and determine its weight using pattern blocks. Have the students convert from the nonstandard units (pattern blocks) to standard units (grams). Have groups record and report to the class what they weighed, how much it weighed in nonstandard units, and how much it weighed in standard units, as well as explain how they figured it out.

14. Remind the students of the boat sinking demonstration. Discuss why the ability to convert weights from one set of units to another is important. What uses can they think of for this skill? Some of the ideas that should be raised are:

    - We can use anything for a weight. If the boat is long and thin, we can find weights that fit the boat.
    - We have alternative weights if there is a shortage of standardized ones, or even if there is a shortage of pattern blocks.
    - We can find values for smaller weights and thus increase our accuracy.

Activity 3: Scientific Method and Carrying Capacity

This lesson sets the stage for all subsequent explorations in this module. Establishing the stages of the scientific method, this activity provides the students with the background on experimental design, data collection, and the beginning of data analysis. It is critical to establish the underlying understanding of variables in an experiment, because these factors affect data accuracy and reliability.

Each student receives 100 grams of clay to make one of the three types of model boats (V-bottomed, flat-bottomed, or round-bottomed). Because the models are each made from the same amount of clay, the variable of shape is isolated and students can readily investigate how shape affects function. Today students start out with the simplest of the three explorations, answering the question: what is the maximum weight that each of the three types of boats can hold before sinking? Students will make a conjecture, then systematically load weight into their boat, observe, record, and compare the load carried by each type of boat. The model design alone will determine differences in load (or carrying capacity) of the boats if all other variables are controlled. This is the scientific method in action,making it accessible to your students.

Teacher Note

All schools are set up differently, and this module can be logistically difficult if there isn't space to leave the water containers and boats sitting out. Teachers should customize the module as necessary to make the logistics work.

This is the first of multiple experiments using the boats students make during the first part of this activity. The students will continue to collect data over the next three activities on the boats and will, in the end, have a chance to optimize the function of their boats based on the data they collect throughout the experiments. **It is important that students do NOT modify their boats until Activity 7 except for small repairs, so they use the same boat formation for all of their data collection.** They will have a chance to optimize their boats by modifying them during Activity 7. They will be gathering performance data on their boats' load capabilities, speed, and stability. They will also be finding performance averages for each boat type. Then, in Activity 7, they will choose one type of boat and attempt to optimize their boat performance to exceed the class averages for that boat type in each function: load, stability, and speed. Early modification will make this much more difficult for the student.

In any classroom, time is of the essence. As the next three experiments are written, each student has the chance to test their boats individually and to do their own data analysis. We recognize that this requires more time due to limited resources and basic logistics. In light of this, here are a few suggestions to help minimize the time required for the experiments if this is a concern for your classroom. The first alternative would be to have the students complete the experiments as a class. Three boats would be tested by small groups of volunteers, and the data shared by the entire class. This option of course reduces student ownership of the process. Another option to consider is having students complete the experiments on their own boats, as described, but during other time openings throughout the day (when they have finished their work or are between activities).

Goals

- To introduce the concept of variables by varying boat types (round, flat, V-shaped)
- To understand how an experiment provides uniform conditions to measure a single variable
- To conduct an experiment holding all but one variable constant
- To introduce the concept of working with several variables
- To create and interpret tables and graphs to address their conjectures
- To use data to inform and optimize their boat's design
- To understand the need to control for extraneous variables—students should be able to explain what a variable is, which variable they are measuring, and for what variables they are controlling

Materials

- Butcher paper or large paper for class graph
- Clay for model boats (100 grams of clay per student, 70 grams for teacher demo)
- Container for floating boats (one per group)
- Handout, Boat Type and Load Table (optional)
- Math Notebooks
- Pattern blocks and other nonstandard weights used from Activity 2
- Sponges or paper towels
- Stickers, three colors
- Storybook, *Kukugyarpuk* (optional)
- Two-pan balances for each group
- Water
- Wax paper for each student, to prevent clay from sticking to tables

Vocabulary

Carrying capacity or load—the amount of weight placed in a boat that can be supported by that craft

Controlling for variables—holding one variable constant so that another variable can be measured

Optimization—to make as effective as possible

Reliable—if an experiment yields the same or very similar data when repeated

Valid—results in an experiment that accurately represents the variable tested

Variable—something that can be changed or controlled in an experiment; also a letter used to represent an unknown quantity in a mathematical expression

Preparation

- Read the Teacher, Math, and Science notes.
- Based on time available, decide if this activity will be done as written, as a class, or by having students perform the tests on their boats during other times throughout the day.
- Have the student-created reference posters from Activity 1 visible for students.
- Weigh out 100g of modeling clay for each student and 70g for the teacher.
- Arrange containers for water and a cold water supply so that when the students are ready to float their boats, the containers can be easily distributed and filled.
- Make sure you have paper towels and/or sponges to clean up spills.
- All student groups should have access to the nonstandard weight sets developed in Activity 2.
- Draw a large x–y axis on butcher paper for use in step 15 (preparation for a graph that will have boat types on the x-axis and weight in grams on the y-axis).
- Decide if you want to use the provided data table or have the students create their own. Make copies if you use the one provided.
- If you are reading *Kukugyarpak* along with the module, pages 12 to 15 closely relate to Activity 3. Pages 16 and 17 can be read before Activity 5.

This symbol means students need their math notebooks.

An audio recording of the optional reader can be found on the *Kukugyarpak* CD.

Duration

Four class periods.

Instructions

1. **Introduction:** Remind students about the different types of boats (skin boats, log rafts, and kayaks) and their functions discussed previously. Explain that today their groups are going to make three different types of boats based on their bottom shapes (round, flat, and V-shaped). They are going to determine which boat will hold the most weight. Have the students discuss the important characteristics of each boat's shape and how the boats will be different.

Teacher Note

Students will be using 100g of clay. The demonstration boat uses less clay so less weight is required to sink it in the demo, as well as to let the students discover for themselves what load their own boat can carry instead of copying the results of the demonstration.

Science Note

Conservation of mass is the principle that states that mass cannot be created or destroyed, although it may be rearranged in space and changed into different types of particles. This means that the boat, despite being rearranged in different shapes, will maintain the same mass throughout the process. Because all of the boats weigh the same, this eliminates the boat's weight as a variable affecting the outcome of the experiments.

Tricia Wick, a teacher in Togiak, Alaska, had students in groups of four list characteristics for each of the boat types in a limited time, before sharing their ideas to the whole group. Adding an element of challenge, she had the students see which group could come up with the most items. Prompts included: What is different about each boat type? What makes a round-bottom boat better than a V-shaped boat? What makes a V-shaped boat better than a round-bottom boat? What are they used for? Where are they used?

2. Take 70g of clay and weigh it in the pan balance in front of the kids to verify its weight. Decide as a class what shape boat to make for the demonstration and make it. Ask the students if making the boat will change the weight. Reweigh the boat to verify, and discuss conservation of mass principles.

3. **Model:** Gather the students around a container large enough to float the boat model. Ask students how much weight they think your boat can hold and write the conjectures on the board. Float your boat. Load your boat with nonstandard weights and while doing so, have students notice things (variables) that would affect how much weight you could put in the boat: location of weights in the boat, how they are put in the boat, water disturbances, etc. Continue adding weights until the boat sinks. With the help of students, calculate how much weight the boat was able to carry just before it sank. Remind students that they will be loading their boats three times to their maximum carrying capacity to ensure that the data is reliable.

4. Recap the idea of variables and have the students explain why they think it is important to pay attention to them. Discuss the difference of reliable and accurate data and how these variables will affect their data (see Science Note). Have students compile a list of variables they need to control so their data will be reliable and accurate.

Science Note

Data that is valid accurately measures the property being tested and is not influenced by outside factors. Validity should not be confused with reliability, which refers to the repeatability of the measurements. Consider if a marble was stuck in the bottom of the boat and the experimenter did not know it. The maximum load measured would be reliable—you'd get nearly the same measurements every time—but not valid because it would not be taking into account the weight of the marble already supported by the boat. An error that is repeatable is called a systematic error. This type of error is hard to find because it is something wrong that the experimenter is doing the same each time.

5. **Scientific Method:** Discuss and explain the stages of the scientific method: conjectures, variables and testing, data collection, analyzing data, and making a conclusion. Point out that they will be using these steps for each experiment, to collect data on their boats and improve their boats in the end. See Science Note for more details.

Science Note

The **scientific method** consists of making conjectures and then testing them. The tests are designed so that outside variables that could affect the test (such as differences in boat weight in this activity) are eliminated. Making sure there are no outside influences ensures that you are only testing for the variable you want (load in this activity). This is called controlling the variables. Initial conjectures being incorrect are important. Many important scientific discoveries were made when scientists discovered what they believed was inaccurate. If a conjecture is wrong, it means something new has been learned. There are no penalties for having a conjecture that was disproven.

6. **Conjectures:** For their first conjectures, students will need to make an educated guess on which type of boat they think will carry the most weight. They need to record their guess and why they picked that boat in their notebooks so they can refer back to it at the end of the experiment.

7. **Set Up Experiment:** Organize students into groups of three students. Give each group a two-pan balance, wax paper, and three 100-gram blocks of clay. They should weigh their clay before making the boats and record the value in grams in their Math Notebooks.

   A teacher in Togiak suggested having one color of clay per group to help easily identify groups of boats at the beginning of each class period. Something else to consider would be to have the more patient students build the kayak because it requires more finesse and patience to build.

8. Allow the groups to decide who will make which kind of boat, making sure all three boats are represented in each group. Have them make their boats. Throughout this process, allow students to individually test their boats' ability to float in a single small container of water in the classroom and to make needed adjustments until they are happy with their boats' basic floating performance. You can make a boat simultaneously, to be used in future demonstrations. See Figures 3.1, 3.2, and 3.3 for possible boat designs.

Teacher Note

Cold water works better for these experiments, because it helps harden the clay helps the boats keep their shape.

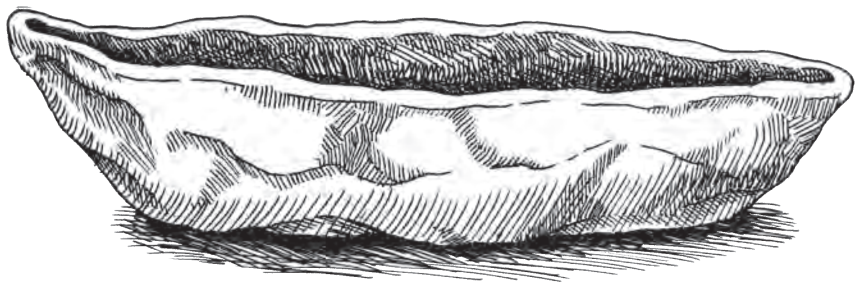


Fig. 3.1: Example of a V-bottom boat made with 100g of clay.

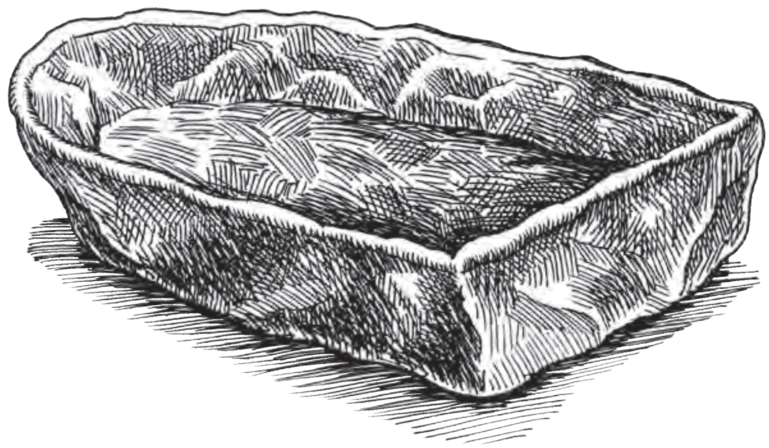


Fig. 3.2: Example of a flat-bottom boat made with 100g of clay.

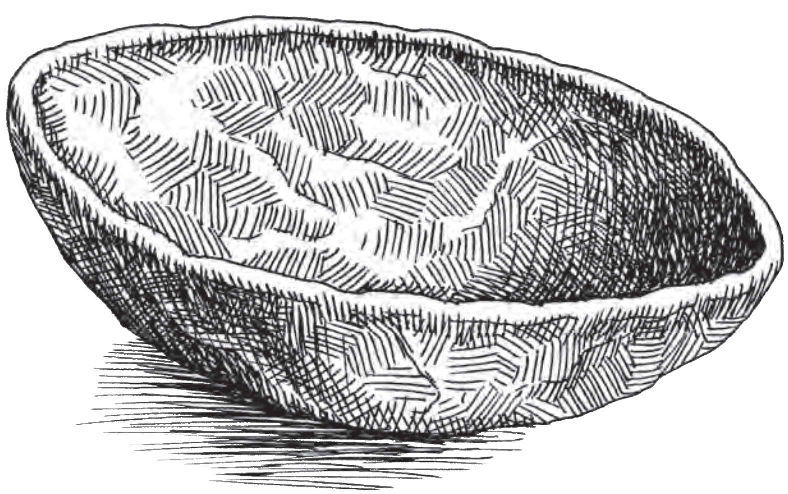


Fig. 3.3: Example of a round-bottom boat made with 100g of clay.

9. Have the students weigh the finished boats and compare the values to the ones entered in their Math Notebooks previously. Did forming the boat change its weight? Why or why not? Have the students discuss this and write their reasoning in their Math Notebooks.

This would be a good place to stop for the day.

10. Remind the students that they will be testing their boats' ability to hold weight, or carry a load. Have students discuss why carrying a load might be important for certain boat types and have them compare that to the form and function of each boat type they learned about at the beginning of the module. Have them review the variables they need to control during the experiment and the conjectures they wrote yesterday.

11. **Organizing Data:** Point out that scientists collect data all the time (just as they have been collecting data in their notebooks) and need to keep it organized so they know what they did. Ask students what type of data they will need to collect in this experiment. Explain that they will repeat the process of loading their boats three times each. Have volunteers share how they might organize their data from this experiment. Discuss the advantages and disadvantages to each approach. (If your students need support in creating a table, the blackline master, Boat Type and Load Table, can be copied and given to the students. The gray area will be filled in during the next activity). Have students use the handout or draw their own table in their Math Notebooks. Figure 3.4 is an example of table with data.

Load Trial	Round Bottom	Flat Bottom	V-Shaped
1	*75 grams*	*60 grams*	*20 grams*
2	*73 grams*	*62 grams*	*24 grams*
3	*74 grams*	*62 grams*	*25 grams*

Fig. 3.4: Boat types and example loads.

Teacher Note

When helping students discover how to **make and use tables**, make note of how the students are doing it on their own. Do they organize the data randomly? By boat type? Do they code it? Can the data be used for comparative purposes? This activity provides an assessment of what students already know and can apply for organizing, displaying, and understanding the purpose for organizing data. The table provided is to guide students' understanding of table creation, and in subsequent activities students will be expected to create their own table or create one as a class.

12. **Experiment:** In their groups, have students take turns at their container floating and incrementally loading their boat three different times, being aware of controlling other variables. They should record what nonstandard pieces it took to sink their boat (see Figure 3.5).

Fig. 3.5: A clay model boat floats while nonstandard weights are added to test for load capacity by students in Dillingham, Alaska.

13. **Practice Mathematical Expressions:** Have students write the value of their load using the same variables they used in Activity 2 to represent their pattern blocks, and then calculate the weight of the load. For example, if a boat held three hexagons and two trapezoids, the mathematical expression representing that weight would be: 3H + 2P. Next the students will use the gram equivalences found in Activity 2 to determine the weight their boat held. For example:

    3 H + 2 P
    3(9g) + 2(4.5g)
    27g + 9g
    36g This is the value that would be entered in their table.

Teacher Note

Students are encouraged to work with and help each other across groups. Monitor and facilitate discussions as students address the process being used and variables they notice affecting the test.

14. **Discussion:** Ask a group to volunteer and share their completed table with the rest of the class. Remind them that scientists use data to find patterns and information about something. Model a discussion on how to interpret this group's data. Ask students to identify the patterns they see, what conclusions they can make, etc.

15. Now, as a class, create a large table to display all of the groups' data by boat type to look for patterns across all of the data. Have students record their individual data on this table. Have students identify any patterns or conclusions they can from the table. (Keep this table to use in Activity 4.)

16. Help students see that a large number of data points in a table can make it hard to read and find patterns; see if they can think of another way that might make it easier to see patterns or trends for large groups of data. Guide them to the idea of graphing.

17. **Graphing Data:** Before making a type of bar graph, review the three important parts of a graph: title, labels, and data. Hang a large x–y plot and as a class fill in the axis labels and title (boat type on the x-axis

and load (grams) on the y-axis). Model graphing one set of data points. Color-code each boat type and give each student three of the appropriate color sticker dots according to their boat type. Have them place their stickers on the graph in the appropriate column. See Figure 3.6 for an example. (Keep this graph for use in Activity 4).

Fig. 3.6: Example boat load graph with class data from Mike McGill's classroom in Fairbanks, Alaska. Data will vary within each boat type due to variable influences, yet trends are apparent.

18. **Analyze Data:** Facilitate the students' discussion about interpreting the class data, using the graph. Have students' explain why there might be variations between boat types and within each boat type (e.g., variables they discovered earlier weren't controlled by each group, different heights of boat sides, dropping the weights rather than gently placing them, water in the boat from a previous trial, etc.). How do the three boat types compare to one another? Ask students what conclusions they can make from this data.

Science Note

Students should be able to make statements such as, "the weights held by each type of boat were different in each trial," or "the round-bottom boat held more weight most of the time." This simple exercise can serve as an instructional assessment, giving you an indication of whether students can make meaningful and accurate statements about data. You could assess students further by posing statements that are true or false based on the students' data and asking them to respond using their data, or a statement that is indeterminate relative to the class data (e.g., "Round-bottom boats are more stable" is indeterminate from this data because the experiment tested for load, not stability. Students will test the boats for stability later in the module).

Teacher Note

At this point, some students may feel that their boat is "bad" because it cannot hold as much weight as another boat. Remind students about **form and function**, and that not all boats are meant to hold a lot of weight but instead might need to move faster, might help them realize that their boat might outperform other boats in future experiments.

19. **Reflection:** Students should record their conclusions about the data analysis just discussed as a class and clearly identify which boat could hold the most, to the least. They should review their conjectures and explain in their entry if the conclusion supported their conjectures. Why or why not?

20. Review and discuss the different purposes that the kayak (*qayaq*: V-shaped bottom), the log raft (*angyarrluk*: flat bottom), and the skin raft (*angayaqatak*: round bottom) are used for as you discussed in Activity 1. Does the form of the boat they just tested serve its function with respect to load? Have the students support their arguments with data.

