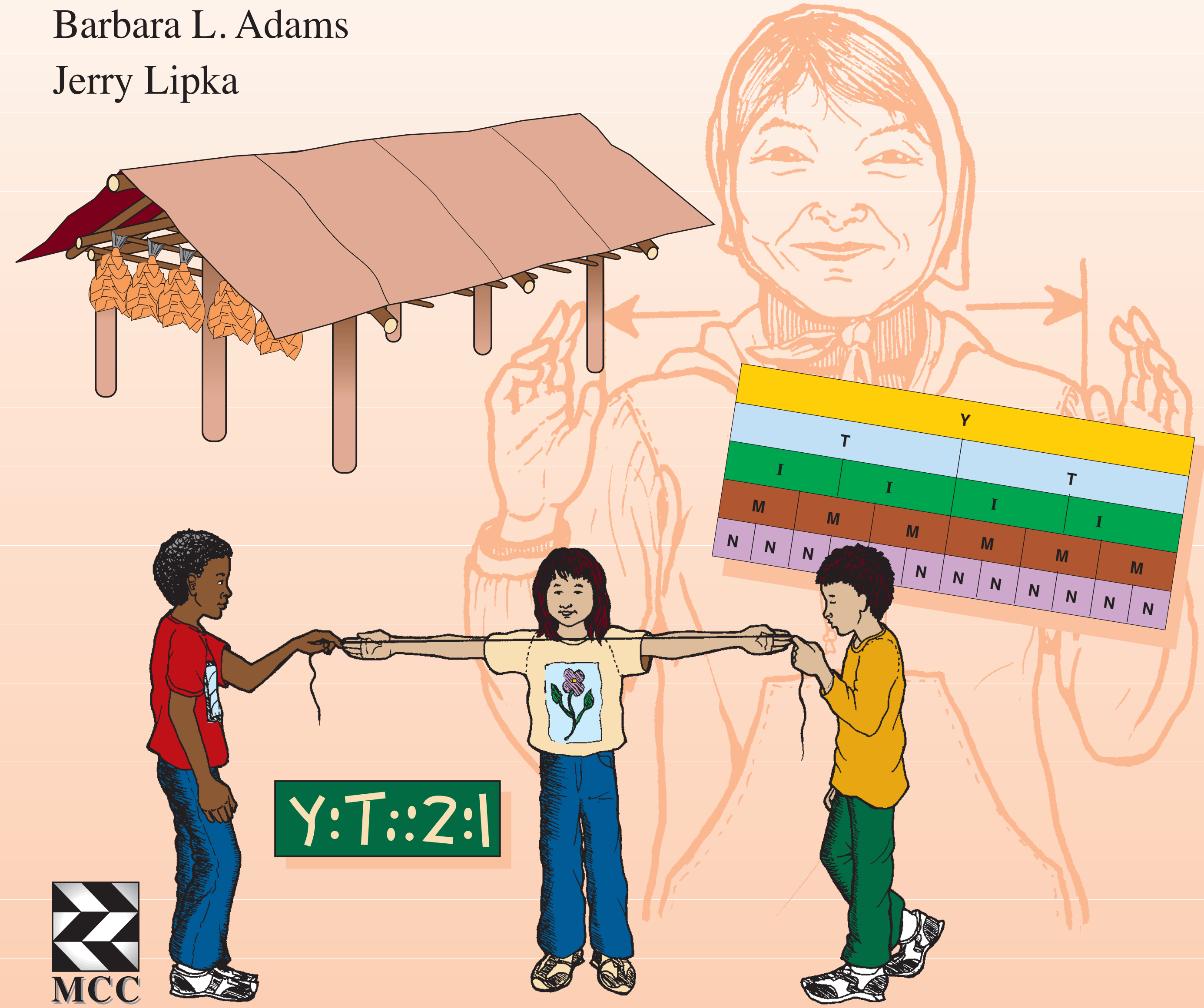
EDUCATION MATHEMATICS ELEMENTARY

Drying Salmon

GRADE LEVEL 6-7
K 1 2 3 4 5 6 7

Journeys into Proportional and Pre-Algebraic Thinking

Barbara L. Adams
Jerry Lipka



MCC

Part of the series *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders©*

Drying Salmon Journeys into Proportional and Pre-Algebraic Thinking

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders is the result of a long-term collaboration. These supplemental math modules for grades K-7 bridge the unique knowledge of Yup'ik elders with school-based mathematics. This series challenges students to communicate and think mathematically as they solve inquiry-oriented problems. Problems are constructed with constraints so that students can explore and understand mathematical relationships, properties of geometrical shapes, place value understanding, conjecturing, and proofs. The modules tap into students' creative, practical, and analytical thinking. Our classroom-based research strongly suggests that students engaged in this curriculum can develop deeper mathematical understandings than students who engage only with a procedure-oriented, paper and pencil curriculum.

Also in this series for Grade 2

Going to Egg Island: Adventures in Grouping and Place Values Students learn to group objects, compose and de-compose numbers using the Yup'ik counting system (base 20 and sub-base 5), and use place value charts in multiple bases. Package includes a storybook, *Egg Island*; five posters; two CD-ROMs; and coloring book.

Picking Berries: Connections Between Data Collection, Graphing, and Measuring Students engage in hands-on activities that help them explore data, graphic representation, and linear measuring. Package includes two storybooks, *Big John and Little Henry* and *Berry Picking*; one poster; a CD-ROM; and coloring book.