21. **Impact of Experiment:** Explain that a scientist uses the information he or she finds to improve or change later on. Have your students determine what, if anything, they would change about their boats to optimize its performance, now that they know how their boat did on load. They should record it in their Math Notebook so they can refer back to it in Activity 7 when they optimize their boats.

Teacher Note

Students are NOT to modify their boats in this activity; they are modifying their boats in Activity 7 based on the outcomes of these experiments.

Boat Type and Load Table

Load Trials	Round-bottom boat	Flat-bottom boat	V-bottom boat
1	_______grams	_______grams	_______grams
2	_______grams	_______grams	_______grams
3	_______grams	_______grams	_______grams
Total for all 3 trials	_______grams	_______grams	_______grams
Average (or mean)	_______grams	_______grams	_______grams
Median	_______grams	_______grams	_______grams
Mode	_______grams	_______grams	_______grams

Activity 4: Data Analysis

This activity introduces students to data analysis and elementary statistics. Students review and learn how to collect, organize, and interpret data. For example, students will use data collected from loading and sinking their model boats and will graph and analyze it. Students will learn to display their data using a line plot and bar graph. Statistics will be used throughout the rest of the module to help students interpret data on boat type and performance. At the end of the module, students will apply the lessons learned from interpreting boat performance data and apply them by redesigning their boats to optimize their performance to exceed the class averages.

Your students are probably familiar with the word "average," which is generally interpreted as computing the arithmetic mean. They may not understand the concept of "central tendency," which includes mean and other frequently used definitions of the word average: the median and mode. This activity is designed to help them understand these concepts by first using a smaller, more manageable set of data before using their load data.

Teacher Note

Students will need calculators throughout the activity.

Goals

- To understand the concepts of central tendency (mean, median, and mode)
- To use and calculate the central tendencies of mean, median, and mode
- To determine which of the central tendencies to use in which situations
- To understand how to read, interpret, and construct a number of different plots, including line plots and bar graphs
- To calculate the range for a data set

Materials

- Adding machine tape (optional)
- Calculators
- Load graph and data table from Activity 3
- Markers
- Math Notebooks
- Poster-sized paper (poster board or butcher paper)
- Post-It notes, small, three colors
- Projector (optional)
- Small strips of paper (1 inch by 4 inches), 15 per student
- Storybook, *Kukugyarpuk* (optional)
- Student load data from Activity 3
- Tape

Vocabulary

Bar graph—a graph consisting of vertical or horizontal bars whose lengths are proportional to amounts or quantities, also called a bar chart or bar diagram

Central tendencies—a number that in some way conveys the "center" or "middle" of a set of data; examples include the mean, median, and mode

Data analysis—process of making sense of and drawing conclusions from data

Line plot—a line plot shows data on a number line with an X or other marks to show frequency

Mean—sum of all data points divided by the number of points

Median—the middle value when data is put in order

Mode—the most commonly occurring value or values in a set of data

Outlier—a data point that is extreme compared to the rest of the data

Stem and leaf plot—a plot where each data value is split into a "leaf" (usually the last digit) and a "stem" (the other digits); for example "32" would be split into "3" (stem) and "2" (leaf)

Preparation

- Read through this activity and the multiple exercises in Appendix A about scaffolding for the concepts of mean, median, and mode.
- Determine what exercises in the appendix, if any, are appropriate and/or needed by your students.
- Create data sets for each student by writing names on small Post-It notes to reflect the number of caribou caught (e.g., Nicolle's name will be on two pieces).

This symbol means students need their math notebooks.

Name	Number of Caribou Caught
Nicolle	1
Karen	2
Jerry	5
Kayt	6
Carrie	1

Fig. 4.1: Caribou data.

An audio recording of the optional reader can be found on the *Kukugyarpak* CD.

- Tape together 15 paper strips, 1 inch by 4 inches, lengthwise to make one long strip.
- If you choose to use *Kukugyarpuk*, you may want to read pages 16 and 17 during Activity 4.

Duration

Four to five class periods.

Instructions

1. Remind your students that yesterday they collected load data from their boats and that we want to determine how their boats performed compared to the class. To do this they need to analyze their data. Mathematicians work with data to find patterns and to find the best way to represent a set of numbers. Explain that today they will be practicing to find measures of central tendency, using a smaller set of data first, and then they will apply what they learn to their load data.

2. Give each student a set of data pieces (see Preparation). Explain that each piece represents a caribou and the name represents who caught the caribou. Ask the students to organize the pieces in different ways. Allow discussion between students and encourage them to share multiple ways to visually represent the data (e.g., vertical bar graph (Figure 4.2), horizontal bar graph, piles, etc.).

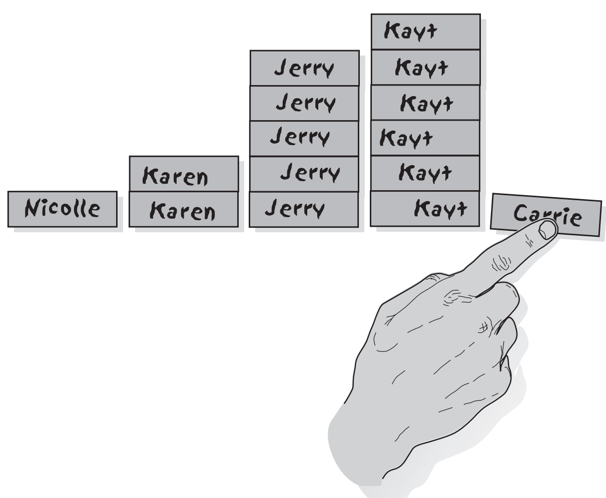


Fig. 4.2: One possible way to organize the data pieces.

3. As a class, have students share the ways they organized the data by having a few volunteers post their pieces on a class poster at the front of the room. As students post their examples, have them explain how they organized the data.

4. **Comparison:** Referencing the poster just created, ask the class to explain what the individual representations tell us about the data. What would provide us with more information (title, labels)? Have students compare each visual representation: how are they similar? How are they different? Does one representation tell us more than another? Can they identify one that is easier to use and understand? Why?

5. **Numerical Representation:** Point out that they have found many ways to physically represent the data. Now ask them to find numbers that could represent the data. As they provide numbers (15, 5, 1, etc.), have them identify what that number explains about the data (total caribou caught, number of people hunting, the number of caribou more than one person caught). As students provide numbers that directly relate to statistics (see Math Note) stress the name that the value represents. Keep a list of all values on the board.

6. Remind them that mathematicians look for the best numbers that explain the data: for example a number that could generally explain how many caribou the hunters caught. Referencing the list of numbers generated by the students, discuss which of the numbers on the board gives the most information about how many caribou each hunter caught. The discussion should lead to students identifying numbers that represent some or all of the central tendencies on their own: median (3) and mode (1) may be the most likely to be referenced.

Math Note

There are three common types of measures called "**central tendencies**." Mean (the average of a group of numbers), median (the middle number in a group of numbers) and mode (the most frequently seen value or values) are official statistical methods of finding numbers to represent a set of data. All of these measures can technically be called averages, though the mean is most commonly referred to as the "average." Note that when calculating the median or middle number, the procedure is slightly different if there are an odd number of data points versus an even number. Once the data are organized in order from smallest to largest value, if there are an odd number of points, the middle is an actual data point with the same number of points above and below it. If there are an even number of points, then to find the middle, select the two middle numbers and find their average. If the two middle numbers are not the same value, then the median will not be a data value but will still represent a point in the middle with the same number of data points above and below it.

Representing Data Graphically and Numerically

Math Note

Numerical values students might identify could include total in set (15), number of groups (5), mode (1), median (2), the most caught (6), the least caught (1), or range (6–1 = 5).

7. **Median:** If students do not demonstrate understanding of median and mean through the previous step, you will now use direct instruction. Ask your students to organize their data pieces from the least to most caught (see Figure 4.3). Ask them which number represents the middle. They should identify Nicolle's column. Explain that the number of caribou caught by Nicolle (2) is the median. Explain that the value of 2, representing two caribou, is a single number that could represent this data. It means that half the people caught more than two caribou and half of them caught less than two caribou. As a class, ask them if they think it represents the data well and to give a reason for their thinking.

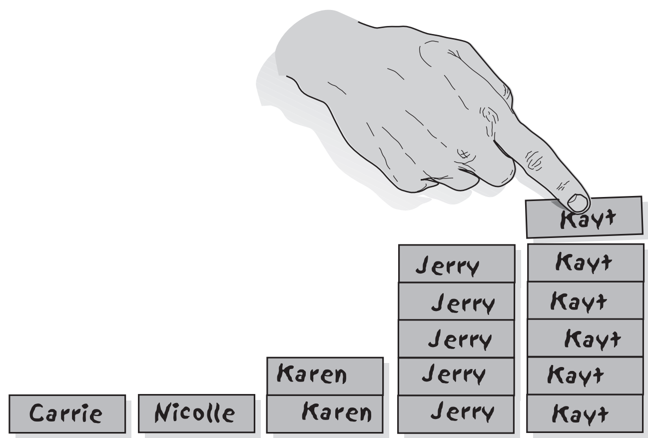


Fig. 4.3: Bar graph organized in increasing order to help find the median.

8. **Mean:** Explain that another value mathematicians commonly use is called the mean. Describe the mean as the number of caribou that every hunter would have gotten, if they all had caught an equal number. Ask students, "How can you rearrange your pieces to show this idea?" Give students five minutes to work in groups to answer the question. If students need help, lead the discussion towards rearranging the strips so all columns have the same number of caribou. See Figure 4.4. Model this with your own pieces if students still need help. Ask the students to identify how many caribou each person would have if they all got the same (3). Explain if everyone had the same amount of caribou, each person would have 3. Ask the students if this represents the data well, and why they believe so.

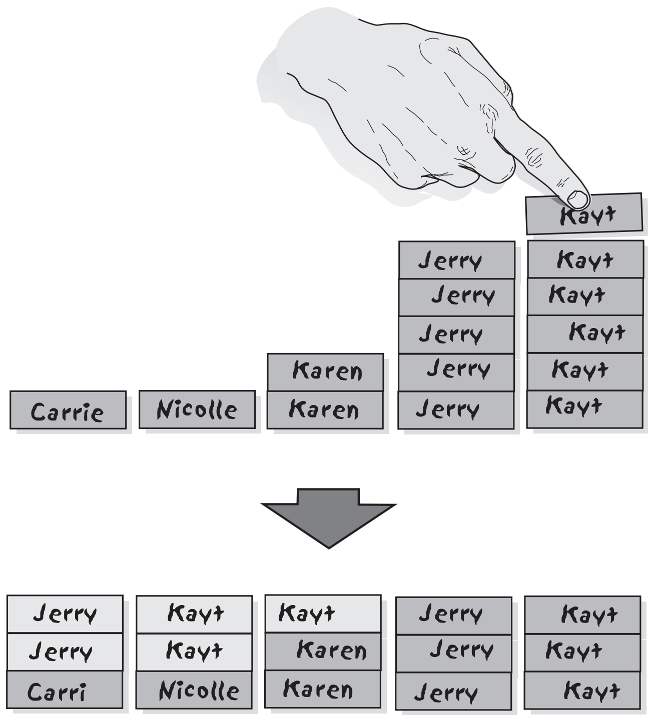


Fig. 4.4: Modeling how to balance the bars in a bar graph to represent mean.

9. Now have the students record their own definition or explanation of mean, median, and mode in their notebooks.

This would be a good place to stop for the day.

10. Remind students of their explorations with the caribou data the previous day and measures of central tendency. Recreate the last caribou graph, which models finding the mean (Figure 4.4). Show students a long strip with 15 paper strips taped together and explain that this represents the same data as the other graph (Figure 4.5). Have volunteers identify what is the same between the two representations (15 caribou). Write the number 15 on the board. Ask for a few volunteers to come to the front of the room to manipulate the strip to show how they would evenly divide the total caribou among the five hunters. As they play with the strip, write the number 5 on the board to represent the number of hunters. Students should discover again, through folding the strip or their own methods, that each person would have three caribou. Write that number on the board as well.

Fig. 4.5: Fifteen small strips taped together to represent the entire number of caribou caught (15).

11. **Discussion:** Ask the students if they can come up with a mathematical way to represent what they just did with the strips to find the mean caribou caught (the numbers on the board will be helpful for them to reference). Help guide the students into discovering and writing the formula for mean and recording it in their notebooks (total # of caribou for the whole group / number in the group = mean number of caribou). Ask students to share why they think someone might use a formula to calculate the mean instead of using strips like they did. (Possible answers might include large data sets, uneven numbers, faster, easier, etc.)

12. Review with the class that they have now found three numbers (mean, median, and mode) and each could represent the number of caribou caught, in general, by the hunters. Record on the board the three values for each of the central tendencies. In small groups, have students discuss which number they think best represents the caribou data. Have groups share their ideas and have the class come to a consensus.

13. Explain to students that they will now have a chance to practice finding the mean, median, and mode for other sets of data [located on the next page (Sample Problem Set)]. As individuals or in small groups, have students find the three values and explain which one they think best represents the data and why. Choose from the problems found in the Sample Problem Set (see next page) or Appendix A. They should record their work and their explanations in their notebooks. You may write problems on the board, photocopy the page to hand out, or write them individually on index cards to distribute to each group or student.

14. **Discussion:** After students work through the problems you gave them, as a class discuss the solutions (mean, median, and mode) and come to a class consensus on which value best represents the data for at least examples 1, 2, 4, and 8.

15. **Picking a Central Tendency:** Remind students that mathematicians like to find values that best represent their data. From the previous examples you gave the students, have them share their ideas on when they would choose each of the different values (mean, median or mode). Are there patterns to follow? See the Math Note regarding measures of central tendency and situations when each is typically used. The Teacher Note also provides an online tool for reinforcing this concept for a classroom discussion.

Sample Problem Set

A few problems to practice finding mean, median, and mode and to help with determining which of the three central tendencies represents the data the best.

1.

Name	# caribou caught
Bob	4
Mike	3
Sue	4
Kim	3
Jerry	1

2.

Name	# caribou caught
Bob	5
Mike	2
Sue	6
Kim	3

3.

Name	# caribou caught
Bob	1
Mike	2
Sue	4
Kim	1

4. 1, 110, 7, 2, 3

5. 23, 30, 27, 25

6. 77, 75, 80, 91, 87, 83

7. 50, 93, 54, 3, 56

8. 15, 68, 12, 57, 12, 3, 48, 12, 57, 57, 65, 6

Solutions to Sample Problem Set

1. Mean = 3 caribou, Median = 3 caribou, Mode = 3 and 4 caribou

   Best choice: any of them make sense

2. Mean = 4 caribou, Median = 4 caribou, Mode = none

   Best choice: mean

3. Mean = 2 caribou, Median = 1.5 caribou, Mode = 1 caribou

   Best choice: mean

4. Mean = 24.6, Median = 3, Mode = none

   Best choice: median

5. Mean = 26.25, Median = 26, Mode = none

   Best choice: mean or median

6. Mean = 82.17, Median = 81.5, Mode = none

   Best choice: mean

7. Mean = 51.2, Median = 54, Mode = none

   Best choice: median

8. Mean = 34.3, Median = 31.5, Mode = 12 and 57

   Best choice: mode (bimodal data: it has two modes)

Math Note

Measures of **central tendency** are used to find one number or group of numbers that best represents a set of data. There are three common types of central tendencies: mean, median, and mode. They are all types of averages. Each is used for different reasons, and below you will find examples and explanations for when to use each.

Mean is the central tendency commonly referred to as the “average” and is the most commonly used measure of central tendency. The mean is the sum of all data points divided by the number of points in the data set. The mean is the balancing point of the data and is used when the data is roughly balanced; in other words, it does not have outliers.

Median, the middle number in a set of data, is found by putting the data points in numerical order and finding the middle value. The median minimizes the effect of outliers: points that don’t fit the pattern of the rest of the data. For example, if there are single data points that are much larger or smaller than the rest of the data, the mean will be drastically changed by them while the median is not influenced by the small or large numbers as easily.

For example, in the data set 45, 50, 53, 60, and 175, the 175 greatly differs from the other numbers and is called an outlier. The mean for this data set is 76.6, and the median is 53. The mean doesn’t represent the data well because it is larger than all but one of the values. This suggests the median represents the data better in this example.

The median is also important because it helps determine how balanced the data is. If the mean and median are compared, and are about the same, the data is well balanced; that is, it has the same number of points above and below the mean. If the mean and median are significantly different, then the data is not well balanced. We call this skewed data.

Mode is most often used when picking what is most popular, such as tallying votes. If you wanted to know what boat everyone wanted to make, you would take a vote. When you counted the votes, the one with the most is the mode as well as the winner. It is also used in numerical data if you have more than one peak in your data distribution. For example, if you had a data set of 1, 12, 12, 12, 56, 65, 65, 65, and 90, the mean is 42, which doesn’t really represent the data well; the median is 56, which is better, but still not very accurate. The data has two modes: 12 and 65. This is the most informative way to describe the data.

You might also have the students come up with example data sets and have the class decide if the mean or the median represents the data the best and why. If they choose median, have them identify the outliers.

16. **Formative Assessment:** To conclude for the day and assess where students’ understanding, give students the following two examples to write down in their notebooks. Have them identify which central tendency represents the data best and explain why.

    Example 1: 23, 25, 27, 30

    Example 2: 3, 50, 54, 56, 73

Teacher Note

http://illuminations.nctm.org/ActivityDetail.aspx?ID=160

This NCTM resource is an **interactive tool** that can be used to model how data can change the mean and median and why one might be a better choice than the other. Modeled with a projector or Smartboard, a class discussion could be very valuable to help students distinguish between these two measures of central tendency.

Example problem: Place five balls on the line with the following values: 17, 20, 25, 29, and 74. This produces a mean of 33 and a median of 25. Discuss which value represents the data the best. Note that the mean doesn't represent similar values in the data set. Have the students predict what will happen to the mean and the median if you change the last ball, 74, to 53. Have the students explain what happened. (It shifts the mean, but the median stays the same.)

This might be a good place to stop for the day.

17. Review as a class what students worked on yesterday and explain that now they will apply their new knowledge to understanding the load data they gathered earlier. Give students ample time to practice finding the mean, median, and mode using their individual boat data from Activity 3. Their values can be entered in the blanks on the previous data table from Activity 3. Encourage students to help one another. They will be coming back to this information after looking at the class data.

18. Once students have finished analyzing their own data within their kayak groups, explain to them that to understand their data in relation to the whole class data they will now work in three groups: all V-bottom boats, all round-bottom boats, and all flat-bottom boats. Allow students to work in these three groups and ask each to calculate the three central tendencies for all of the trials for the specific boat type. Students should record their work in their notebooks.

19. Now as a class, record and discuss the central tendencies found for each of the three boat types in a new table (Figure 4.6). Encourage students to discuss which central tendency is best for this data. Keep in mind that for the load data, **all groups must use the same central tendency in order to be able to compare values** in Activity 7, so be sure students come to this conclusion.

Math Note

Mean, the average of a group of numbers, is used when data is generally evenly distributed. **Median**, the middle number in a group of numbers, is used when outliers exist. And **mode**, the most frequently seen value, is generally used if there are two values that happen frequently, or if you are working with categories. There are other factors that impact which central tendency to use as well. These include, but are not limited to, sample size and the type of data.

	V-bottom	Flat bottom	Round bottom
Mean			
Median			
Mode			

Fig. 4.6: An example data table for collecting the central tendencies for the load data. Students can circle the value that best represents the data.

20. Have students record in their Math Notebooks the central tendency the class agreed upon to represent the data and the values for each boat type and explain why they are the best choice.

21. **Reflection:** Have students refer back to their own boat data and the calculated central tendencies. Have them compare their averages to the one chosen to represent the class data. Students should record their conclusions about their boat performance compared to the class overall preformance. More prompts could include: Could my boat hold more, less, or about the same weight as the other boats of the same type?

22. **Impact of Experiment:** Have your students determine what, if anything, they would change about their boat to optimize its performance. Record it in their Math Notebook. They can refer back to this in Activity 7 when they optimize their boats.

This might be a good place to stop for the day.

23. Review yesterday's work as a class and explain that today they will be using another method to represent data: graphing. Hang up the class load graph from Activity 3 and remind them this is how they first graphed the load data.

24. **Line Plot:** Introduce a new way to graph the data: line plots. With the original caribou data (Figure 4.7) demonstrate on the board how to make a line plot (Figure 4.8). Explain as you draw this on the board how the names are now missing, how the numbers have become the categories, and how the X's now represent the number of people.

Name	Number of Caribou Caught
Nicolle	1
Karen	2
Jerry	5
Kayt	6
Carrie	1

Fig. 4.7: Caribou data.

Fig. 4.8: Example of line plot with caribou data.

25. As a class, create a line plot of the load data. Using butcher paper or adding machine tape, create the number line, and use color coded markers (by boat type) to represent data points. See the Teacher Note for helpful hints.

26. As a class, discuss the patterns or trends seen in the line plot. Encourage students to find patterns in the data and explain what they observe. Although data will vary between classrooms, data that is bunched up tightly by color means that the data are probably more reliable than if it is spread out. Ideally three groups of colored points signifying the three boat types makes it easy to identify which boat held the most. As the data becomes more overlapped between boat types, reasons might include variation in how the experiment was conducted, differences in boat design, lack of accuracy in measurement tools, etc.

27. Introduce another way of graphing called a stem and leaf plot. Model making a stem and leaf plot with the provided example (see Math Note).

28. As a class, create a stem and leaf plot of the load data. Use color-coded markers to write the values so that the different boat types are still represented. If you have a large class you may decide to have students use their measure of central tendency, or in smaller classes, using all of their data points might allow the trend to be more obvious.

Teacher Note

When graphing the load data on a line plot, or a stem and leaf plot, depending on your class and time, you may either have students graph their three individual data points or just their personal measure of central tendency (the one agreed upon by the class earlier). If you have a smaller class, it would be helpful to plot more points to allow trends to be more obvious.

Math Note

Line plots and **stem and leaf plots** can be useful when you have a lot of data points or a large range of numbers. These types of graphs will display how data is grouped, the range of the data, and some idea of central tendencies (mode on the line plot). When displaying data using the stem and leaf plot, digits are used to separate values. For example, the tens digit could be represented by the numbers on the left hand side, and the ones digit by the numbers on the right side, as in the example below. The data from a possible load table might be: 20 g, 24 g, 25 g, 60 g, 62 g, 62 g, 73 g, 74 g, 75 g. As a stem and leaf plot, it would look like this:

2	0 4 5
3	
4	
5	
6	0 2 2
7	3 4 5

Key: 2 | 0 = 20

29. As a class, discuss the patterns or trends they see in the stem and leaf plot. Similar trends should be evident on the stem and leaf plot and the line plot. Data that is bunched up tightly by color means that the data are probably more reliable than if it is spread out. The stem and leaf plot reduces the spread of data, potentially making it easy to identify which boat held the most. As the data becomes more overlapped between boat types, reasons might include variation in how the experiment was conducted, differences in boat design, lack of accuracy in measurement tools, etc.

30. **Discussion:** Referencing the three different graphs (modified bar graph, line plot, and stem and leaf plot), have students share why and when they think each type of representation would be useful. Think out loud to model how to look for trends and patterns in each graph, within one boat type and between boat types. Share one pattern you noticed in the data. Give students a chance to look at the graphs and ask them to share patterns and trends they see in the class data.

31. **Reflection:** Explain to students that they now have multiple ways of analyzing and displaying data to use with all the data they will collect on their boats. These methods should help them understand their data, make decisions, and be able to describe their results to others. In their notebooks, have students record the multiple ways of representing the data and list the conclusions reached regarding boat design and load capacity. Remind them that they will want to refer back to this throughout the module when working with the various boat data.

Activity 5: Speed Trials

In this activity, students continue to use the scientific method to explore the relationship between form and function of boats. Students will test the speed of their boats to determine if the boat's hull shape affects its speed. Students may notice that their boats are good at one function or another. Features such as how fast a boat travels or how much it can hold could be viewed as trade-offs: neither good nor bad, but a characteristic of the boat's function. Yup'ik Eskimo and other indigenous people learned through experience how form and function related to trade-offs. Your students are using their in-classroom experiments to model performance by varying form and tasks.

Mathematically, students will discover the formula for speed through physical movement. By calculating their own speed in a running test, students begin to conceptually understand that speed is the ratio of distance and time (d/t). Once they have this understanding, they will test their boat's speed by collecting, organizing, and analyzing data from the trials to determine the class average speed for each boat type.

As part of this activity, students will have an opportunity to create a number of graphs and will analyze the speed data using scatter plots for each type of boat. They will also be introduced to the mathematical idea of inverse relationships between variables as they discover that as a boat's speed increases, the time it takes to travel decreases.

Teacher Note

Aside from making necessary repairs, **students are NOT to modify their boats until they are told to do so in Activity 7**. They will be gathering performance data and finding averages with their current boats. Then, in Activity 7, they will modify their boats so their performance exceeds the averages. **Early modification will make this much more difficult for the student**.

Goals

- To use the scientific process
- To understand the need to control variables in an experiment—students explain what a variable is, which variable they are measuring, and what variables they are controlling
- To conduct an experiment (speed trials) to determine the relative speed performance of the three types of model boats
- To understand how the algorithm or formula for speed was derived
- To collect and organize speed, time, and distance data in a table for the three types of boats
- To analyze the data using statistical concepts of range, median, and mean, and meaningfully and accurately interpret the speed data and boat performance by boat type
- To graph data both in a bar graph and in a scatter plot

Teacher Note

Most schools will not have the equipment to allow all the students to do their speed tests at the same time. You can split the class and have the half that is not speed testing work on another project, such as developing a skit, poster, or game that would explain speed to third graders. These projects can be shared at the end of the testing with the best one presented to a third-grade class or their parents at the end of the module. Some classes prefer to just cheer on the experimenters. This depends on the class and teacher.

Materials

- Calculators
- Container with water for floating kayak at least 3.5 feet long

- Graph paper
- Markers
- Masking tape
- Math Notebooks
- Paper, large sheets (2 feet by 3 feet) so student groups can display and share data and scatter plots (three for class graphs and one for each group)
- Rulers
- Safety pin (one per group)
- Sinker weight, 1/8 oz. (one per group)
- Stickers (four different colors) or small Post-It notes for scatter plots
- Stopwatches (one per group)
- Storybook, *Kukugyarpuk* (optional)
- String (cut in approximately 4-foot lengths for each group)
- Students' clay model boats
- Yard or meter stick

Vocabulary

Algorithm—mathematical process or formula used for calculation or problem-solving

Controlling for variables—holding variables constant so that another variable can be measured

Inversely related variables—when the value of one variable goes up and the other goes down

Outlier—a data point that is extreme compared to the rest of the data

Range—difference between the smallest and largest data values

Ratio—the relationship between two units expressed in one of three ways: a fraction (1/2), "1 to 2," or "1:2"

Relative speed performance—how fast or slow a boat moves, in comparison to other boats

Reliable—if an experiment yields the same or very similar data when repeated

Scatter plot—a graph that displays how two variables in a data set are related

Second—unit to measure time

Speed—distance per unit of time

Valid—results in an experiment that accurately represents the variable tested

Variable—something that can be changed or controlled in an experiment; also a letter used to represent an unknown quantity in a mathematical expression

Preparation

- Read the instructions for the activity, especially the Math, Science, and Teacher Notes.
- Based on time available, decide if this activity will be done as written, as modeled to the class, or by having students perform the tests on their boats during other times throughout the day.
- Day 1: Arrange for a place where the students can run a distance of about 100 feet. This could be the gym, outside, or a hallway. Measure off and mark the distances of about 100 feet and about 50 feet.
- Day 2: Determine how many experimental stations will work for your class. It is suggested that you have at least one for every group of three or four students. Set up the experimental stations as follows:
    - Cut strings about 4 feet in length, one for each group.
    - Put a long (about 3 feet) water container at each experimental station. The container should be placed so that one edge overhangs the table. The weight that pulls the boat will need to fall freely to the floor.
    - Fill the water containers with enough water to float the model boats.
    - Attach a safety pin to one end of a string and the 1/8 oz. (3 to 4 gram) sinker to the other; the string should be long enough to stretch from one end of the container to the other end, with the weight hanging over the edge of the container. (Note that weights much more or less than 1/8 ounce may either move the boat too quickly or slowly for students to collect reliable data.)
    - Place a stopwatch and either a string and yard/meter stick or tape measure at each experimental station.
- Determine if your containers are the same size or not. See Math and Teacher Notes pertaining to specific issues concerning container size.
- If you are reading *Kukugyarpak* along with the module, pages 17 to 20 closely relate to Activity 5. Pages 21 through 25 can be read before Activity 6.

Duration

Four to five class periods.

This symbol means students need their math notebooks.

Math Note

Technically, students are not measuring **speed** in their trials, but elapsed time. In particular, they are measuring and recording the amount of time it takes for each boat to travel the same distance from one end of the container to the other, and calculating the speed from the time. Because the distance is the same in each case, a shorter time means a faster speed and a longer time means a slower speed. This means time and speed are inversely related variables. This makes sense because speed is a ratio of distance and time: speed = distance/time. This connection between the concept of speed as a ratio and their actual data collection during the speed trials can help students conceptually understand this ratio.

An audio recording of the optional reader can be found on the *Kukugyarpak* CD.

Teacher Note

Depending on time and student preferences, you may choose to allow all students to run and time each other in pairs. One student in the pair will run 50 feet and the other will run 100 feet. They will need to record their distance and time for themselves and their partners.

Instructions

1. Remind the students that the Yup'ik needed boats for many different purposes. They solved this dilemma over many generations by creating boats that could be used in different situations and purposes. Today students will determine which boat design will go the fastest. Before working with the boats, explain that they will be talking about and calculating their own speed by running and timing themselves.

2. Take students to a location where they can run about 100 feet (the gym, hallway, or outside). Be sure they have their Math Notebooks, a writing utensil, and two stopwatches for the class. Mark 50 feet and 100 feet from the same starting line and have two student volunteers run the two distances while two other students use stopwatches to time them. Have all students record the times and distances for both volunteer runners.

3. **Conjecture:** Declare the student with the shortest time the fastest. Ask your students if they agree with your statement. Ask them to write in their Math Notebook who they think is the fastest and why they chose that person.

Teacher Note

Use some of the following **prompts to foster the conversation about speed**: How do we determine who is fastest? How can we compare their speeds? Why does distance matter when determining who is fastest? How can something with one value (speed) need two pieces of information (distance and time)? If speed depends on both distance and time, how do we put them together? Ask students to give examples of speed. If they give a number, such as 60, ask them to identify the units (for example miles per hour, feet per second, etc.). Ask them to explain what the units mean.

4. **Discussion on Speed:** Return to the classroom and have students share their responses about who is the fastest and their reasons. Ask them, "What is speed?" Give students five to ten minutes to come up with an agreed-upon definition for speed. See the Teacher Note for helpful prompts.

5. **Discovering the Formula:** Using the students' definition of speed, ask them to work with a partner to try and write it as a math equation. Remind them if necessary that there are two elements to speed. After a few minutes, have the pairs share their equations for speed on the board. As a class identify the one(s) that correctly model speed. If students need help, lead the students to discover that speed is represented by the formula $s = d/t$ and is another example of a ratio. By the end of the discussion students should also understand that both the unit of time used (seconds, hour, etc.) and the unit of distance used (feet, meters, etc.) have to be the same to compare speeds between objects.

Mike McGill's class in Fairbanks, Alaska also enjoyed converting their running speeds to miles per hour.

6. Have students record their own definition of speed and have them explain why speed is a ratio in their notebooks. Have them include the formula in their entry. They will use this during the experiment.

7. **Calculating Speed:** Now that students have an idea of speed, and the formula, ask the students how they might verify who the fastest runner is between the first two volunteers. Using the times and distances recorded in their journals, have students work in small groups to calculate the speeds of the two volunteer runners and determine the fastest runner. Have students share their results, how they determined it, and justify their reasoning. Ask students what these values tell them about the speed of the two runners.

8. **Model Speed:** At the front of the room, have both runners stand at the appropriate distance they ran in one second from a common point (e.g., if Bob's speed was 10 feet/second, he should walk 10 feet from the starting line and if Sue's speed was 11 feet/second she would walk 11 feet from the starting line). Have the class compare their two locations. Ask the students how they can tell who ran the fastest and explain their reasoning.

This may be a good place to stop for the day.

9. Review the concept of speed as a ratio and the formula the students derived the previous day. Explain that today they will be testing their boats' speeds.

10. **Set Up Experiment:** Gather the students around one large container. Show the students how to attach the end of the string to a boat with the safety pin (see Figure 5.1) and the other end of the string to a 1/8 oz. sinker weight. The weight should hang over the edge of the container and table so that it can fall freely toward the floor when released. Explain to the students that each of them will have a role in conducting the experiment. The Captain says go and releases the boat. The Recorder tracks the time with a stopwatch, and the Watchman releases the string to let the weight fall and stops the boat when the weight hits the floor. (These positions can rotate within the groups.) Remind them that they will be repeating this process three times with each boat type.

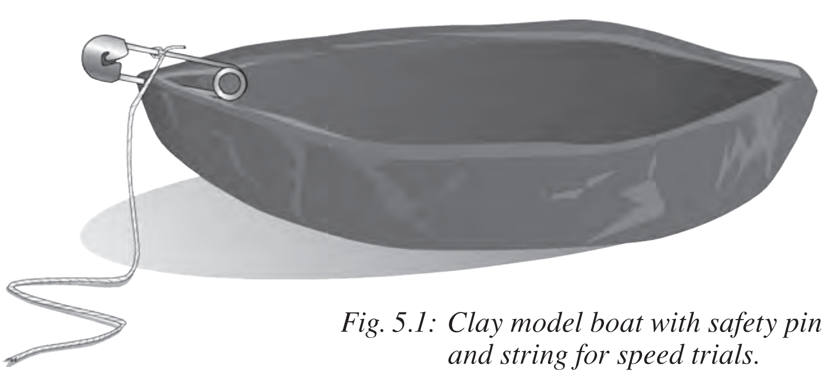


Fig. 5.1: Clay model boat with safety pin and string for speed trials.

Ratios and Algebraic Thinking

Math Note

An extension of the concept would be to have students understand that **a formula can be manipulated** in several ways to find information, as long as you have two of the three parts of information. For example, could the students determine how far each volunteer would be after another second of running? What would their distance be? After 10 seconds or 30 seconds? Students can see how the distance is a multiple of the value of speed. If a student who ran 15 ft/sec, and he ran 45 feet, how long was he running?

Teacher Note

If a 1/8 oz. sinker is not available, the equivalent weight is 3 to 4 grams. Tell the students that they should not let their sinker hang or pull on the string when they are not timing it, because this could tear a hole in the boat.

Teacher Note

Discuss with students that when conducting their speed trials, to ensure that they are collecting **reliable data** they need to release the weighted string and the boat at the same instant (this will require some practice) and not push the boat or pull on the string. This way, in all trials for all three of the model boat types, the time reflects the same force of the weight pulling the string and not a variable push or pull on the boat (i.e., leading to unreliable data). Also, in mathematical terms, by timing the boats over the same distance with the same force, students are controlling for other variables. This allows the shape of the model boat to determine the relative elapsed time and speed.

11. **Model:** Get volunteers for two of the roles and take the third role yourself. Demonstrate taking a speed measurement by measuring the distance the boat will travel and the time it takes for the boat to travel the distance. Record the time and distance values on the board and have the students calculate the speed of the test run (Figure 5.2). During the modeling process, have students identify variables that might influence their results. Have the students create a list of the conditions in their notebooks that everyone needs to follow to ensure that extraneous variables won't impact the validity of the test. If necessary, point out the importance of the scientific method (repeated testing helps with reliable data, and controlling for all variables helps with validity).

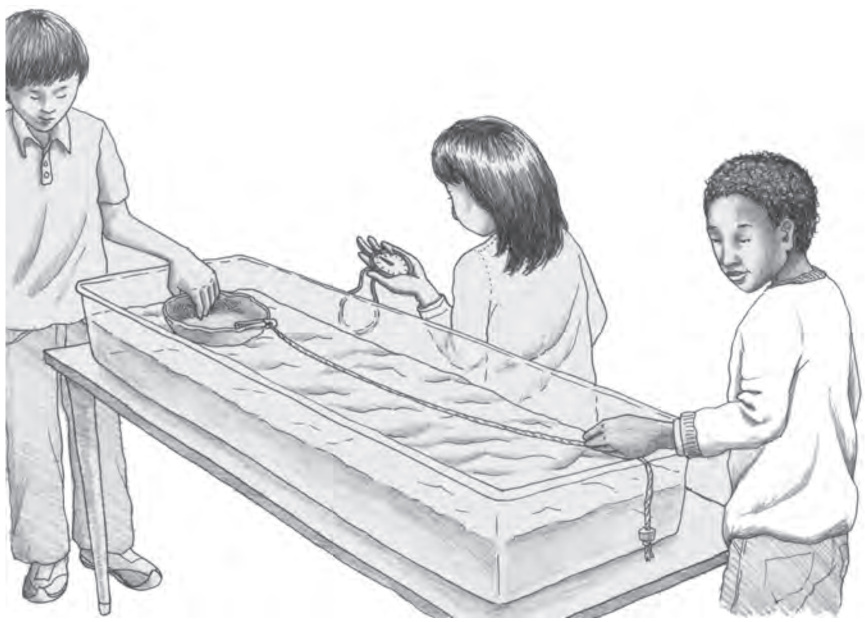


Fig. 5.2: Students running the speed experiment.