Patterns and Parkas: Investigating Geometric Principles, Shapes, Patterns, and Measurement Students learn about the properties of squares, rectangles, triangles, and parallelograms. They learn a variety of ways to make these shapes and how Yup'ik elders use these shapes to create patterns on their traditional winter parkas. Through a hands-on approach to making these shapes, students learn about symmetry, congruence and proof. Package includes a DVD documenting elders making shapes and patterns; *Iluvaktuq*, a storybook about a famous warrior; and seven posters.

Also in this series for Grade 6-7

Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area Following an elder demonstration, students must lay out a rectangular base and prove it is a rectangle. In so doing, they explore various properties of quadrilaterals, including measurements of perimeter and area. They further investigate the key relationship between area and dimensions of a rectangle when perimeter is held constant. Package includes three posters and a CD-ROM.

Salmon Fishing: Investigations into Probability Students use activities based on subsistence and commercial fishing in Southwest Alaska to investigate various topics within probability, such as experimental and theoretical probability, the law of large numbers, sample space, and equally and unequally likely events. Package includes two posters, a CD-ROM, and an interactive Excel spreadsheet.

Building a Smokehouse: The Geometry of Prisms Students learn to generalize properties of rectangles to three-dimensional rectangular prisms through constructing models. Further investigations into triangles and triangular prisms arise while designing roofs for the structures. The hands-on activities lead students through comparing and contrasting models to understand prisms in general. Package includes a CD-ROM and one poster.

Star Navigation: Explorations into Angles and Measurement Students learn ways of observing, measuring, and navigating during the day and at night, including specific details of the location and orientation of the Big Dipper and Cassiopeia. They refine their understanding of angle measurements and how they differ from linear measures throughout the activities. Package includes *The Star Navigation Reader* with traditional stories and personal accounts related to navigating; a CD-ROM; two posters; and a DVD with the "Morning Star" story, song and dance.

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MCC The *Supplemental Math Modules* curriculum was developed at the University of Alaska Fairbanks

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Drying Salmon:

Journeys into Proportional and Pre-Algebraic Thinking

Part of the Series

Math in a Cultural Context:

Lessons Learned from Yup'ik Eskimo Elders

Grades 6–7

Barbara L. Adams
Jerry Lipka

Developed at University of Alaska Fairbanks, Fairbanks, Alaska

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders© was developed at the University of Alaska Fairbanks

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Other Modules in the MCC Series

Going to Egg Island: Adventures in Grouping and Place Values (Grade 2, also appropriate for grade 1)
Storybook—*Egg Island*

Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area (Grade 6, also appropriate for grades 5 and 7)

Picking Berries: Connections Between Data Collection, Graphing, and Measuring (Grade 2, also appropriate for grade 3)
Storybook—*Berry Picking*
Storybook—*Big John and Little Henry*

Salmon Fishing: Investigations into Probability (Grade 6–7)

Designing Patterns: Exploring Shapes and Area (Grade 3–5)
Storybook—*Iluvaktuq and Paluqtalek: Two Yup'ik Warrior Stories*

Star Navigation: Explorations into Angles and Measurement (Grade 6, also appropriate for grades 5 and 7)
Storybook—*The Star Navigation Reader*

Patterns and Parkas: Investigating Geometric Principles, Shapes, Patterns, and Measurement (Grade 2, also appropriate for grade 1)
Storybook—*Iluvaktuq*

Building a Smokehouse: The Geometry of Prisms (Grade 6, also appropriate for grades 5 and 7)

Contents

Acknowledgements

From Jerry Lipka

The supplemental math series *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders* is based on traditional and present-day wisdom and is dedicated to the late Mary George and her late father, George Moses, of Akiachak, Alaska; and to the late Lillie Gamechuk Pauk of Manokotak, Alaska. Mary contributed to every aspect of this long-term project, from her warm acceptance of people from all walks of life to her unique ideas and ways of putting together traditional Yup'ik knowledge with modern Western knowledge. Mary's contribution permeates this work. Without the dedication and perseverance of Mary and her husband, Frederick George, who tirelessly continues to contribute, this project would not be possible. George Moses was always eager and willing to teach us and share his knowledge of the land and river. He was particularly concerned with the well-being of the next generation and hoped that this project would help connect community knowledge to schooling. Lillie cheerfully worked with this project even when she was ill. She always made sure that she first told her story to the group before she attended to personal concerns. Her dedication, laughter, and spirit of giving formed the foundation for this project.

This module, like all the others in this series, developed out of a deep and continuing working relationship with Yup'ik elders and teachers. Sometimes we have magic moments when an idea crystallizes in an instant, but it may take years to actualize the concept. This was exactly the case with this module. We were meeting with Pacific Island navigators who were sharing their methods of sea navigation with Henry Alakayak and Frederick George. While hurrying to the gate at the San Francisco airport, Mary George and I talked about how she used her body to measure the different types of salmon she dried. As we walked with Henry Alakayak and Mary's husband, Frederick, she further explained her system of body proportions—how she used her body measures in relation to the length of her fish rack pole and partitioned the fish rack's pole length by body measures that related to the width of different salmon. In effect, she used her body proportions as a system of measurement. Mary had unique ways of connecting Yup'ik knowledge and Western knowledge and her system of body proportions became the inspiration behind this module. Sadly, Mary succumbed to breast cancer before this module was completed, but her influence continues to be felt. I thank her and am glad to have had the pleasure of working with her. This module is dedicated to Mary George.

For decades now, I have had the pleasure of working with and learning from Evelyn Yanez of Togiak, Alaska; Nancy and Ferdinand Sharp of Manokotak, Alaska; Sassa Peterson of Manokotak, Alaska; and Dora Andrew-Ihrke of Aleknagik and Anchorage, Alaska. Their contributions are immeasurable, as is their friendship. Our long-term relationships with elders who embraced this work wholeheartedly have made this difficult endeavor pleasurable. In particular, I would like to acknowledge Henry Alakayak of Manokotak, Alaska, and Annie Blue of Togiak, Alaska, whose dedication and commitment to cultural continuity has much inspired our project. They have set examples for our own perseverance in creating ways for students to learn and become flexible thinkers through culturally-relevant school curricula. Mary Active and Mary E. Bavilla from Togiak, Alaska, and Anuska Nanalook from Manokotak, Alaska, also contributed unselfishly. They came with stories that enriched us. The late Sam Ivan of Akiak, Alaska; Joshua Phillip of Akiachak, Alaska; and Anecia and Mike Toyukak of Manokotak, Alaska, provided knowledge about many aspects of traditional life, from storytelling and storyknifing, to how kayaks and other traditional Yup'ik crafts were made and used, to countless stories about how to survive. This valuable information contributed to all the modules in the series.

We would like to thank our dedicated staff, especially Flor Banks, for her highly refined organization skills, determination to get the job done, and motivation to move this project forward—from reading and editing manuscripts to holding the various pieces of this project together—she has been a pleasure to work with, and she has done it all with a smile. To Putt Clark, graphic artist extraordinaire, who kept up with every demand and produced more and better artwork than anyone could have hoped for and who has worked with this project from its inception—thank you. To Barbara Adams, who has done an outstanding job as project mathematician and so much more. Her clear-headed thinking and mathematical insights contributed so much depth to these modules, and we appreciate her perseverance and dedication to seeing this project through. Thanks to Eliza and Ben Orr for all their hard work and for producing the *Yup'ik Glossary*, an outstanding piece of work that continues to evolve and that accompanies this project. To Joan Parker Webster, for her literacy and pedagogical insights. Particularly, her work has opened up new aspects of this project—connecting indigenous literacy to the modules and connecting the mathematical and literacy pedagogy. Thank you to Anthony Rickard, who has brought a wealth of experience in reform-oriented math, adding to the connections between math and the Alaska context within the modules. Also thanks to Wendy Wood and Kellie Oxie for their continued support behind the scenes, which enables products like to this to come to fruition. Also, thanks to previous staff members, Jessica Potrikus, Kristy Nicholas, Linda Sheldon, and Alice Atuk for their work on supporting the daily activities that allowed progress on the modules and research.

A special thanks to the Lower Yukon School District, the Yupiit School District, the St. Marys School District, the Kuspuk School District, the Anchorage School District, the Yukon Flats School District, the Fairbanks North Star Borough School District, and to the Southwest Region Schools for their cooperation in piloting modules. Thanks also to all the other writers and editors, project and pilot teachers, and elders who have assisted this project.

Last, but not least, thank you to my loving wife, Janet Schichnes, who has supported me in countless ways that allowed me to complete this work, and to my children, Alan and Leah.

From Barbara Adams

This module has been one of the most challenging and dear to my heart. When I started working with Jerry Lipka in early 2000, my first task was to work on the surface area and volume activities in this module. It was apparent at that time that the content in this module was more challenging than that of other modules and deserved special respect both mathematically and culturally. Over these last seven years, we talked and worked with many consultants, teachers, editors, and colleagues—trying to understand how to build algebraic thinking by analyzing their thinking, especially during problem-solving. Based on these collaborations, we continually narrowed the scope of the module to focus on the core concepts necessary for students to bridge the gap between arithmetic and algebraic thinking. As a result, our goals for how this module can bridge that gap in student thinking have been quite modest. Thank you to all of the people who have helped to form the ideas found within this module.

Although I never had the pleasure of meeting Mary George, I am grateful to her for what she has taught me through the stories of her experiences. I would also like to thank Frederick George, her husband, for his commitment and belief in this project and for all of his time and continued efforts toward completing this work. Thank you also to Helena Williams and Wassilie Evan both from Akiak, Alaska, who along with Frederick, helped as elders in gathering the body measure and fish cut information specific to the Kuskokwim region. Thank you to Elizabeth Lake for translating during the elders meeting in Akiak, Alaska, as well as for providing her own knowledge on the topic of fish cutting. Thank you to Ferdinand Sharp and Evelyn Yanez who have helped me to understand the elders and what they shared throughout many meetings and for their own expertise in body measures, fishing, and drying and smoking salmon. I have enjoyed all of our trips, meetings, and time together. *Quyana*. Also, thanks to Steve and Anna Jacobson for sharing their expertise with the Yup'ik language.

I need to thank several of the teachers who supplied more than their share of understanding and suggestions to previous drafts of this module, in particular, Scott Darter, Elizabeth (Kent) Payne, and Janet Speed. Their experiences and expertise have helped to shape the content of this module. Most importantly, thank you to Stacy Clark for the countless hours we worked together revising activities and finding alternative ways of helping students understand proportional thinking. Stacy's time with her students and as a consultant provided valuable information to me while I wrote and I can't thank her enough.

Thank you to the people who have helped me complete this module especially during the tedious end stages—Katherine Mulcrone for her time and commitment to consistency in editing; Elizabeth (Putt) Clark for her tremendously beautiful illustrations and for giving so much time to each detail; Paula Elmes for her professionalism in graphic design and layout; and Flor Banks who is invaluable in maintaining order, accuracy, professionalism, and integrity to the project and the modules. I would like to express deep appreciation to Jerry Lipka. I have been incredibly privileged to have had all the opportunities this project has afforded me. Also, thank you, Jerry, for all of the knowledge that you have shared with me, for your patience and trust over the years, and for your commitment to the Yup'ik people and the work that you began so many decades ago. Thank you for bringing me into the project family; I will always cherish these relationships and this experience.