12. **Conjectures:** Ask students to make conjectures about which boat will be the fastest and which one will be the slowest. Have them write their conjectures in their Math Notebooks and include the reasoning for their choice.

13. **Organizing Data:** Before the experiment, discuss as a whole class what data will be collected and how they think they need to set up the data table. Remind them, if necessary, what they learned about the variables affecting speed (time and distance) and the number of trials they will be doing. Ask students, in their groups, to create a table for collecting and organizing their data in their notebooks. Figure 5.3 is a sample data table.

Math Note

When using **containers of a different size** your students' time data will likely be different from the above sample data, but should be organized in a similar way (by boat type and individual trials) and should show similar patterns. If the students use different length containers, the times will be different, but the speeds should be comparable.

distance 1 m	V-Bottom		Round Bottom		Flat Bottom	
	Time (seconds)	Speed (m/s)	Time (seconds)	Speed (m/s)	Time (seconds)	Speed (m/s)
Trial 1	4.32	0.23	6.89	0.15	4.68	0.21
Trial 2	4.05	0.25	7.09	0.14	5.00	0.20
Trial 3	4.35	0.23	6.47	0.15	4.68	0.21
Mean	4.24	0.24	6.82	0.15	4.79	0.21
Median	4.32	0.23	6.89	0.15	4.68	0.21

Fig. 5.3: Example speed data table.

Students organizing data by creating their own table.

14. **Experiment:** Have the students get into their groups of three and do a few trial runs to practice the process before collecting their data. When they are ready, have them conduct the experiment, repeating it three times for each boat. They will need to record their data in tables in their notebooks.

15. **Speed Calculations:** Once the times are collected, the students can use their calculators to calculate the speed (distance/time) for each trial, referencing the formula they recorded in their notebooks the previous day. They should record their work in their notebooks and put the speed values in the data table. Students who are struggling should be encouraged to visit others for help.

16. **Model Graphing:** Once your students have completed their speed trials and their calculations, as a class determine the best way to graph the time data for the boat information. As a class decide on a title and the x and y axis labels. Model graphing one student's set of data on the class graph. Then, have students practice graphing their group data in their notebooks (see Figure 5.4). Encourage them to help one another.

Graphing—students take data they collected in a table and create an appropriate graph

Math Note

Range is the difference between the greatest and smallest data point. It shows how spread out the data is. For example if the fastest time on V-bottom boats was 2 seconds, and the longest time was 6 seconds, the range is 4 seconds. The range can also determine the scale and spacing for values on the y axis. Determining the range helps when drawing the graph so that the data is not clumped together in one corner.

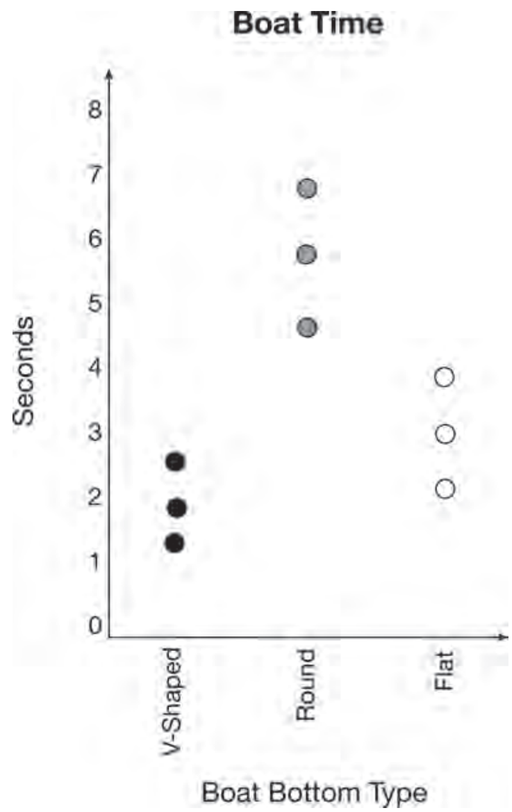


Fig. 5.4: Graph of the three time trials for one group.

17. **Graph Time Data:** As students finish their own graphs, have them post their three data points (time) for their boat on the class graph at the front of the room, using color-coded stickers or Post-It notes.

18. **Analyze Data:** Once all the students have graphed their data, refer to the class time graph and help students analyze the data by identifying and interpreting patterns they see by modeling verbally some conclusions that can be made. This discussion might include topics from the following list: outliers, range, difference between boat types, difference within a boat type, tank size, reliability, other variables that influenced outcomes, etc. From this discussion, have students record what they found out about each boat type with respect to time in their notebooks (e.g., the V-bottom boat took the least amount of time).

19. **Graph Speed Data:** Explain that they will now graph the other set of data they calculated, speed. Have students create their own speed graphs in their notebooks using their group data, while you do the same on a new x/y plot with a sample data set. Again, as students finish their own graphs, have them post their three speed data points for their boat on a new class speed graph (Figure 5.5).

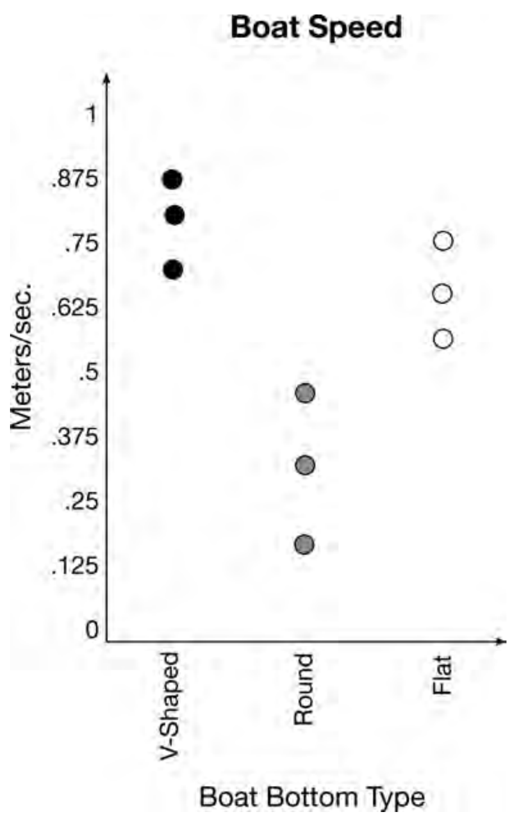


Fig. 5.5: Sample class graph of speed data, with three different speeds represented for each boat type.

20. **Analyze Data:** Once all of the students have graphed their data on the class graph, refer to the class speed graph and help students analyze the data by identifying and interpreting patterns they see. This discussion might include topics from the following list: outliers, range, difference between boat types, difference within a boat type, tank size, reliability, other variables that influenced outcomes, etc. From this discussion, have students record in their notebooks what they found out about each boat type with respect to speed (e.g., the V-bottom boat had the highest speed).

21. **Analyzing Data Sets:** Ask your students to compare their conclusions from the two previous graphs (e.g., the V-bottom had the shortest time and the highest speed). Ask them to explain how the kayak can be the lowest on one variable and the highest on another. Have students compare and look for patterns or trends they see between the two graphs, class time and speed graphs (Figure 5.6). Discussion points might include the inverse relationship between the graphs (e.g., V-boat has a short time and a high speed), range, differences between boat types, differences within a boat type, etc.

Teacher Note

Below are some helpful **prompts while analyzing data** with your students.

Ask students to think about which boats have greater speed, the ones that have shorter elapsed times or the ones with longer elapsed times (shorter times mean greater speed because less time is needed to cover the same distance; also students can see that for the shorter times, the model boat moves faster in the water). This is a good place to introduce the concept of inversely related variables.

If the tanks are different sizes, what quantities can you compare and which ones don't work? Speed can be compared, but time and distance can't.

What can you say about how the shape of a model boat helps to determine its speed? In our data, the V-shaped boat is the fastest, the flat-bottom is next fastest, and the round-bottom is the slowest. In general, your students should find the same pattern and relative speeds in their data.

How close are the mean and median data values for each type of boat and what does that tell you? If the mean and median are very close in value for each boat type, this says that the data is relatively balanced.

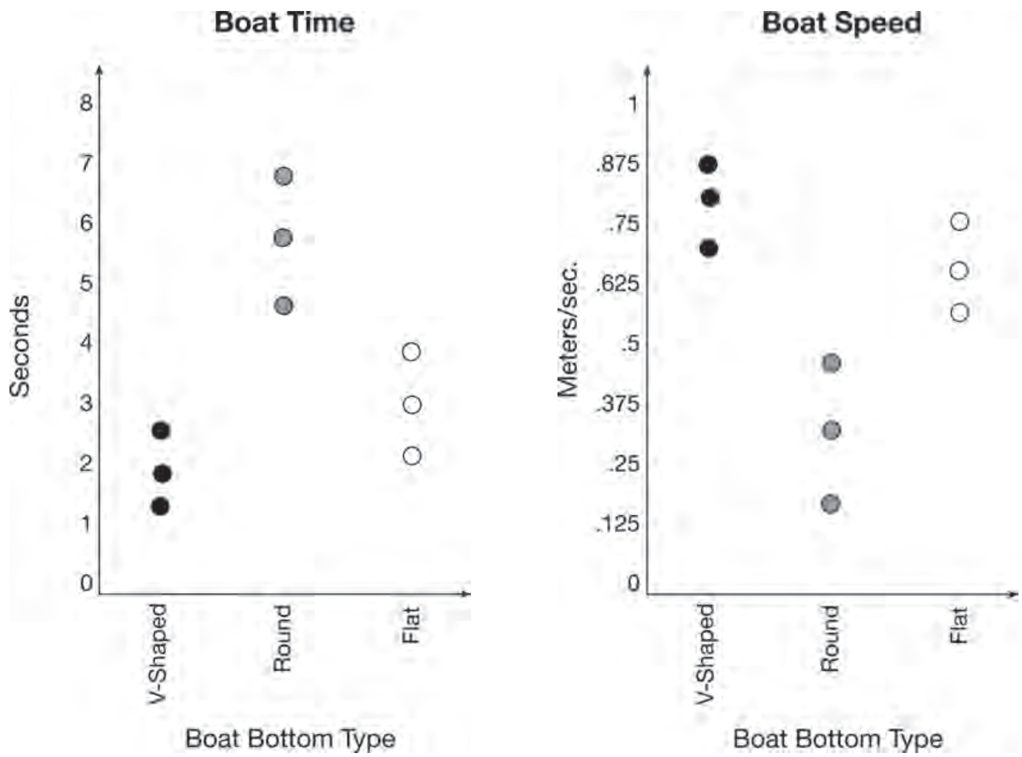


Fig. 5.6: Class graphs of time and speed, modeling the inverse relationship between the variables.

22. **Graphing Challenge:** Remind them that scientists always look for different ways to represent the data, to find other patterns. In their small groups, ask students to create one more graph that shows speed and time on the same graph. Provide materials for graphing they may need (stickers, graph paper, markers). Figure 5.7 is an example of the data graphed.

Teacher Note

Writing the formula s = d/t on the board might help point out why the **relationship between distance (the numerator) and time (the denominator)** exists. As the denominator gets larger, the value (speed) gets smaller. This shows why the graphs are inverses of each other.

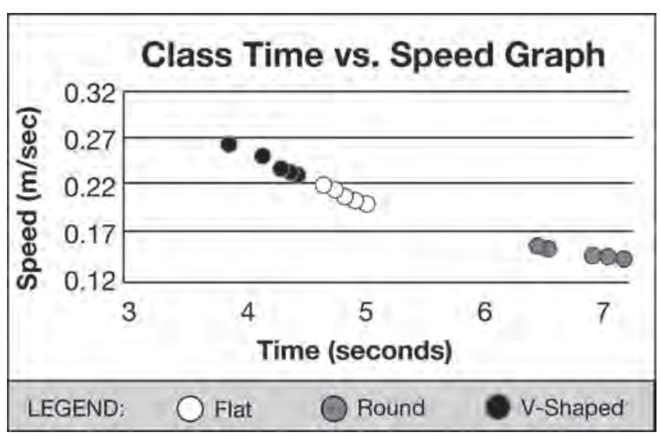


Fig. 5.7: Time versus speed graph includes all three boat types and five trials.

23. Once the graphs are completed, have student groups share them with the class. As a class review these graphs and look for patterns and trends in the graphs. What happens if the students put speed on the opposite axis? Help students analyze these graphs and find the inverse relationship between the variables.

24. As a summary, have students describe how speed and time are related to each other (inverses) in their Math Notebooks.

This may be a good place to stop for the day.

25. **Central Tendencies:** Now that students have looked at the data graphically, they will use central tendencies to analyze the data. Ask your students why they need to find a value that best represents each boat.

26. **Model Central Tendencies:** Ask a student to model how to find the mean and median time and speed of a small sample data set. Students will need to do the same for their own boat data and finish filling in the data table (Figure 5.2) in their notebooks.

27. **Finding Class Central Tendencies:** To find the class averages, have students record their mean speeds for their boats on a class table (see Figure 5.8) so all of the data is in one location. Assign students, by their boat type, into groups to calculate the mean and median class boat speed using the class data table for their specific boat type.

Boat Speeds			
	V-Bottom	*Round Bottom*	*Flat Bottom*
Group 1			
Group 2			
Group 3			
Etc.			
Class Mean			
Class Median			

Fig. 5.8: Example of class data table for mean boat speeds.

28. Once the mean and median speeds are calculated for the class data and recorded on the data table, have volunteers place contrasting stickers for the two values on the speed graph the class made in step 21. Have the volunteers label each dot as the mean or median.

Form and Function

29. **Picking a Central Tendency:** Now that the two central tendencies are represented on the graph, as a class have students discuss and determine which one best represents the speed data. Have them justify their answers and come to a consensus as a group on which value they will use to represent their speed data. Once a value is decided on, they should be recorded in the student notebooks, and they should record a statement about speed performance of each boat type (e.g., the V-bottom went the fastest, the round bottom went the slowest, and the flat bottom was between them).

30. **Reflection:** Have students reread their conjectures from the first day and answer the question: Does the data support their conjectures? Why or why not? Have them explain why a particular boat form may have performed better than other boat types in the speed trial.

31. As a class, review and discuss the different purposes of the kayak (*qayaq*: V-bottom), the log raft (*angyarrluk*: flat bottom), and the skin raft (*angyaqatak*: round bottom). Does the form of the boat serve its function with respect to speed? Have students support their arguments with data from their notebooks.

32. **Impact of Experiment:** Have your students determine what, if anything, they would change about their boat to optimize its speed performance and explain why in their Math Notebook. They can refer back to this in Activity 7 when they optimize their boats.

Activity 6: Testing the Stability

In the previous experiments, students found that the three primary boat designs have different properties with respect to load and speed. Another aspect of boat performance is stability: a measure of a boat's ability to withstand tipping. In this activity, students will test the stability of their model boats by floating them in water and hanging weights over the side, measuring both the amount of weight used and the degree (angle) of tilt before tipping over. Students will only perform the experiment on their boats once, to minimize data collection for ease of analysis.

Students will make a personal protractor to gain an intuitive grasp of angles. They will use a commercial protractor to measure the angle of tilt. The personal protractor comes from the Yup'ik method of folding used while constructing stars and circles for pattern making. As in other trials, students will use the scientific process by making conjectures, testing their boats, and using statistics and graphing (scatter plot) to determine the relative stability of their boats.

Teacher Note

Students are NOT to modify their boats until Activity 7. They will be gathering performance data and finding central tendencies with their current boats. Then, in Activity 7, they will modify their boat to optimize its performance, in an attempt to exceed the class averages. Early modification will make this much more difficult for the student.

Goals

- To use the scientific process and control for variables
- To construct and use a protractor to measure angles
- To compare the stability of the three different types of boats
- To understand the need to control variables in an experiment—students explain what a variable is, which variable they are measuring, and what variables they are controlling
- To conduct an experiment to determine the relative stability performance of the three types of model boats
- To collect and organize weight and angle measurements in a table for the three types of boats
- To analyze the data using statistical concepts of range, median, and mean, and meaningfully and accurately interpret the stability data and boat performance by boat type
- To construct a scatter plot of weight versus the degree of tilt in a boat

Materials

- Balances
- Graph paper
- Handout, Circle Outline for Protractor Construction (1 per student)
- Large pieces of paper to display data and scatter plots
- Markers
- Math Notebooks
- Paper and pencils

- Pattern Blocks
- Paper clips to make weight clips for model boats
- Protractors
- Scissors
- Small amount of clay
- Small water containers with water, same ones as Activity 3
- Storybook, *Kukugyarpuk* (optional)
- Straight edge
- Straws or popsicle sticks, one for each boat
- Students' model boats
- Tape
- Washers of various weights

Resources

These resources show or demonstrate stability in watercraft.

- http://www.youtube.com/watch?v=jKEEzKYxfpE
- http://www.youtube.com/watch?v=1vX2wmuZUV0

Vocabulary

Angle—the amount of rotation between two lines meeting at a point

Degree—unit used to measure angles; 360 degrees make a full circle

Ordered pair—a pair of numbers used to locate a point on a coordinate plane, written in the form (x, y) where x is the x-coordinate and y is the y-coordinate

Protractor—device used to measure angles in degrees

Scatter plot—a graph that displays how two variables in a data set are related

Stability—tendency of a boat to return to level or neutral position after a roll or tilt

Variable—something that can be changed or controlled in an experiment; also a letter used to represent an unknown quantity in a mathematical expression

Δ Notation—the Δ symbol is the Greek letter delta and is used to indicate change in measurement, variable, or other quantities

Preparation

Day 1 of this activity will be using paper to construct a protractor. From Day 2 onward, students will be using the boats, so the preparation is split into two parts.

- If time is a concern, remove the work with angles. Students can collect data by keeping track of the number of washers that the boat can hold before tipping over. This would allow for data analysis regarding stability, but would eliminate the use of angles and protractors.
- If you are reading *Kukugyarpak*, pages 25 to 30 closely relate to Activity 6. Pages 31 through 41 can be read before Activity 7.

An audio recording of the optional reader can be found on the *Kukugyarpak* CD.