Thank you to my friends and family for continuing to love me when life has been so busy that I may not have been able to be there for you. Thank you, especially, to my husband, Todd, for being so patient, listening so often, and continuously supporting me in my endeavors.

Introduction

Math in a Cultural Context:

Lessons Learned from Yup'ik Eskimo Elders

Introduction to the Series

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders (MCC) is a supplemental math curriculum based on the traditional wisdom and practices of the Yup'ik Eskimo people of southwest Alaska. The kindergarten to seventh grade math modules that you are about to teach are the result of more than a decade of collaboration between math educators, teachers, Yup'ik Eskimo elders, and researchers to connect cultural knowledge to school mathematics. To understand the rich environment from which this curriculum came, imagine traveling on a snowmachine over the frozen tundra and finding your way based on the position of the stars in the night sky. Or, in summer, paddling a sleek kayak across open waters shrouded in fog, yet knowing which way to travel toward land by the pattern of the waves. Imagine building a kayak or making clothing and accurately sizing them by visualizing or using body measures. These are a small sample of the activities in which modern Yup'ik people engage. The mathematics embedded in these activities formed the basis for this series of supplemental math modules. Each module is independent and lasts from three to eight weeks.

From 2001 through spring 2006, with the exception of one urban trial, students who used these modules consistently outperformed students who used only their regular math textbooks at statistically significant levels on MCC tests. This was true for urban as well as rural students, both Caucasian and Alaska Native. We believe that this supplemental curriculum will motivate your students and strengthen their mathematical understanding because of the engaging content, hands-on approach to problem-solving, and the emphasis on mathematical communication. Further, these modules build on students' everyday experience and intuitive understandings—particularly in geometry, which is underrepresented in school.

A design principle used in the development of these modules is that the activities allow students to explore mathematical concepts semiautonomously. Through the use of hands-on materials, students can "physically" prove conjectures; solve problems; and find patterns, properties, shortcuts, or generalizations. The activities incorporate multiple modalities and can challenge students with diverse intellectual needs. Hence, the curriculum is designed for heterogeneous groups with the realization that different students will tap into different cognitive strengths. According to Sternberg and his colleagues (1997, 1998), by engaging students creatively, analytically, and practically, students develop a more robust understanding of math concepts. This approach allows for shifting roles and expertise among students rather than only privileging those students with analytical knowledge.

The modules explore the everyday application of mathematical skills such as grouping, approximating, measuring, proportional thinking, informal geometry, and counting in base twenty and then present these skills in terms of formal mathematics. Students move from the concrete and applied to more formal and abstract math. The activities are designed to meet the following goals:

- Students learn to solve mathematical problems that support an in-depth understanding of mathematical concepts.
- Students derive mathematical formulas and rules from concrete and practical applications.
- Students become flexible thinkers because they learn that there is more than one method of solving a mathematical problem.
- Students learn to communicate and think mathematically while they demonstrate their understanding to peers.
- Students learn content across the curriculum, since the lessons comprise Yup'ik Eskimo culture, literacy, geography, and science.

Beyond meeting some of the content (mathematics) and process standards of the National Council of Teachers of Mathematics (2000), the curriculum design and its activities respond to the needs of diverse learners. Many activities are designed for group work. One of the strategies for using group work is to provide leadership opportunities to students who may not typically be placed in those roles. Also, the modules tap into a wide array of intellectual abilities—practical, creative, and analytical. We assessed modules that were tested in rural Alaska, urban Alaska, and suburban California and found that students who were only peripherally involved in math became more active participants through the use of these modules.

Students learn to reason mathematically by constructing models and analyzing practical tasks for their embedded mathematics. This enables them to generate and discover mathematical rules and formulas. In this way, we offer students a variety of ways to engage the math material through practical activity, spatial/visual learning, analytical thinking, and creative thinking. They are constantly encouraged to communicate mathematically by presenting their understandings, while other students are encouraged to provide alternate solutions, strategies, and counterarguments. This process also strengthens their deductive reasoning.

Pedagogical Approaches Used in the Modules

The concept of third space is embedded within each module. Third space relates to a dynamic and creative place among school-based knowledge, everyday knowledge, and knowledge related to other non-mainstream cultural groups. Third space also includes local knowledge such as ways of measuring and counting that are distinct from school-based notions, and brings these elements together in a creative, respectful, and artful manner. Within this creative and evolving space, pedagogical forms can develop creatively from both Western schooling and local ways. In particular, this module pays close attention to expert-apprentice modeling because of its prevalent use among Yup'ik elders and other Alaska Native groups.

Design

The curriculum design includes strategies that engage students:

- cognitively, so that students use a variety of thinking strategies (analytical, creative, and practical);
- socially, so that students with different social, cognitive, and mathematical skills use those strengths to lead and help solve mathematical problems;
- pedagogically, so that students explore mathematical concepts and learn to reason and communicate mathematically by demonstrating their understanding of the concepts; and
- practically, as students apply or investigate mathematics to solve problems from their daily lives.

The organization of the modules follows five distinct approaches to teaching and learning that converge into one system.