Day 1

- Based on time available, decide if this activity will be done as written, as a class, or by having students perform the tests on their boats during other times throughout the day.
- Have a supply of pencils, paper, protractors, and scissors available.
- Make copies of the blackline master circle for each student.
- Practice folding a 90-degree angle into three 30-degree angles.

Day 2+

- Make two hooks for the demonstration (Figure 6.3) and a straw or popsicle stick mast for a boat (Figure 6.5).
- Set up one water station completely (with protractor and paper) for modeling the process with the students (Figure 6.3).
- Fill small containers with enough water that the bottom of the boats will be a couple inches above the bottom of the container.
- Have protractors and tape available.
- Make sure washers have a variety of weights (1 or 2 to 5 or 6 grams).

This symbol means students need their math notebooks.

Duration

Four class periods.

Instruction

1. **Introduction:** Ask your students what happens if you step into a small boat and put all of your weight on one side. Have students share their ideas. Tell the cultural story told by the late elder Sam Ivan of Akiak (below), and discuss the idea of stability.

Using a Kayak

By Sam Ivan, translated by Eliza Orr

Sam Ivan, a deceased elder of Akiak, said that when he was thirteen or fourteen, his father told him that the first thing he had to learn about his kayak was how to get into and out of it safely. They had a way of stabilizing the kayak using the small boat hook stuck into the ground vertically beside the kayak. Before learning those skills he had to learn to make sure that his shoes or footwear were clean, so he had to wash them to keep the kayak clean inside. He also had to learn to tie up the kayak so it wouldn't float away.

When he got into the kayak, he was told not to put his knees together, because if it tipped, he would just kind of tip over with it. Instead he should have his knees spread far apart so he could balance the kayak (personal communication with Sam Ivan at a workshop in Fairbanks, Alaska, 2002).

An audio clip of this story can be found on the *Kayak* CD.

2. Ask students to explain in their own words what stability is and share their ideas with the class. Explain that Sam Ivan was referring to stability: a measure of how hard it is to tip a boat over. Explain that they will be investigating the stability of their model boats.

3. **Conjectures:** Ask students to make conjectures about which boat they think will be the most stable and which one will be the least stable. Have them write their conjectures in their Math Notebooks and include the reasoning for their choice.

4. Have students brainstorm possible ways to measure a boat's stability. Help the students realize that they need a tool to measure it. What tool could measure how much a boat tips over? Lead the discussion to the idea of a protractor.

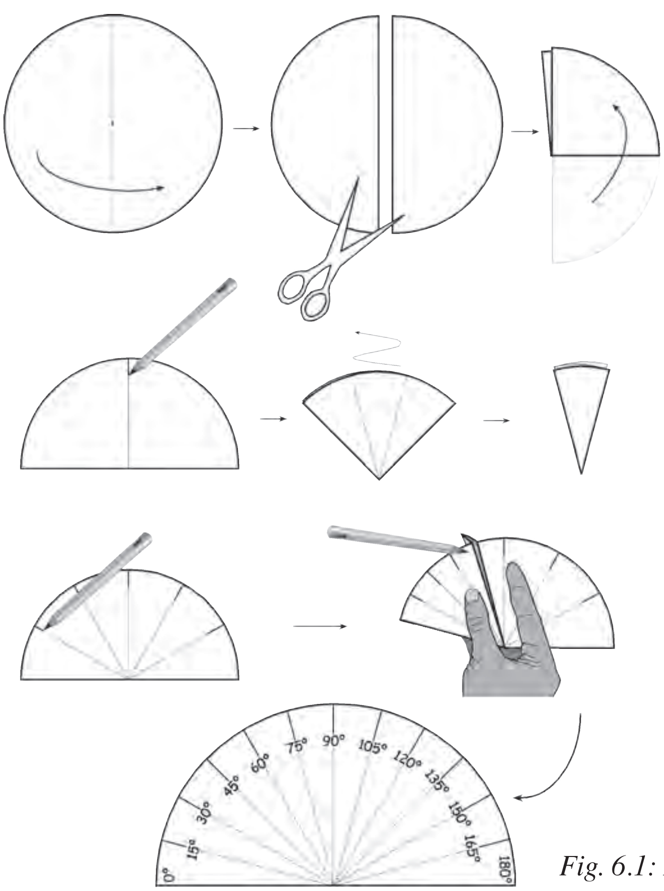
5. **Model Making a Protractor:** Explain that by using a method of folding paper, learned from Dora Andrew-Ihrke, you make your own protractor to measure angles. As students watch, model the process in silence, except to stress that all folds must go through the center and each fold will make equal partitions of the whole. See "how to fold a protractor" in the Math Note.

Math Note

The process of making a protractor uses concepts of equal partitions, halving, and fractions. As you follow the directions below, stress the importance of the center point. The folded sections represent equal partitioning of the whole (as long as they pass through the center point), and by helping stress this to the students it will help them fold more accurately. The center point is also a valuable concept when measuring angles, because the measurement always begins from the center.

How to fold a protractor:

1. Fold the circle in half, through the center. Open it up again.
2. Cut the circle in half along the fold line. Set one half aside.
3. Take one semicircle, fold it in half, open it up, and add a tick mark at the edge of the circle on the fold line (90°).
4. Refold and now make a trifold using a snake fold (the sections made should be even and folded through the center point).



5. Open up the semicircle and add tick marks at the edge of the circle at the crease marks (these represent the 30° increments on the protractor).
6. While open, make crease lines between the existing tick marks by folding the edges of the circle to align two adjacent tick marks (you are now adding 15° increments to the protractor). Open and add tick marks.
7. Continue this process until you have all 15-degree increments marked off around the semicircle (there will be 11 tick marks around the edges when complete).

Fig. 6.1: How to fold paper to make a protractor.

Dora Andrew-Ihrke

Dora, originally from Aleknagik, is a Yup'ik consultant who works with the MCC program. She learned a method of folding from her late mother, and she has applied the same process and logic to make angles for an accurate protractor.

6. Show the completed protractor with tick marks. Hand out a copy of the Circle for Protractor Construction and challenge your students to make their own protractor, using this method or folding their own way, but remembering the importance of the center and folding into equal sections. Have them determine and label the angle values for each tick mark on their protractor as accurately as possible. Engage in joint productive activity by making another protractor so others can watch and follow the process a second time.

7. When students are done, have them compare their protractors with a partner. Ask students to reflect on their protractor: have them explain to their partner how they determined the angle measurements. How are the protractors the same? What is different? Is one protractor more accurate than the other? How can you tell? Is there another way to fold it? Can you change what you did to make it more accurate? Try it!

8. **Discussion:** As a class, have a few students share how they made their protractors and how theirs compared to their partner's. If students used a different but equally effective method of folding, encourage them, and use it to discuss the accuracy of the protractors. For example, repeated half folds might make increments of 22.5°, where folding by thirds and halves may result in 15° increments.

9. Have students compare their protractors to a commercial protractor to see how accurate their folding was. They may refold a new one to make improvements. (Figure 6.2)

Fig. 6.2: Checking the personal protractor measurements against a commercial protractor.

10. Once students are satisfied with the accuracy of their protractor, have them record in their notebooks the steps they used to make their own

protractors, how they adjusted it to be more accurate, and how they determined the degrees each fold line represented.

Estimation and Angle Measurement

11. **Formative Assessment:** Give out several blank sheets of paper and have students put away all protractors. Ask students to close their eyes and visualize their protractor. Call out angle measurements (for example 60°), then have them open their eyes and by estimating, draw it on their paper with a pencil and straight edge. Have them check their angles with their personal protractors. Repeat as necessary. Continue practice with other angles: 45°, 30°, and 120°. Optional: If they have mastered angles up to 180°, you can try angles greater than 180 degrees. For example, 270° is 360° minus 90°, so they would make a 90° angle downward.

"I like this step because it gives me a quick assessment to see if the kids understand the concept."

—Laura Scholes, a 6th grade teacher from Juneau, Alaska

This might be a good place to stop for the day.

12. Ask the kids to explain why they made a protractor and how it is going to help them with their boat experiment. Explain that they will now test the stability of their boats by measuring how far the boats tip before taking on water.

13. **Set Up Experiment:** Use a boat and demonstrate how to attach a mast to the boat by using a small piece of clay on the end of a straw or a popsicle stick vertically to the center of the boat. Then add the paperclip hooks (Figure 6.3) on each side. Ask students to explain why two hooks are used, if they are only hanging weights on one side of the boat. Ask them to identify other variables they will need to control throughout the stability experiment (mast through zero, the angle they read the protractor from, how they adjust the boat, etc.). Place the boat in the water and adjust the mast to align with the 90° mark on the protractor (Figure 6.4). Ask students why it is important that the mast needs to pass through the center of the protractor when measuring angles.

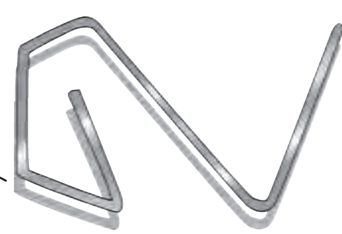


Fig. 6.3: Paper clip bent into the shape needed to hang weights on the boat.

Science Note

In this experiment students are only finding the angle of tilt on their boat once, instead of three times as in the other experiments. Because students are going to be asked to graph the change in the angle of tilt, it will be simpler to have only one trial of data points to work from. If time allows, have students do three trials using the three final sinking angles to calculate an average for their boat. They can choose which trial to use for the graphing exercise.

14. **Model:** Explain that each student in the group will have a job. The Captain's job is to keep the boat aligned with the protractor through little nudges. The use of a pen or pencil can help make minor adjustments to the boat's position. They should not hold the boat. The Watchman reads the protractor angle by aligning the mast with the hole in the center of the protractor and by bending down to read the protractor from the same spot each time to minimize errors. The Recorder gently places washers on the hook and records the weight they added and the angle the Watchman reports. With two volunteers, model how to add washers as weights on one hook and how to record the tilt of the boat by measuring the angle the mast shows. Model how to find the change in the angle by doing the calculation. Continue adding weights and recording the angle until the edge of the boat goes underwater. Have students practice reading the mast angle and calculating the change.

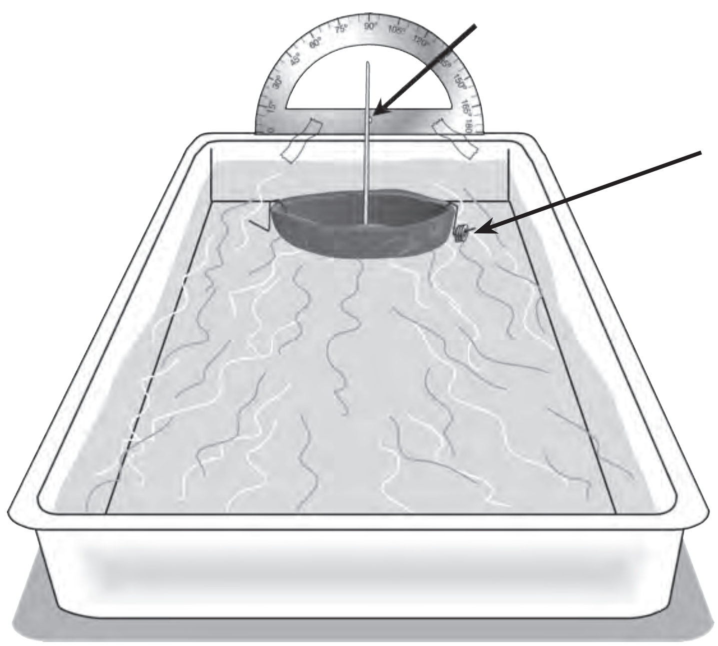


Fig. 6.4: The stability experiment setup, including washers loaded on one side of the boat. Note how the straw and the center point of the protractor are aligned to measure the angle accurately.

15. **Organize Data:** Once the students have an idea about the experiment, as a class determine the table they need to create to collect the data (see Figure 6.5 for an idea).

16. At this point the students might recognize that they do not know the weights of the washers. Have them share ideas on how they might solve this problem and allow them to calculate the weight(s) when they are setting up for the experiment.

V-Bottom Boat		
Weight (g)	*Angle*	*Change (Δ) in Angle from 90°*
0	90	0
4.5g	81	9
9g	79	11
13g	73	17
18g	70	20
22.5g	66	24
24.5g		sinks

Fig. 6.5: Example data table for stability data for one boat.

Math/Science Note

When we are looking for a change rather than the exact reading, scientists use the **Delta notation**. Delta is a greek letter that looks like a triangle (Δ), and it is used to mean 'change in,' so Δ Time is the change in time and Δ Angle is the change in the angle. In this case, we want to know how much the boat tilted, not what the final angle is, so we subtract the larger from the smaller and say that this is the change in the angle due to the weight applied. Determining the change is actually subtracting the initial number from the final number, but in this case that could give a negative number if the weights are put on the left side rather than the right. The sign of the change (negative or positive) tells us which direction the boat is tilting. Since we don't care which direction, we just use the difference between values, e.g., the larger angle minus the smaller angle.

17. **Preparation for the Experiment:** Provide small groups of students with pattern blocks, balances, and a set of washers so they can use their algebraic reasoning to find the weight of the washers (as in Activity 2). Have them determine the weight(s) of the washer(s), recording the value(s) and how they figured it out in their notebooks for reference. They also need to make two hooks and a mast for the group.

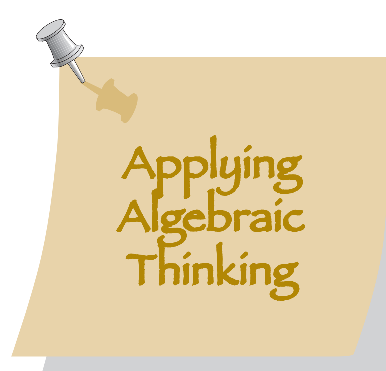
18. **Experiment:** Allow students to set up their stations: containers with water, protractors attached to the container, and a piece of paper behind it to help when reading the angles. Have the small groups conduct the experiment, switching roles for each boat.

Teacher Note

Measuring the angle of change requires two steps by the students. They will need to record the angle the mast lies at after the weight is added, and then they will need to calculate the change in degrees from the starting point of 90 degrees.

Teacher Note

Students should add enough weight to get the mast of the boat to tilt at least 5° on their first measurement. The weights are small enough that the first washer or two has very little effect on the boat's tilt. Note that the angle could be greater or less than 90 degrees, depending on which side of the boat the weight is added. If a weight is accidentally dropped into the water, it should be left until the trial is done and the boat has sunk, so as not to disturb the boat.



At the end, each student should have a copy of their boat tilt data in their Math Notebook. They will then need to calculate the change in degrees.

Science Note

When the boats are close to sinking, an observant student may notice that the water's edge is slightly higher than the boat's edge, but the water is still not coming in. This is due to the **surface tension** of the water. This is why dew forms drops rather than running off of a surface. Drops from a faucet seem to stretch out and then, once free of the faucet, become round because of surface tension. Many bugs and some small lizards use this tension; it allows them to run across the surface of the water rather than to swim.

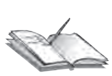
Teacher Note

Remind your students to find the range of the whole data set to help them determine what numbers should be on the axis. Point out to your students that they should use three different colored markers, one for each type of boat. Students have a set of data for each of the three boat types, so each should be distinct on the scatter plots, making it easier to compare the relative stability of each of the three boats. They should include a key for their scatter plot. Students can think of their stability data as ordered pairs in (x, y) form, with the first coordinate being the weight and the second being the angle of tilt, e.g., (weight, angle).

Math Note

The data collected consists of two distinct measurements that are linked—a weight and an angle measure. One way of thinking of the data values for stability is as an ordered pair where the first value (independent variable) is the weight and the second value (dependent variable) is the corresponding angle of tilt—e.g., (weight, angle). Because **the data values for stability are ordered pairs, they lend themselves well to being displayed as a scatter plot.**

19. **Model Graphing:** Explain that they will be using a similar method (scatter plots) to display this data as they did in the speed trials. Remind them of the idea of having two variables and ordered pairs, and how the stability data can be represented on a scatter plot with one variable on each axis: weight and change in angle. (See Math Note above.) Draw an x/y plot on the board, having the students provide labels for the axes and a title for the graph, reminding them if necessary that the independent variable (weight) goes on the x-axis. Display some student data and model plotting a few points on the graph. Ask volunteers to practice plotting points using this data. Include a key for the graph.



20. **Graph Data:** Now that they have had some practice, have students make a graph to display the data from all three boats in their group. Provide markers and butcher paper. Display the completed graphs. See Figure 6.6 for an example scatter plot.

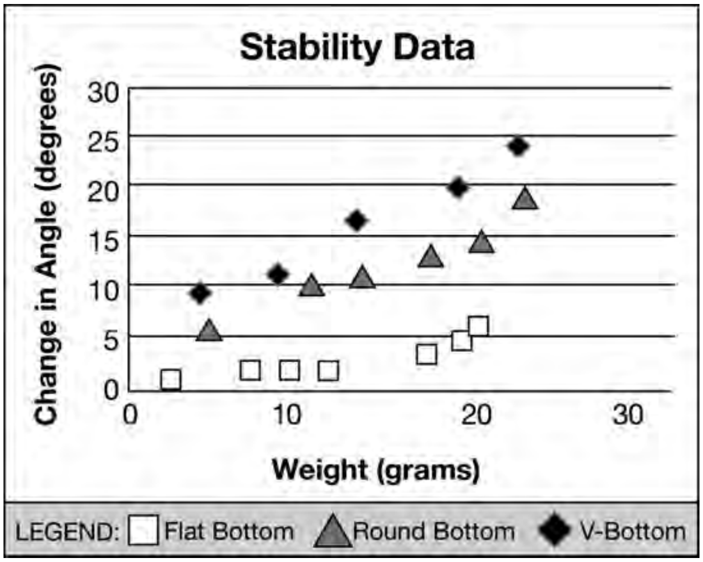


Fig. 6.6: Stability scatter plot for all three boat types from one group.

Math Note

The data for one or more of the boats on the scatter plots may approximately form a line, which means that the relationship between the weight on one side of the boat and the angle of tilt have a positive linear relationship (e.g., as the weight increases, so does the angle the boat is tilted). Mathematically, this also shows that the **relationship between the weight and the angle of tilt are proportional**. Not all boats or scatter plot data will show a linear relationship, but point it out if this occurs.

Individual groups may have different results, but try to bring the class to a shared understanding about which design tends to be most stable. Mathematically, if the x axis is weight and the y axis is angle, the boat that creates the most horizontal line (the flat-bottom boat in Figure 6.6 represents the boat with the most stability. Traditionally the independent variable (weight in this case) is graphed on the x axis, and the dependent variable (tilt) is graphed on the y axis. If the axis is reversed, with weight on the y axis and angle on the x axis, the results will be reversed, with the most stable boat being the data that is the steepest on the graph. If your class has graphs with opposite axes, ask the students why the lines are reversed. Turn one of the graphs so the axes match if necessary to show that they are showing the same thing.

Cultural Note

In general, boats that are less stable tend to be more maneuverable than more stable boats. More maneuverable boats are often preferred for hunting when quick movement can be critical. More stable boats might be necessary when hauling large loads in rough waters.

21. **Analyze Data:** Facilitate a class discussion (see previous Math Note) and encourage students to analyze and interpret the graphs from each group. Ask students to share any patterns or trends they see in their graphs. Have them identify patterns they see between group graphs. Can they identify which boat has the greatest stability? The least stability? How do they know? Are they surprised by the results? Why or why not?

22. From this discussion, have students record what they found out about each boat type with respect to stability in their notebooks (e.g., the V-bottom boat was the least stable boat).

23. **Central Tendencies:** Now that students have looked at the data graphically, they will explore how to use central tendencies to analyze the data. On a class data table, collect the tilting angle that was last measured before each boat sank, for example, 24° for the V-bottom boat (see Figure 6.7 for a reference). Ask a student volunteer to model how to find the mean and median using the V-bottom boat data. Have students find the mean and median for the other two boat types, using the data in the class table. Share and discuss these values for each boat type. Compare the mean and the median—how close are they?

	V-Bottom Sinking Angle	Flat Bottom Sinking Angle	Round Bottom Sinking Angle
	24°	7°	19°
Group 1			
Group 2			
etc.			
Class Mean			
Class Median			

Fig. 6.7: The class data table with examples of angles the boats reached before they sank.

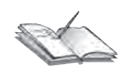
24. **Picking a Central Tendency:** With the two central tendencies calculated, have students discuss and determine which one best represents the data (using the rationale they discussed in Activity 4). Have them justify their answers and have the class come to a consensus. The one they decide on will be used for reference when redesigning their boats in Activity 7.

25. **Reflection:** Have students record the central tendencies for each of the boat types and record what this means about the relative stability for

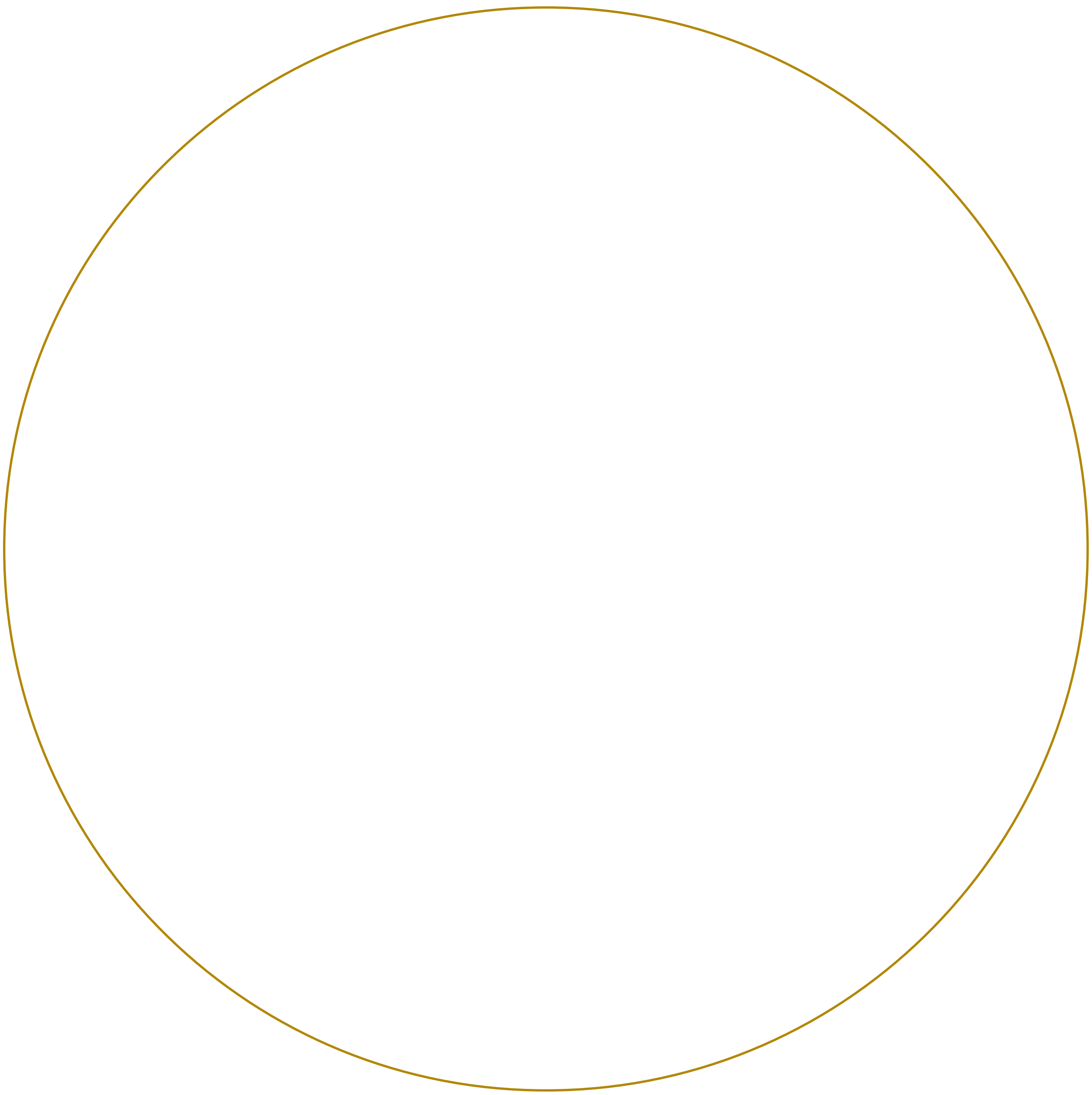
each of the boats (which one is more stable, which one is least stable). Have students reread their conjectures from the first day and answer the question: Does the data support their conjectures? Why or why not? Have them explain why a particular boat may have performed better than other boats in the stability trial.

26. Review and discuss the different purposes of the kayak (*qayaq*: V-shaped bottom), the log raft (*angyarrluk*: flat bottom) and the skin raft (*angyaqatak*: round bottom). Does the form of the boat serve its function with respect to stability? Have the students support their arguments with data.

27. **Impact of Experiment:** Have your students determine what, if anything, they would change about their boat to optimize its stability and write it in their Math Notebook. They can refer back to this in Activity 7 when they optimize their boats.



Circle for Protractor Construction



Activity 7: Redesigning the Model Boats

In this final activity, your students will review and synthesize their findings about key characteristics of boat performance (load, speed, and stability) by using the data from their explorations to improve on boat design. They will put together a performance profile for each of the three boat types with the average load, speed, and stability performances they determined in activities 3, 5, and 6. Students will then have an opportunity to redesign their clay models to optimize their boat's performance on its capacity, speed, and stability. For example, if a student wants to make the round-bottom boat, their new boat needs to go at least as fast as the class average for the round-bottom boat, as well as carry as much and tilt as far as the class averages found in the respective activities.

Refining their models engages students in a mathematical process (optimization) using the performance measures as dependent variables. Students get to decide which of the three boat types they would like to recreate, with the intent of having their new boat exceed the class averages that have been collected in the earlier activities for that specific boat type. For example, if a student decides to build the kayak now, they are attempting to build it to hold more than the kayak class average that was determined in Activity 4, go faster than the kayak average found in Activity 5, and be more stable than the kayak average found in Activity 6.

After the students have redesigned and retested their new boats and created summary posters, they can invite the community into the classroom. As a final assessment and celebration for completing this module, your class may decide to put on an event open to the community to display their clay model boats and the data they have collected about the performance of the boats. In hosting a community event, your students can demonstrate how the experiments were conducted and explain their graphs and statistical analysis by sharing their new and improved boats. In addition to students and parents, this assessment and celebration could include elders who might be willing to share their own boat stories or traditional stories about boats.

Goals

- To summarize the load, speed, and stability data for each boat type into a performance profile
- To use summarized data to decide how to redesign their model boat and explain their choice using reasoning
- To plan a performance assessment and celebration for the module that involves the community, shares students' models, and explains the data

Materials

- Data tables and plots used in earlier activities
- Experiment stations (materials), at least one for each performance criteria (load, speed, stability) from earlier activities
- Markers
- Math Notebooks
- Poster paper
- Storybook, *Kukugyarpuk* (optional)
- Students' clay model boats

Vocabulary

Optimization—to make as effective as possible

Performance profile—a summary of each boat's performance for all three performance tests: load, speed, and stability

An audio recording of the optional reader can be found on the *Kukugyarpak* CD.

Preparation

- Read through the steps below.
- Determine the time it will take for your class to finish the optimization and testing and schedule a time for the students to present to the community. Send invitations home with students, announcing the date for the community event. Invite elders who are interested.
- Locate in your own notes the class averages that were previously determined.
- If you are reading *Kukugyarpak* along with the module, pages 41 to 46 closely relate to Activity 7. Pages 47 through 49 can be read to finish the story.

Duration

Three to four class periods.

This symbol means students need their math notebooks.

Instructions

1. Explain that students will now have the opportunity to build a new boat and to optimize its performance. Explain that this means their new boat is trying to outperform (exceed) the means from the previous experiments. For example, if a student wants to build a V-bottom boat this time, they are trying to build it so it beats the previous class averages in load, speed, and stability for the V-bottom boats previously tested.

2. Introduce Henry Alakayak's story *Sea Otter Hunters* to the students as a Yup'ik story of authentic optimization of a kayak. Read the story to the class.

3. As a class, have students list the different environmental conditions experienced in the *Sea Otter Hunter* story and the purposes of the hunter's kayak (e.g., time of year, the purpose of travel, available resources, personal knowledge and know how, length of travel, and the

An audio clip of this story can be found on the *Kayak* CD.

Sea Otter Hunter

By Henry Alakayak (left)

Additional information by Mike Toyukak (right)

Translated by Anecia Lomack and Evelyn Yanez

May 2003, Manokotak Meeting

The white man like to hear this story about how we went sea otter hunting from Bristol Bay to Anchorage. It was a story back then. I will tell it as I heard it, since I didn't see it but only heard it.

When money became part of our lives sea otters were worth a lot of money. The otter pelt was worth a thousand dollars (*tiissicaaq*).

In the spring, the hunters who floated a lot in the river would begin to get ready for their trip to Anchorage. They first have to take their things to their destination before they can actually leave, that is how I've seen them do it. They brought what they would need and a few dogs to pull them, not a lot. They brought along what they needed to come home with too: enough material to make a kayak if they did put them together before coming home.

continued

So, one day in spring, in a village—the name of the village was not mentioned—several of the men began to prepare to go hunt for sea otters. As I imagined it, they wanted to begin before traveling by land became impossible [before breakup]. Not just one person, but two or three of them would get ready to go while the trail was good. And since they knew each other they probably got everything ready they would need before leaving. And just like in fishing, they would give a percentage of their earnings to their partners; they paid their companions instead of just bringing them along.

Cultural Note

Always getting ready is an important Yup'ik value (*upterrlainarluta*).

John Yuni Nelson and Carl Evon's dad [Apaayaq] used to travel from Dillingham to Anchorage. They would leave Dillingham, traveling on the Nushagak River to Portage Creek. They would continue to travel north on the Nushagak River to a place called Aqumgalugurnarli. This is a place that is not named on maps. From here, they would travel across the tundra, portaging from river to river, to Levelock. Levelock was a new village at this time. It was not called Levelock back then. From Levelock they would travel on the Kvichak River to Igiugig, an older village, on the shores of Lake Iliamna. Crossing Lake Iliamna, they would reach the village Kokhanok. From Kokhanok Bay they would portage across the tundra to Bruin Bay. Sometimes they would use a sled to help when crossing ice and snow and sometimes the sled was placed on the kayak. On their way to Anchorage they would subsistence fish and catch otter.

Fig. 7.1: Map of the route taken by hunters in the Dillingham region to get to Anchorage.

Cultural Note

An important Yup'ik belief and value is that the hunter is in a relationship with the animal and it is the animal that gives itself up to the deserving hunter. There are many stories that stress this point.

These men who hunted sea otters were called great hunters (*nukalpiat*), because of their ability to bring home money.

When a person shoots an arrow at a sea otter, and his arrow breaks, it meant that the sea otter did not want to be caught by him. But the otter wouldn't do that to the person he wanted.

continued

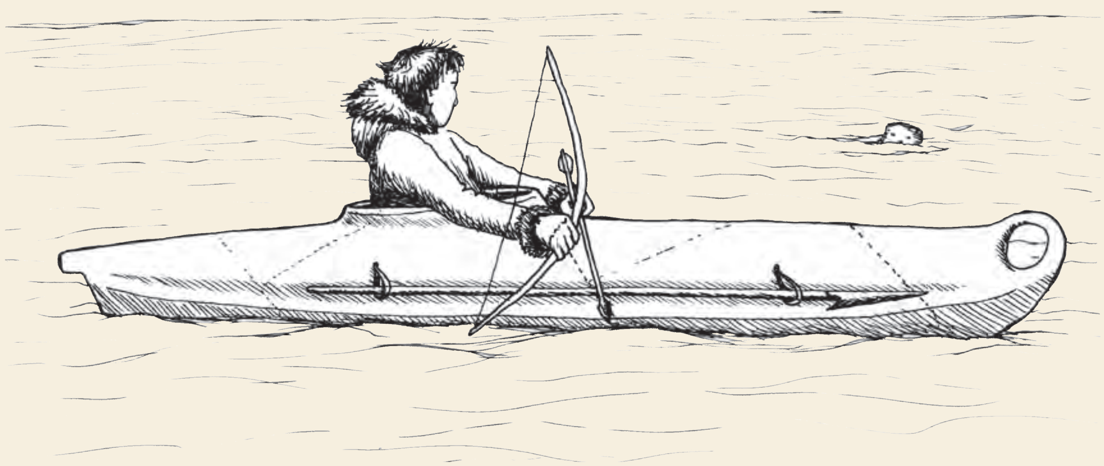


Fig. 7.2: Hunting otter from the kayak.

Whoever wounded it first, the sea otter would belong to that person, even if his partner helped finish it off. That was the way they did it. But he could give half of his earnings to his partner.

The people in Anchorage would receive them well when they arrived. They may not have spoken the same language or used the same gestures, but they still knew each other.

So, in the fall, after being in Anchorage for some time, they would work on their kayaks, if there were four of them they would make two kayaks. Two could go in one kayak. When it was time to go home from Anchorage, they would go to the huge lakes corner and over the land from there and head towards the village. They went over land in a lot of places when they traveled in those days. When they came to the end of a river they would pull their kayaks over land to the next river because they knew what they were doing.

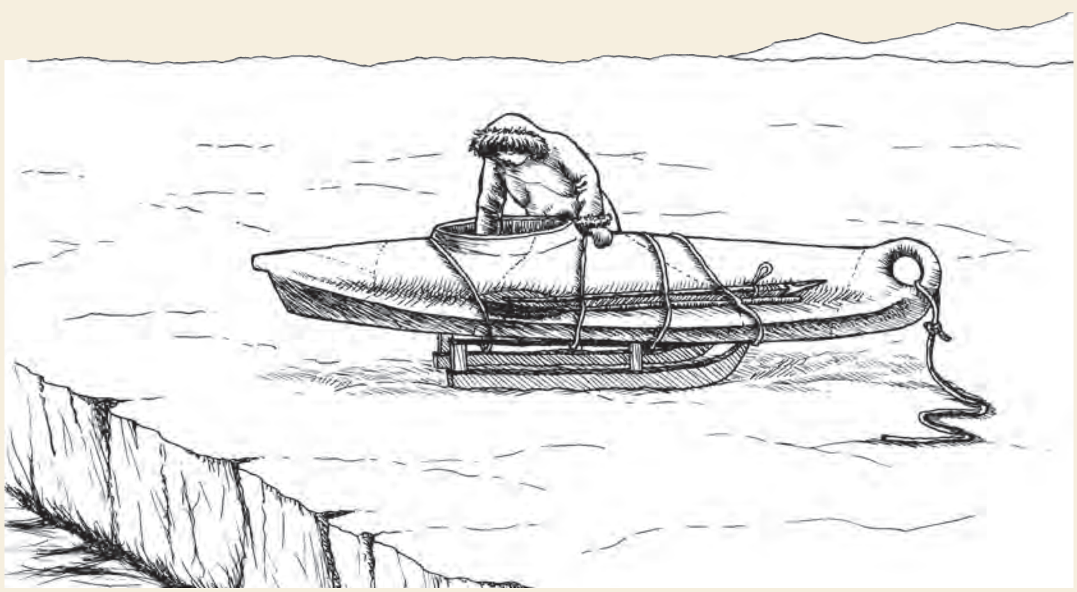


Fig. 7.3: Hunter traveling across the ice, using a sled to transport his kayak.

continued

The men who went away to hunt were hard-working men who were trying to make a living. They went after what they could get from the land so that they could live a better life and have money to buy what they want in the future.

At this time, when money became a part of our lives, people started to buy things and the men who had hunted sea otters would buy the things they wanted. There were other people who were not sea otter hunters but hunted for other fur bearers at home, and they would buy whatever the man who hunted for sea otters bought. The sea otter hunter thought he was the only rich man in the village. Whenever he bought something, the other hunters from the village would buy the same item.

The sea otter hunter began to wonder about this, "Why is he doing what I'm doing; where did he get his money to buy things?" One time the sea otter hunter watched him without letting him know that he was watching. The other hunter was using muskrats to buy what he wanted. Muskrats are very cheap. They are called *tevyuli*.

I was thinking, "How was he able to do as well as the sea otter hunter even though he was not a sea otter hunter himself?" By hunting the land persistently, he was able to make as much money as the sea otter hunter did and copy him.

At that time when they used to have feasts and things were scarce, so they collected things that they would give away to the other people: nice things such as guns and other nice things around that time. Both the land hunter and the sea otter hunter could not do this.

I think this is where this story ends because I haven't heard anything more.

number of people, animals, and cargo involved). Have students reflect on why Henry's story models real-world optimization of a kayak. Have students connect the story to the three experiments they conducted on their own boats.

4. Ask students to consider how they would use a boat. What do they want the boat to do? Have students record their ideas for their boat in their notebooks.

5. Remind your students that they have collected data on three different boat types and have analyzed how each performs with respect to load, speed, and stability. To figure out which boat would work the best for their situation, they need to gather the information that has been collected and summarize it.