Expert-Apprentice Modeling

The first approach, expert-apprentice modeling, comes from Yup'ik elders and teachers and is supported by research in anthropology and education. Many lessons begin with the teacher (the expert) demonstrating a concept to the students (the apprentices). Following the theoretical position of the Russian psychologist Vygotsky (cited in Moll, 1990) and expert Yup'ik teachers (Lipka and Yanez, 1998) and elders, students begin to appropriate the knowledge of the teacher (who functions in the role of expert), as the teacher and the more adept apprentices help other students learn. This establishes a collaborative classroom setting in which student-to-student and student-to-teacher dialogues are part of the classroom fabric.

More recently, we have observed experienced teachers use joint productive activity—the teacher works in parallel with students modeling an activity, a concept, or a skill. When effectively implemented, joint productive activity appears to increase student ownership of the task as well as responsibility and motivation. The typical authority structure surrounding classrooms changes as students take on more of the responsibility for their learning. Social relations in the classroom become more level. Further, the connections between out-of-school learning and in-school learning are strengthened through pedagogical approaches such as expert-apprentice modeling and joint productive activity when those are the approaches of the community.

Reform-Oriented Approach

The second pedagogical approach emphasizes student collaboration in solving "deeper" problems (Ma, 1999). This approach is supported by research in math classrooms and particularly by recent international studies (Stevenson et al., 1990; Stigler and Hiebert, 1998) strongly suggesting that math problems should be more in-depth and challenging and that students should understand the underlying principles, not merely use procedures competently. The modules present complex problems (two-step, open-ended problems) that require students to think more deeply about mathematics.

Multiple Intelligences

Further, the modules tap into students' multiple intelligences. While some students may learn best from hands-on, real-world-related problems, others may learn best when abstracting and deducing. This module provides opportunities to guide both modalities. Robert Sternberg's work (1997, 1998) influenced the development of these modules. He has consistently found that students who are taught so that they use their analytical, creative, and practical intelligences will outperform students who are taught using a single modality, most often analytical. Therefore, we have shaped our activities to engage students in this manner.

Mathematical Argumentation and Deriving Rules

The purpose of math communication, argumentation, and conceptual understanding is to foster students' natural abilities. These modules support a math classroom environment in which students explore the underlying mathematical rules as they solve problems. Through structured classroom communication, students learn to work collaboratively in a problem-solving environment in which they learn both to appreciate alternative solutions and strategies and to evaluate these strategies and solutions. They will present their mathematical solutions to their peers. Through discrepancies in strategies and solutions, students will communicate with and help each other to understand their reasoning and mathematical decisions. Mathematical discussions are encouraged to strengthen students' mathematical and logical thinking as they share their findings. This requires classroom norms that support student communication, learning from errors, and viewing errors as opportunities to learn rather than to criticize. The materials in the modules (see Materials section) constrain the possibilities, guide students in a particular direction, and increase their chances of understanding mathematical concepts. Students are given the opportunity to support their conceptual understanding by practicing it in the context of a particular problem.

Familiar and Unfamiliar Contexts Challenge Students' Thinking

By working in unfamiliar settings and facing new and challenging problems, students learn to think creatively. They gain confidence in their ability to solve both everyday problems and abstract mathematical questions, and their entire realm of knowledge and experience expands. Further, by making the familiar unfamiliar and by working on novel problems, students are encouraged to connect what they learn from one setting (everyday problems) with mathematics in another setting. For example, most sixth-grade students know about rectangles and how to calculate the area of a rectangle, but if you ask students to go outside and find the four corners of an eight-foot-by twelve-foot-rectangle without using rulers or similar instruments, they are faced with a challenging problem. As they work through this everyday application (which is needed to build any rectangular structure) and as they "prove" to their classmates that they do, in fact, have a rectangular base, they expand their knowledge of rectangles. In effect they must shift their thinking from considering rectangles as physical entities or as prototypical examples to understanding the salient properties of a rectangle. Similarly, everyday language, conceptions, and intuition may, in fact, be in the way of mathematical understanding and the precise meaning of mathematical terms. By treating familiar knowledge in unfamiliar ways, students explore and confront their own mathematical understandings and begin to understand the world of mathematics. These major principles guide the overall pedagogical approach to the modules.

The Organization of the Modules

The curriculum includes modules for kindergarten through seventh grade. Modules are divided into sections: activities, explorations, and exercises, with some variation between each module. Supplementary information is included in Cultural Notes, Teacher Notes, and Math Notes. Each module follows a particular cultural storyline, and the mathematics connect directly to it. Some modules are designed around a children's story, and an illustrated text is included for the teacher to read to the class.

The module is a teacher's manual. It begins with a general overview of the activities ahead, an explanation of the math and pedagogy of the module, teaching suggestions, and a historical and cultural overview of the curriculum in general and of each specific module. Each activity includes a brief introductory statement, an estimated duration, goals, materials, any pre-class preparatory instructions for the teacher, and the procedures for the class to carry out the activity. Assessments are placed at various stages, both intermittently and at the end of activities.

Illustrations help to enliven the text. Yup'ik stories and games are interspersed and enrich the mathematics. Transparency masters, worksheet masters, assessments, and suggestions for additional materials are attached at the end of each activity. An overhead projector is necessary. Blackline masters that can be made into overhead transparencies are an important visual enhancement of the activities, stories, and games. Such visual aids also help to further classroom discussion and understanding.

Resources and Materials Required to Teach the Modules

Materials

The materials and tools limit the range of mathematical possibilities, guiding students' explorations so that they focus upon the intended purpose of the lesson. For example, in one module, latex sheets are used to explore concepts of topology. Students can manipulate the latex to the degree necessary to discover the mathematics of the various activities and apply the rules of topology.

For materials and learning tools that are more difficult to find or that are directly related to unique aspects of this curriculum, we provide detailed instructions on how to make those tools for the teacher and students. For example, in *Going to Egg Island: Adventures in Grouping and Place Values,* students use a base twenty abacus. Although the project has produced and makes available a few varieties of wooden abaci, detailed instructions are provided for the teacher and students on how to make a simple, inexpensive, and usable abacus with beads and pipe cleaners.

Each module and each activity lists all of the materials and learning tools necessary to carry it out. Some of the tools are expressly mathematical, such as interlocking centimeter cubes, abaci, and compasses. Others are particular to the given context of the problem, such as latex and black and white geometric pattern pieces. Many of the materials are items a teacher will probably have on hand, such as paper, markers, scissors, and rulers. Students learn to apply and manipulate the materials. The value of caring for the materials is underscored by the precepts of subsistence, which is based on processing raw materials and foods with maximum use and minimum waste. Periodically, we use food as part of an activity. In these instances, we encourage minimal waste.

Videos

To convey the knowledge of the elders underlying the entire curriculum more vividly, we have produced a few DVDs to accompany some of the modules. For example, the *Going to Egg Island: Adventures in Grouping and Place Values* module includes videos of Yup'ik elders demonstrating some traditional Yup'ik games. We also have footage and recordings of the ancient chants that accompanied these games. The videos are available on DVD and are readily accessible for classroom use.

Yup'ik Language Glossary and Math Terms Glossary

To help teachers and students get a better feel for the Yup'ik language, its sounds, and the Yup'ik words used to describe mathematical concepts in this curriculum, we have developed a Yup'ik glossary on CD-ROM. Each word is recorded in digital form and can be played back in Yup'ik. The context of the word is provided, giving teachers and students a better sense of the Yup'ik concept, not just its Western "equivalent." Pictures and illustrations often accompany the words for additional clarification.

Yup'ik Values

There are many important Yup'ik values associated with each module. The elders counsel against waste. They value listening, learning, working hard, being cooperative, and passing knowledge on to others. These values are expressed in the contents of the Yup'ik stories that accompany the modules, in the Cultural Notes, and in various activities. Similarly, Yup'ik people as well as other traditional people continue to produce, build, and make crafts from raw materials. Students who engage in these modules also learn how to make simple mathematical tools fashioned around such themes as Yup'ik border patterns and building model kayaks, fish racks, and smokehouses. This way, students learn to appreciate and value other cultures.

Cultural Notes

Most of the mathematics used in the curriculum comes from our direct association and long-term collaboration with Yup'ik Eskimo elders and teachers. We have included many Cultural Notes to describe and explain more fully the purposes, origins, and variations associated with a particular traditional activity. Each module is based on a cultural activity and follows a Yup'ik cultural storyline along which the activities and lessons unfold.

Math Notes

We want to ensure that teachers who may want to teach these modules, but feel unsure of some of the mathematical concepts, will feel supported by the Math Notes. These provide background material to help teachers better understand the mathematical concepts presented in the activities and exercises of each module. For example, in *Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area*, the Math Notes give a detailed description of a rectangle and describe the geometric proofs one would apply to ascertain whether or not a shape is a rectangle. *Building a Smokehouse: The Geometry of Prisms* explores rectangular prisms and the geometry of three-dimensional objects; the Math Notes include information on the geometry of rectangular prisms, including proofs, to facilitate the instructional process. In every module, connections are made among the "formal math," its practical application, and the classroom strategies for teaching the math.

Teacher Notes

The main function of the Teacher Notes is to focus on the key pedagogical aspects of the lesson. For example, they provide suggestions for how to facilitate students' mathematical understanding through classroom organization strategies, classroom communication, and ways of structuring lessons. Teacher Notes also make suggestions for ways of connecting out-of-school knowledge with schooling.

Literacy Counts!: Developing Language and Literacy in MCC

As MCC has developed over the years, the importance of the role of literacy has also grown. The inclusion of culturally-based stories has proven to contribute to students' engagement with the math modules as well as provide cultural grounding for the module activities. MCC modules have also made use of literacy-based activities, such as journaling, to further students' understanding of math concepts and vocabulary.

Assessment

Assessment and instruction are interrelated throughout the modules. Assessments are embedded within instructional activities, and teachers are encouraged to carefully observe, listen, and challenge their students' thinking. We call this active assessment, which allows teachers to assess how well students have learned to solve the mathematical and cultural problems introduced in the module.

Careful attention has been given to developing assessment techniques and tools that evaluate both the conceptual and procedural knowledge of students. We agree with Ma (1999) that having one type of knowledge without the other or not understanding the link between the two will produce only partial understanding. The goal here is to produce relational understanding in mathematics. Instruction and assessment have been developed and aligned to ensure that both types of knowledge are acquired; this has been accomplished using both traditional and alternative techniques.

The specific details and techniques for assessment (when applicable) are included within activities. The three main tools for collecting and using assessment data follow.

Notebooks

In recent years, the National Council of Teachers of Mathematics (NCTM) has promoted standards that incorporate math journals as part of math instruction. Journaling has most often occurred as a tool for reflecting on what was learned. In contrast, math notebooks, which are incorporated in Strand one of Literacy Counts!, are used by students to record what they are thinking and learning about math concepts before, during, and after the activities in the modules. Through the use of math notebooks, students build their content knowledge while at the same time developing their literacy skills through reading, writing, drawing, and graphic representations. Math notebooks also play an important role in helping students develop math vocabulary.