6. **Organize Data**: Ask students to create a single table that will show the numerical results (averages) of all three boat types for all three tests. If they need help, explain that this table, a performance profile, will include the one central tendency identified to represent each boat type for each of the performance tests they previously conducted: load, speed, and stability. Have students work together, or provide an example similar to Figure 7.4.

7. **Gathering Data**: Once they have the table, have students look back in their notebooks to fill in the table with the appropriate values (see Figure 7.4 for an example).

	V-Bottom	Round Bottom	Flat Bottom
Load	23 grams	74 grams	61.3 grams
Speed	0.24 m/s	0.21 m/s	0.14 m/s
Stability	24°	19°	7°

Fig. 7.4: A performance profile data table with example class average data.

8. Note that scientists summarize their data by using descriptor words to explain what they found. Help students reach a consensus about how the three boat types perform relative to each other for the three criteria. Below is one way to organize the data; the students may find a way that they like better. Have students create a second table in their notebooks that clearly summarizes in words the performance profile for the three different kinds of boat (see Figure 7.5).

Boat Type	V-bottom	Round-bottom	Flat-bottom
Load	Small	Large	Moderate
Speed	Fast	Moderate	Slow
Stability	Least Stable	Moderately Stable	Most Stable

Fig. 7.5: The written translation of boat performance based on the numerical data from the profile information collected in Figure 7.4.

9. **Picking a Boat**: Have your students reread their entry about what they want a boat to be able to do. Based on that criteria, have them choose which boat type would best fit their needs. Explain that they will build the boat type and try to beat the class averages for the boat type they chose. (For example, if a student wants to haul a lot of weight between places, they may pick the round-bottom boat. They are trying to beat the class average for the three tests on the round bottom boat). If students want to improve their current boat, have them refer back to the notes they made about optimizing their boats at the end of each performance experiment. Each student should explain, in their notebooks, why they chose a particular boat type, using the data collected to justify their reasoning (e.g., I choose a flat-bottom boat because I like to hunt, and

I want to carry several seals at a time and still be stable on the wavy water conditions.)

10. **Final Experiments**: Students should build their boat of choice, make a table in their Math Notebooks, and collect data by putting their boat through three tests again. Have students perform the load, speed, and stability trials, repeating each test three times.

11. **Analyze Data**: Students need to calculate their averages for all three performances and compare them to the performance profile data table they created in step 1. They need to check to see if they, in fact, optimized their boat's performance to meet or exceed the previous class average for each test.

12. **Conclusions**: Have students make posters or other visual aids to document what they changed, why they changed it, their data and analysis, and their final results. This visual aid may be displayed during the final community sharing event.

13. The class should now turn to planning and organizing the celebration for the module, specifically sharing their model boats, data, and analyses with parents, community members, and other students.

Appendix A:
Scaffolding and Practice with Central Tendency

This appendix provides five individual activities to help supplement student practice with and understanding of the different central tendencies.

Activity 1A: Central Tendency Practice

Part 1: How Many Letters in Our Names?

First, ask students if they have ever heard the word "average" and ask those who have what they think it means. Connect students' prior knowledge about average (e.g., if something is average it is typical, or the average somehow represents an entire group of data) to the mathematics term for average, which is mean, median, or mode, but usually mean. In this segment of the activity, students will learn what the mathematical mean is.

To begin this activity, ask your students to write down their first and last names on a Post-It note. Ask them to count the total number of letters in their first and last names. (For example, Evelyn Andrew would have a word count of 6 + 6 = 12 total letters.) Now, draw a line on the board with the numbers 1 to 16 marked off (your line may be longer depending on the length of your students' names). Have the students put the Post-It notes above the number that represents the number of letters in their name on the line plot. Below is a sample line plot for a class of 12 students and one teacher, with Evelyn Andrew's Post-It note shaded.

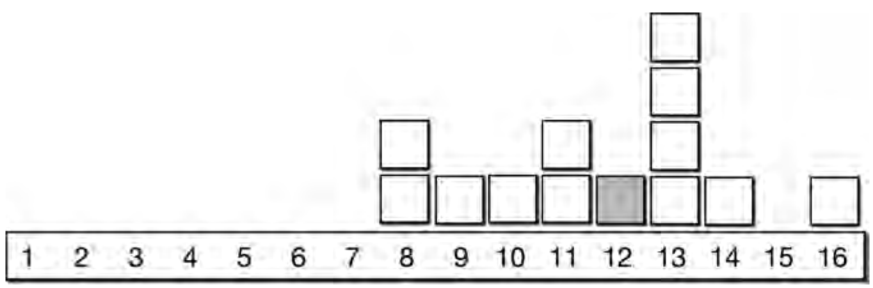


Fig.A.1: Example of a line plot with the number of letters in a name, data.

After all of your students have placed their Post-It notes, explain to the class that they have just organized their information into a line. Explain that the mathematical term for information is data and that the class data about length of names is organized into what is called a line plot (NOTE: the line plot may be given a title, like "Length of Our Names" or "Total Number of Letters in First and Last Names"). Ask your students what the line plot tells them about the length of names in the class and record students' responses on the board. Then, ask the students the following questions or use the questions to highlight or clarify students' responses:

- What is the length of the shortest name in the class? (In the sample line plot it is 8.)

- What is the length of the longest name? (In the sample line plot it is 16.)
- How many people have a name that is 11 letters long? (In the sample line plot it is 2.) Repeat this question for another name length.
- What name length occurs the most often? (In the sample line plot it is 13.)
- What name length seems to be the most typical? (For the sample line plot students would probably say 11, 12, or 13 because they are "in the middle.")
- From looking at our line plot, how many people were in the class today? How do you know? (The number of data points, or Post-It notes, equals the number of people because each person put up one Post-It note. For the sample line plot there are 13 people in class.)
- Ask your students to comment on the shape of the data distribution and what it might tell them about the length of people's names—e.g., most people have names that have similar numbers of letters.

Part 2: Range, Mode, and Median

Next, have your students construct an ordered data set from the line plot. An ordered data set is simply listing all of the data points in increasing value from smallest to largest. For the sample line plot of length of students' names, the ordered data set would be:

8, 8, 9, 10, 11, 11, 12, 13, 13, 13, 13, 14, 16.

An ordered data set is another way to organize and display data. Emphasize to your students that the data have not changed, only the way we are displaying it has. By ordering the data set we can more easily see the numerical data, whereas the line plot is typically intended to emphasize the shape of a data set rather than numeric value. Using the line plot and ordered data set, as well as your students' answers to the above questions, introduce the following statistics concepts:

- The difference between the smallest and largest data points in a set of ordered data is called the range of the data. Range is used to find the scale needed on a graph and can be used to compare the spread of data between two data sets. A data set with a range of 3 and another comparable set with a range of 20 represents a drastic difference in the spread of the data. Ask your students to find the range for the class data on length of names (in the sample data, the range is 16–8 = 8).
- The data point that occurs the most frequently in a set of data is called the mode. Ask your students to identify the mode for their data (note: students probably found the mode already in answering the fourth question about the line plot or the last question about what name length is most typical; for the sample set of data the mode is 13, which occurs four times). A set of data can have more than one mode if more than one

data value occurs with the same highest frequency; a set of data that has no data point that occurs most frequently (i.e., they all appear an equal number of times) is said to have no mode.

- In an ordered set of data, the data point that is in the exact middle is called the median. Ask your students to identify the median for the class data. For ordered data sets, which have an even number of data points, the median is halfway between the two data points in the middle (e.g., for an ordered data set with 14 data points, the median would be halfway between the 7th and 8th data points).

Ask your students which way of displaying the data they think is better and why—e.g., students like the line plot better because it conveys a picture of what data look like, whereas others may like the ordered data set better because it's easier to just write down the ordered data than to make the line plot. Both the line plot and the ordered data are legitimate ways of organizing and displaying data.

Part 3: Names of Alaska Villages and Finding the Mean

This segment of the activity introduces the concept of mathematical mean and provides an opportunity for students to make their own line plots.

Explain to your students that, just like we made a line plot for the number of letters in our names, we can make a line plot for the number of letters in the names of communities in Alaska. Put these names of communities in Alaska on the board: Hooper Bay, Kodiak, Manokotak, Mountain Village, Nightmute, and Togiak. Ask your students to make an ordered set of data of the number of letters in the name of each community and record the data set in their Math Notebooks. Your students should have the following ordered data set: 6, 6, 9, 9, 9, 15.

Ask your students to find the range and mode for this data set. Your students should find that the range is 15–6 = 9 and that the mode is 9.

Next, ask your students to each make their own line plot of this ordered data set in their Math Notebooks using Post-It notes (distribute six Post-It notes to each of your students), like the line plot made earlier on the board of the total number of letters in first and last names. Students may decide on a name for their line plot (e.g., "Number of Letters in Alaska Town Names"). When completed, students should each have a line plot made with Post-It notes that looks like this:

Explain that the mean is the mathematical center of the data. If everyone had the same number of letters in their names, that number would be the mean. To demonstrate this, ask your students to each move one of the Post-It notes from the 6 pile up to the 7 pile, and, at the same time, move a Post-It note down from the 16 pile to the 15 pile. The process would look like this:

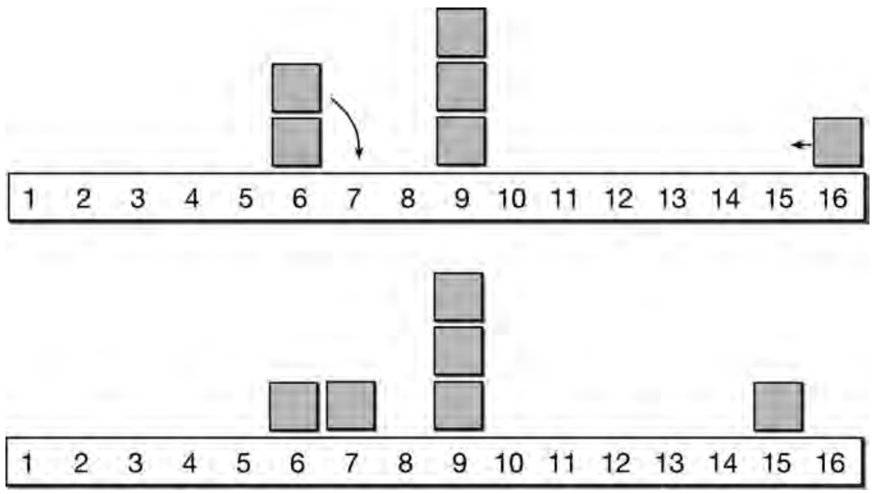


Fig.A.2: Example of how to rearrange Post-It notes on a line plot to find a mean.

Ask your students if the number of data points has changed (no, there are still six data points) and if the total sum (or value) of the data points has changed (no, one data point got increased by 1 and another got decreased by 1, so they "cancel"—i.e., add 1 and then subtract 1—so the total sum of the data have not changed). Have your students continue to move Post-It notes from each end of the line plot up by 1 and down by 1 and ask them to describe what is happening. Your students should describe that the Post-It notes "pile up" on 9 and that, eventually, their line plot will look like this:

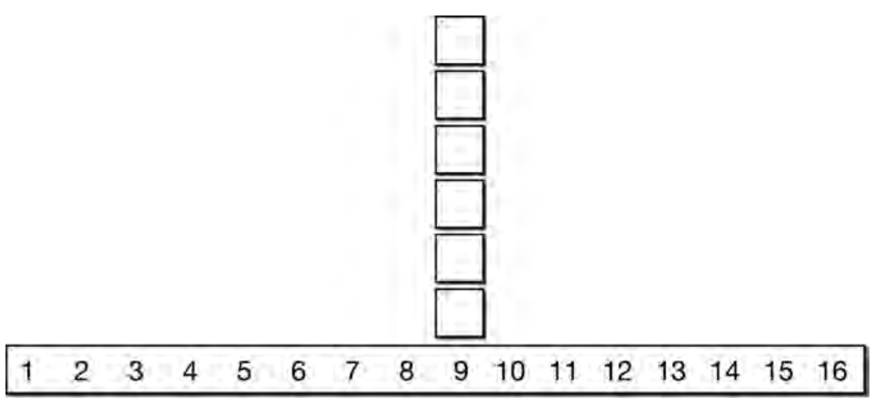


Fig. A.3: Example line plot showing the mean of 9 for the original data set.

The point on which all of the data points "pile up" is called the mean and is sometimes also called the average (note: the median is sometimes described as the physical center of an ordered set of data, whereas the mean is the arithmetic center). Therefore, the procedure of moving the Post-It notes shows that the mean number of letters in names of some Alaska communities is 9. Explain to your students that we could do the same procedure with the line plot of total number of letters in our first and last names. Be sure to point out that, while the mean, median, and mode can all be the same number, there is nothing that says they have to be.

Next, ask your students to multiply the mean in the above line plot by the number of data points and see what they get. The mean is 9 and there are 6 data points, so your students should get 9 × 6 = 54. Now ask your students to find the sum of the original ordered set of data they used to make the line plot. Your students should find that the sum of the ordered data set is 6 + 6 + 9 + 9 + 9 + 15 = 54. Point out to your students that this comparison makes sense because, as we discussed earlier, moving the Post-It notes to find the mean did not change either the number of data points or the sum of the data (i.e., increasing by 1 and decreasing by 1 each time results in no change to the sum of the data). Therefore, for any set of data, the following relationship between the mean, number of data points, and sum of the data points holds:

(Number of Data Points) × Mean = Sum of all the Data Points

This relationship also shows how we can find the mean for a set of data without making a line plot:

Mean = (Sum of all the Data Points) ÷ (Number of Data Points)

Teacher Note

The above relationship is what is typically taught to calculate the mean of a data set, i.e., "add up all the data points and then divide by the number of data points." Ask your students to use their calculators and the above relationship to find the mean number of letters in their first and last names from the class data collected earlier (note: for the sample data on total number of letters in first and last names provided earlier, the mean is 151 ÷ 13 = 11.6).

It is likely that your students will also calculate a class mean for the number of letters in their names that is not a whole number. Ask your students if it makes sense to have, in the case of the sample data, "point 6" letters in a person's name. Explain that, since the mean is the arithmetic center of a set of data, it does not have to be a whole number. In the case of the sample data, one way of interpreting the mean is that it shows that the mean number of letters for the class is 11 or 12 letters. You can also think of the mean as a balancing point for the data. Look back to the original line plot and imagine a fulcrum below the bar at 11.6. You can see that the boxes on the bar would balance.

Activity 2A: Finding a Mean with Blocks

Part 1: Village Names

Another way to help your students understand the concept of mean is to make cube towers of the data for the number of letters in the names of the six Alaska communities. Using Unifix cubes, have your students each build the following six cube towers, each representing one of the Alaska communities (each cube tower is labeled for clarity):

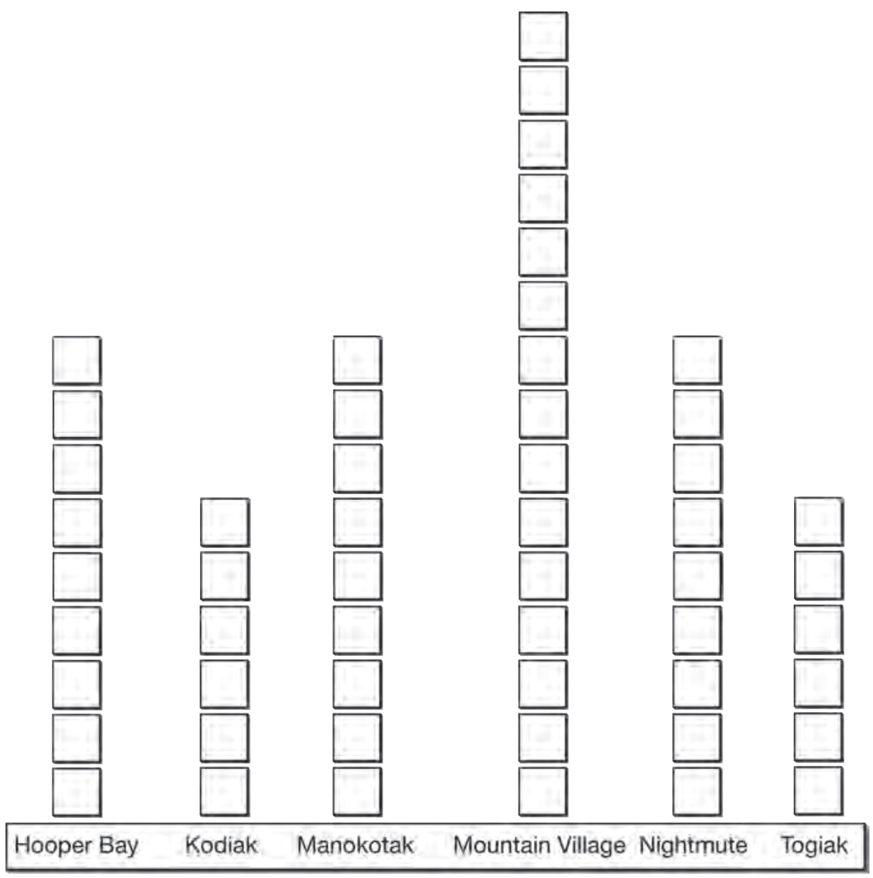


Fig. A.4: Example of cube model with the number of letters in village names.

Because each cube tower represents the number of letters in the name of the Alaska community, moving the cubes from one tower to another won't change the total number of cubes (or the total sum of the data). Ask your students to move cubes among the towers so that all the towers are the same height. Your students should find that if they remove 6 cubes from the Mountain Village tower and add 3 of those cubes to the Kodiak tower and the remaining 3 to the Togiak tower, all six towers will be the same height and will look like this (the shaded cubes show the cubes from the Mountain Village tower):

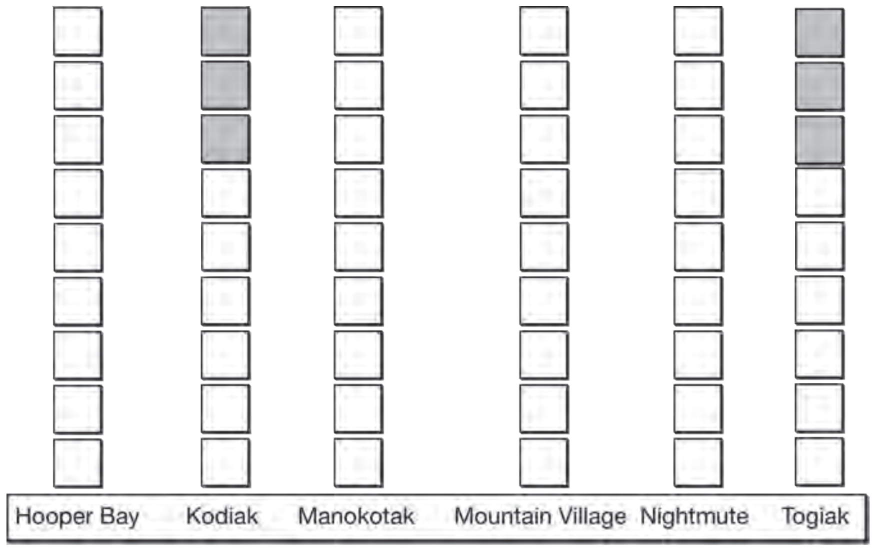


Fig. A.5: Example of cube model with the number of letters in village names rearranged to make balanced stacks.