Observation

Observing and listening to students lets teachers learn about the strategies that they use to analyze and solve various problems. Listening to informal conversations between students as they work cooperatively on problems provides further insight into their strategies. Through observation, teachers also learn about their students' attitudes toward mathematics and their skills in cooperating with others. Observation is an excellent way to link assessment with instruction.

Adaptive Instruction

The goal of the summary assessment in this curriculum is to adapt instruction to the skills and knowledge needed by a group of students. From reviewing journal notes to simply observing, teachers learn which mathematical processes their students are able to effectively use and which ones they need to practice more. Adaptive assessment and instruction complete the link between assessment and instruction.

An Introduction to the Land and Its People, Geography, and Climate

Flying over the largely uninhabited expanse of southwest Alaska on a dark winter morning, one looks down at a white landscape interspersed with trees, winding rivers, rolling hills, and mountains. A handful of lights are sprinkled here, a handful there. Half of Alaska's 600,000-plus population lives in Anchorage. The other half is dispersed among smaller cities such as Fairbanks and Juneau and among the over 200 rural villages that are scattered across the state. Landing on the village airstrip, which is usually gravel and, in the winter, covered with smooth, hard-packed snow, one is taken to the village by either car or snowmachine. Hardly any villages or regional centers are connected to road systems. The major means of transportation between these communities is by small plane, boat, and snowmachine, depending on the season.

It is common for the school to be centrally located. Village roads are usually unpaved, and people drive cars, four-wheelers, and snowmachines. Houses are typically made from modern materials and have electricity and running water. Over the past twenty years, Alaska villages have undergone major changes, both technologically and culturally. Most now have television, full phone systems, modern water and sewage treatment facilities, an airport, and a small store. Some also have a restaurant, and a few even have a small hotel and taxicab service. Access to medical care and public safety are still sporadic, with the former usually provided by a local health care worker and a community health clinic, or by health care workers from larger cities or regional centers who visit on a regular basis. Serious medical emergencies require air evacuation to either Anchorage or Fairbanks.

The Schools

Years of work have gone into making education as accessible as possible to rural communities. Almost every village has an elementary school and most have a high school. Some also have a higher education satellite facility, computer access to higher education courses, or options that enable students to earn college credits while in their respective home communities. Vocational education is taught in some of the high schools, and there are also special vocational education facilities in some villages. While English has become the dominant language throughout Alaska, many Yup'ik children in the villages still learn Yup'ik at home.

Yup'ik Village Life Today

Most villagers continue to participate in the seasonal rounds of hunting, fishing, and gathering. Although many modern conveniences are located within the village, when one steps outside of its narrow bounds, one is immediately aware of one's vulnerability in this immense and unforgiving land, where one misstep can lead to disaster. Depending upon their location (coastal community, riverine, or interior), villagers hunt and gather the surrounding resources. These include sea mammals, fish, caribou, and many types of berries. The seasonal subsistence calendar illustrates which activities take place during the year (see Figure 1). Knowledgeable elders know how to cross rivers and find their way through ice fields, navigating the seemingly featureless tundra by using directional indicators such as frozen grass and the constellations in the night sky. All of this can mean the difference between life and death. In the summer, when this largely treeless, moss- and grass-covered plain thaws into a large swamp dotted with small lakes, the consequences of ignorance, carelessness, and inexperience can be just as devastating. Underwater hazards in the river, such as submerged logs, can capsize a boat, dumping the occupants into the cold, swift current. Overland travel is much more difficult during the warm months due to the marshy ground and many waterways, and one can easily become disoriented and get lost. The sea is also integral to life in this region and requires its own set of skills and specialized knowledge to be safely navigated.

The Importance of the Land: Hunting and Gathering

Basic subsistence skills include knowing how to read the sky to determine the weather and make appropriate travel plans, being able to read the land to find one's way, knowing how to build an emergency shelter and, in the greater scheme, how to hunt and gather food and properly process and store it. In addition, the byproducts of subsistence activities, such as carved walrus tusks, pelts, and skins are made into clothing or decorative items and a variety of other utilitarian arts and crafts products that provide an important source of cash for many rural residents.

Hunting and gathering are still of great importance in modern Yup'ik society. A young man's first seal hunt is celebrated; family members who normally live and work in one of the larger cities will often fly home to help when the salmon are running, and whole families still gather to go berry picking. The importance of hunting and gathering in daily life is further reflected in the legislative priorities expressed by rural residents in Alaska. These focus on such things as subsistence hunting regulations, fishing quotas, resource development, and environmental issues that affect the well-being of game animals and subsistence vegetation.

Conclusion

We developed this curriculum in a Yup'ik context. The traditional subsistence and other skills of the Yup'ik people incorporate spatial, geometric, and proportional reasoning as well as other mathematical reasoning. We have attempted to offer you and your students a new way to approach and apply mathematics while also learning about Yup'ik culture. Our goal has been to present math as practical information that is inherent in everything we do. We hope your students will adopt and incorporate some of this knowledge and add it to their learning base.

We hope you and your students will benefit from the mathematics, culture, geography, and literature embedded in the *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders* series. The elders who guided this work emphasized that the next generation of children should be flexible thinkers and leaders. In a small way, we hope that this curriculum guides you and your students along this path.

Tua-ii ingrutuq [This is not the end].