The common height of the cube towers is the mean for the data. Just as with using the Post-It notes in the line plot, the mean of the number of letters in the six Alaska communities is 9. The same relationship also holds between the mean, number of data, and the total sum of the data: The mean multiplied by the number of data sets equals the sum of the data, i.e., $9 \times 6 = 54$.

Part 2: More Examples with Blocks

Here is another visual model of mean, mode, and median: If you have 3 stacks of 6 blocks (see Figure A.6), ask the students, what is the mode and median? Reiterate that to find the mode, you look for the numbers that appear more than once. For the median, you look for the middle number. The answer for each is 6. The mean (balance point or "smoothing out") is also straightforward. To transition students from the model to the mathematical procedure, write the formula on the board as you ask students the following questions. How many piles? (3) How many blocks in each pile? (6) How many blocks are there total? (18) Do you see a relationship between these two numbers? Possible answers are: $(6 + 6 + 6) \div 3 = 6$, $3 \times 6 = 18/3 = 6$, or you add all the numbers up and divide by the number of piles. So the formula accurately represents the mean.

Fig. A.6: Three piles of six blocks.

To deepen the transition from the model to the procedure or equation of mean, change the numbers in each pile to 8, 3, and 7 as shown below. Ask the students to find mode (none), median (7), and mean (6). Ask the students, how it is possible that the mean is the same if our piles of blocks changed? Have the students move the blocks to "smooth out" or "balance" the data (see Figure A.7).

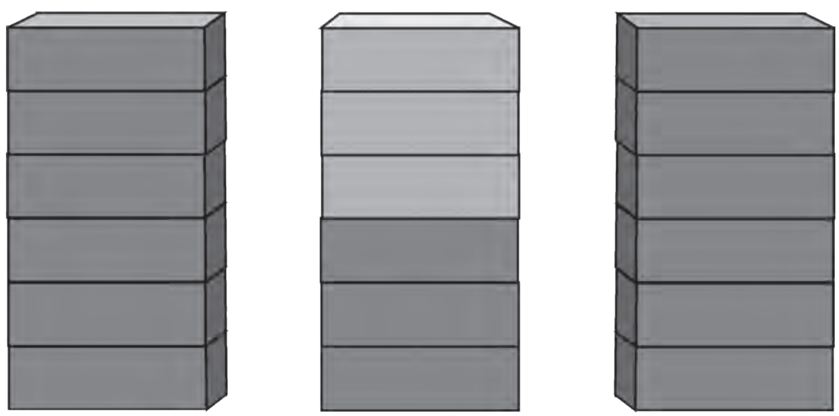


Fig. A.7: The piles of blocks representing a different problem set.

Pay attention to the connection between the "smoothing" process and how it relates to the formula for calculating mean. See if any student has an "ah ha" experience. Students should notice that the total number of objects (18) does not change regardless of the distribution and that there are three distinct collections. Average = sum of elements / number of elements. For the mean, you have to count all the blocks and divide by the number of stacks (3), as in the above example (8 + 3 + 7 = 18: 18 / 3 = 6). So you can define the mean as how high the stacks will be if they are all the same height. You can repeat the exercise again where the stacks are 1, 2, and 15. Show that there is no mode, the median is 2, and the mean is 6. Since the median and mean are far apart, we know the data is spread out, or has a wide range.

Activity 3A: A Variation on the Game of Kakaanaq

In the MCC module *Salmon Fishing: Investigations into Probability* students learn about the traditional Yup'ik game of Kakaanaq, as shared by elder Henry Alakayak. This activity uses a variation of Kakaanaq to provide students with an opportunity to collect data and then apply statistics to analyze their results from playing the game. Like Kakaanaq, this game involves tossing circular tokens or other small disks at a target, with each of the players keeping track of their own score. This game is very similar to Kakaanaq, but the method for keeping score is different.

For this game, each student takes 10 disks or tokens (counting chips from a math manipulative kit, pennies, or any small circular objects that are all uniform in size and will fit on the target could be used). Students should play the game in pairs or in small groups. Each pair or group of students should get a copy of the target (see blackline master) and receive 10 disks for tossing. Each student then takes a turn tossing the 10 disks onto the target. Each disk receives points based on the target circle it lands on, and the tossed disks should remain on the board until the end of a player's turn. After the student has tossed the 10 disks and recorded her or his score, the next student takes a turn. This continues until all the students in the group have tossed the 10 disks and recorded their scores.

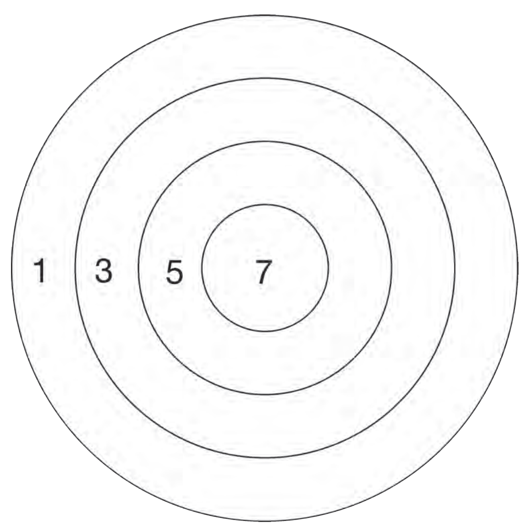


Fig. A.8: Target for central tendency practice.

After each player has taken a turn playing the game, students should make an ordered data set with their scores. Here is an example of what ordered data sets for two students might look like:

Student 1: 1, 3, 3, 3, 5, 5, 5, 5, 5, 7

Student 2: 1, 1, 1, 1, 1, 3, 7, 7, 7, 7

After ordering their data, ask students to find the range, mean, median, and mode for their data. For example:

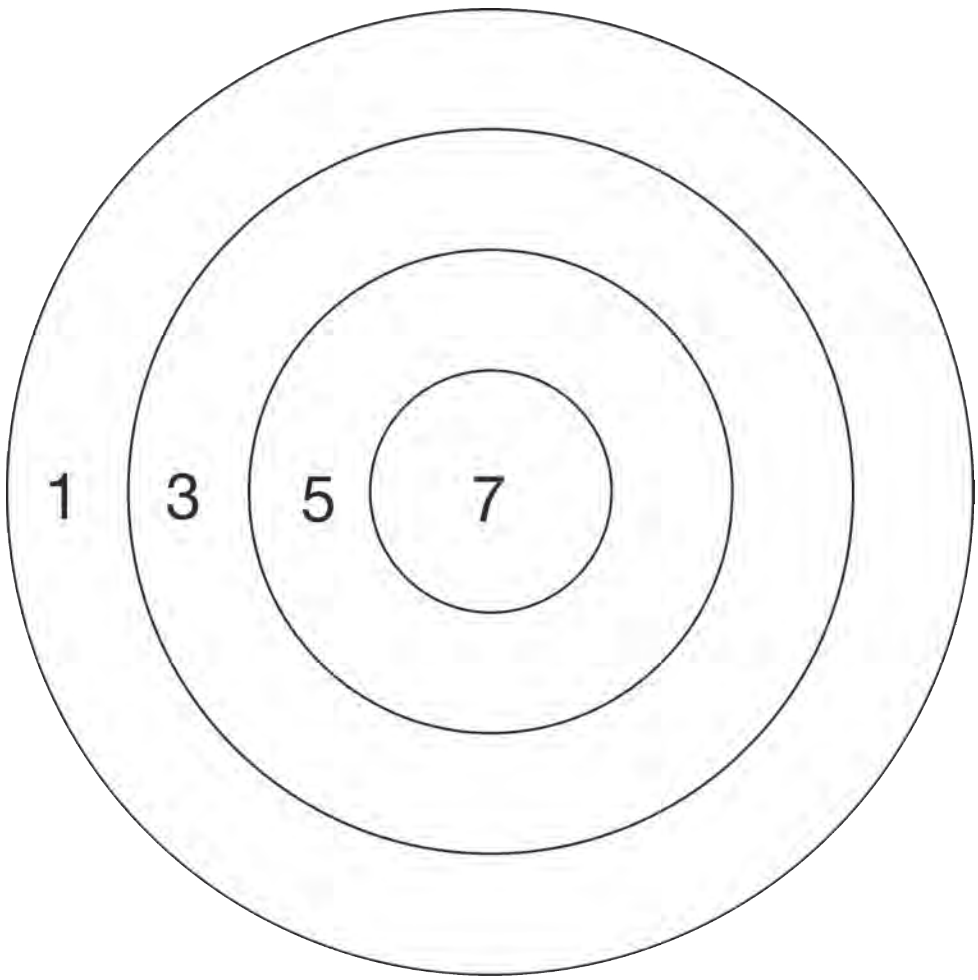
Student 1: range = 7–1 = 6; mean = 42 ÷ 10 = 4.2; median = 5; mode = 5

Student 2: range = 7–1 = 6; mean = 36 ÷ 10 = 3.6; median = 2; mode = 1

After your students have determined the range, mean, median, and mode for their ordered data set, ask them to compare their results with the student(s) they played the game with and think about what the statistics tell them. For example, comparing Student 1 and Student 2 above, Student 1 would win the game because she had a total score of 42 versus a total score of 36 for Student 2. However, Student 2 scored many more 7s than Student 1, suggesting that he might be better at hitting the center of the target. Also, while both students' scores have the same range, their respective means, medians, and modes are all different. Finally, Student 1 has a mean, median, and mode that are all very close numerically; Student 2's data has mean, median, and mode that are very different. This suggests that Student 1 is more consistent in her aim than Student 2.

Note that the total score is only one way to decide who wins the game. Other criteria for "winning" the game could be who scores the most 7's or who is the most consistent (e.g., as measured by mean, median, and mode being close to each other). The game can also be played with no competition at all and students simply try to improve their total score the more times they play.

Target for Central Tendency Game



Activity 4A: Mean Mobile

To visually represent the concept of mean this activity takes the students through the process of making a mean mobile. If the students don't yet understand mean, explain that the mean is where the data will balance.

If you have students that are artistically inclined, get them to draw a picture of each type of boat to be used as the top support, mount it on cardboard, and punch a hole in the bottom at the center.

a. Give out one strip and three squares of cardboard to each student.
b. Each student should find their data range, add 1, and cut their strip that many inches long.
c. They should use a ruler and make a mark 1/2 inch in, then every inch until they have a mark for every number in their range.
d. Have them write the range of their data on the strip, one number at each point marked in part c, and the individual values on both sides of the squares. They can put their names on the back of the strips.
e. Punch a hole in the top of each square and one on the range strip where each of the data points lie. Use light string, yarn, or thread to tie it on the strip (see picture below).
f. Punch a hole in the top of the range strip where the mean is and tie a string there. The strip should balance. This shows that the mean is the balancing point of the student's data.
g. Take one long strip of cardboard (or maybe a dowel) and put the full range of the class data (one for each type of boat) on it, using the same 1-inch spacing. Punch holes where the student means are.
h. Connect the student strips with tape. Punch a hole in the class range strip at the point calculated for the class mean. Use a string to connect that with the top picture. The mobile should balance.

For example: looking only at round boats, if we had 3 groups, and group 1 had values of 75 g, 73 g, and 74g (mean = 74 g); group 2 had values of 87 g, 83 g, and 85g (mean = 85 g); and group 3 had values of 76 g, 81 g, and 80g (mean = 79 g), the class mean is 79 1/3 and the mobile would look like this:

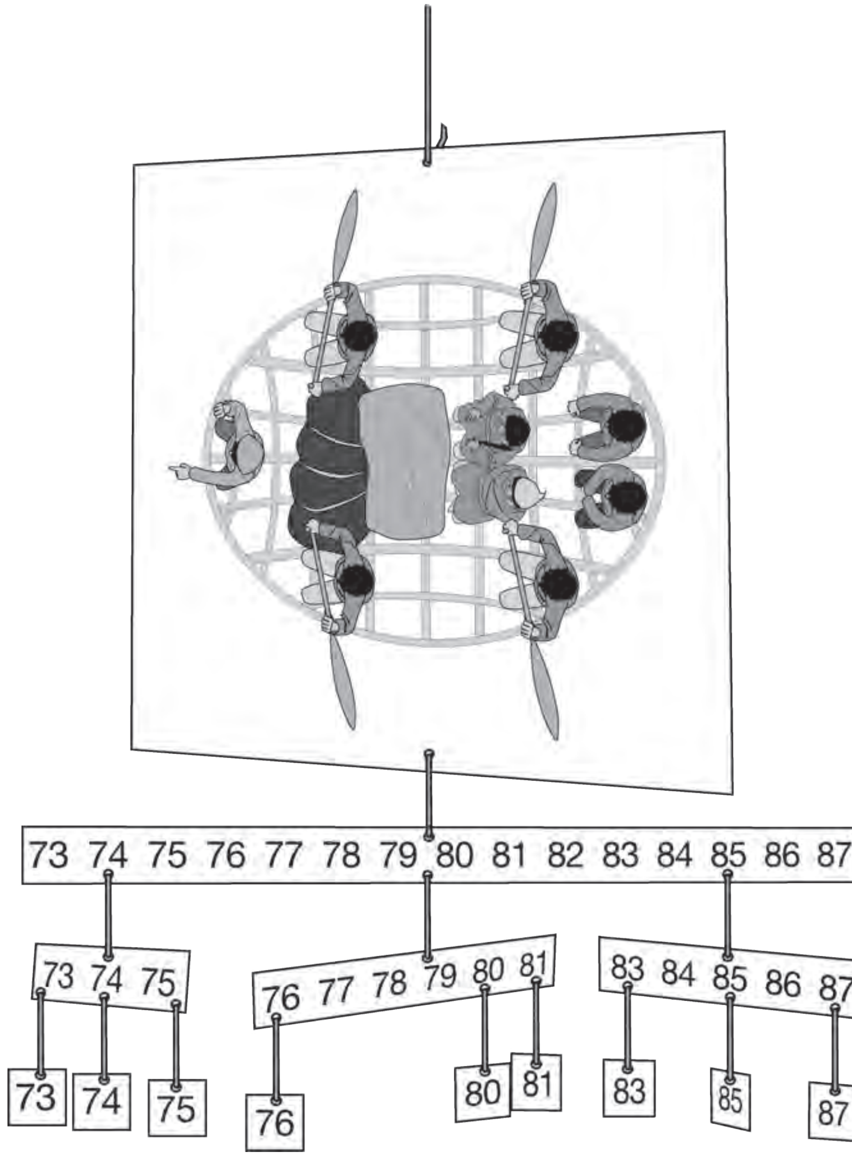


Fig. A.9: Mean mobile modeling a class mean of 79.33g for the load data.

Whether we use all the data points or just the student means, we get the same class mean.

Activity 5A: Additional Practice Problems

1. Use the data in the table below to do the exercises that follow:

Populations of some Alaska villages (2005–2006)

Alaska Village	Estimated Population (2005-2006)
Akiak	311
Hooper Bay	1,090
Manokotak	403
Mountain Village	812
Nightmute	209
St. Mary	538
St. Michael	368
Togiak	816
Tuluksak	438

Fig. A.10: Populations of some Alaska Villages (2005–2006).

a. Make an ordered data set from the population data in the table.
b. Find the range of the data.
c. What is the mode of the data? If there is no mode, explain why.
d. What is the median for the data?
e. What is the mean population of the nine communities?
f. If you were asked what the typical population of the nine communities is, what would you say? Explain your reasoning.

2. Use the data in the table below to do the following exercises:

Length of Alaska Rivers in Miles

Alaska River	*Approximate Length in Miles*
Chena	60
Kobuk	350
Kuskokwim	724
Naknek	35
Nushagak	285
Tanana	659
Togiak	58

Fig. A.11: Lengths of some Alaska rivers in miles.

a. Make an ordered data set from the data in the table.
b. Find the range of the data.
c. What is the mode of the data? If there is no mode, explain why.

d. What is the median for the data?
e. What is the mean length of the rivers?

3. The Yukon River is about 2,300 miles long. If the Yukon were added to the river data in the above data table,

    a. What would be the new range of the lengths of the rivers?
    b. What would be the new median of the lengths of the rivers?
    c. What would be the new mean for the lengths of the rivers?
    d. Explain why the Yukon affects the median and mean differently.

Solutions to Practice Problems

1. See answers below:

    a. 209, 311, 368, 403, 438, 538, 812, 816, 1,090
    b. 881 1,090–209= 881
    c. No mode
    d. 438
    e. 553. 89 4,985 ÷ 9 = 553.89, so the mean population is about 553 or 554.
    f. Students' answers will vary. A good sample response would be, "The median 438 is typical because it's the population of an actual village (i.e., Tuluksak) and the mean is not the population of a real village."

2. See answers below:

    a. 35, 58, 60, 285, 350, 659, 724
    b. 689 724–35 = 689
    c. No mode
    d. 285
    e. 310.14 2,171 ÷ 7 = 310.14, so the mean river length is about 310 miles.

3. See answers below:

    a. 2,265 2,300 —35 = 2,265
    b. 317.5 miles The median would be the mean of 285 and 350 (the two data points in the middle of the ordered set of data) which is 635 ÷ 2 = 317.5.
    c. 558.88 miles
    d. Adding the Yukon to the data on lengths of some Alaska rivers does not change the median very much (i.e., from 285 miles without the Yukon to 317 miles with the Yukon). However, the mean with the Yukon changes a lot (i.e., from 310 miles without

the Yukon to 559 with the Yukon). The reason that the Yukon does not change the median very much, but changes the mean a lot, is because the length of the Yukon is so much larger than the length of any of the other rivers (note: statistically, the length of the Yukon, as a data value, is an outlier relative to the other data; outliers are data values that are generally a lot smaller or larger than most of the other data in an ordered set of data). For the median, adding one data value, no matter how large or small, will not affect it much because most of the data around the middle of the ordered set of data do not change; however, for the mean, a single data value that is much larger or smaller than most of the other data can significantly affect the sum of the data and therefore the mean. For these reasons, the median is said to be a measure of central tendency that is resistant to (or less affected by) outliers, whereas the mean is influenced by (or can be strongly affected by) outliers.