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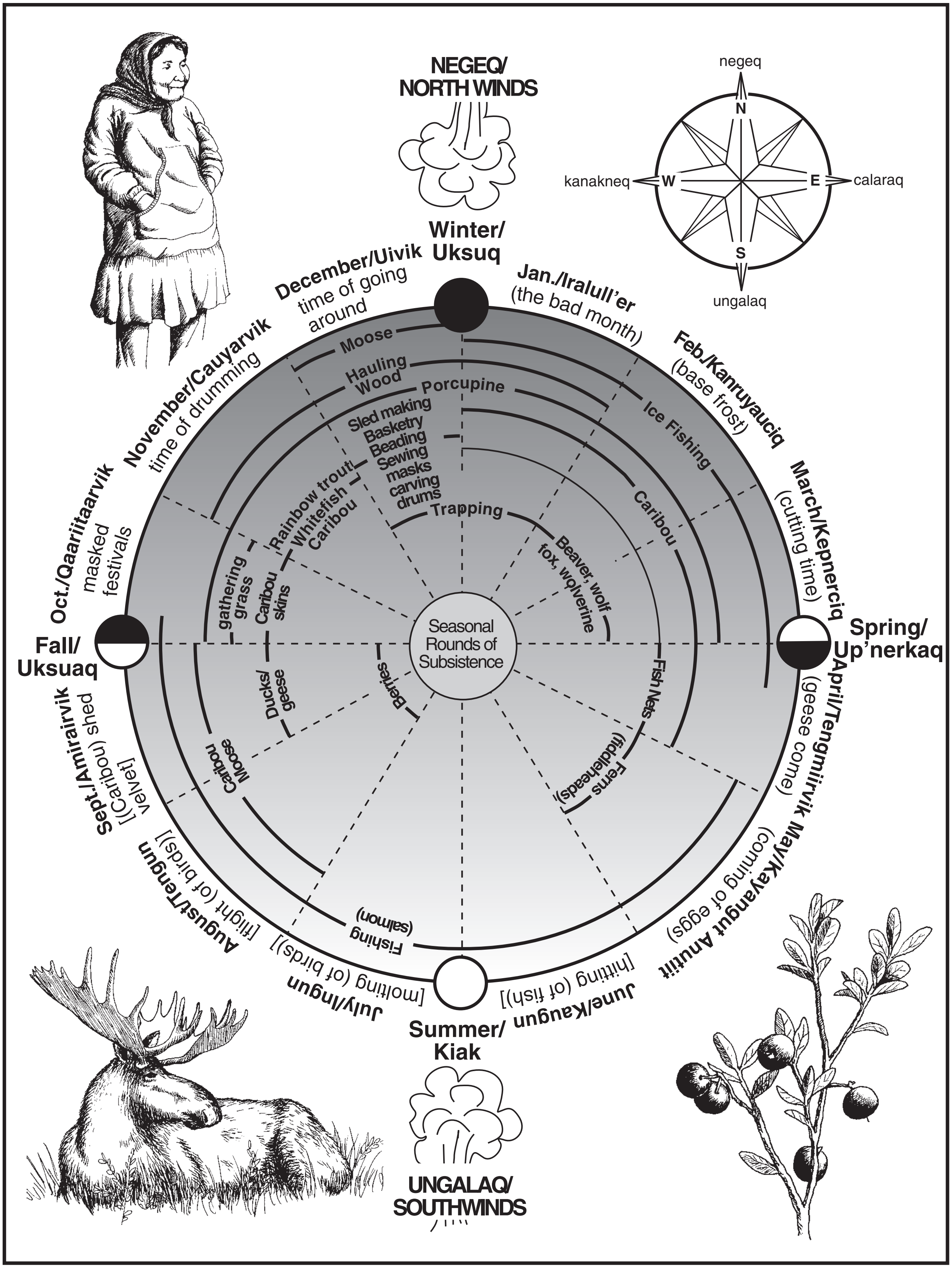

Fig. 1: Yearly subsistence calendar.

Introduction

Drying Salmon:

Journeys into Proportional and Pre-Algebraic Thinking

Introduction to the Module

For millennia, the Yup'ik people, as well as other people throughout the world, have faced the hazards of food spoilage from bacteria, flies, and maggots. To preserve their food supply for prolonged future use, many cultures have developed techniques to reduce food waste and spoilage, increasing its shelf-life. Some methods include drying or smoking foods and storing them in cool places, such as in containers submerged in cold water.

Mary George, a Yup'ik teacher from Akiachak, Alaska, lived a subsistence lifestyle including catching, cutting, and hanging salmon to dry as a way of preserving food for the long winters. This sixth and seventh grade module explores pre-algebraic and proportional thinking over a four- to six-week time period using the unique system of body measures that Mary George used when hanging her salmon to dry. These body measures provide a distinctively different approach to building students' proportional and pre-algebraic thinking than standard math books as well as connect the math to a cultural practice of food preservation used around the world.

Body Measures across Cultures

The use of body measures is not unique to the Yup'ik culture. For example, the ancient Egyptians used the cubit, a measurement from the elbow to the fingertips, and the digit, the length of the middle finger. By applying these measures, they constructed the pyramids, which continue to be one of the world's wonders. Other measures in other cultures include using the hand to measure horses and the stride to measure a soccer field. Inherent in any system of measures is a series of relationships—how one measure relates to another.

This module was developed by a team of University of Alaska Fairbanks faculty and Yup'ik Eskimo people who live in southwest Alaska. In particular, Mary and Frederick George supplied most of the background knowledge for this module. Mary George used a series of proportional body measures to build her fish drying racks (a structure built for drying salmon). Through years of fishing experience, Mary learned how to proportion the dimensions of the rack to her body size and to the size of different salmon and their cuts. Her techniques allowed her to determine how many of each species of salmon would fit on a pole.

Over the last twenty years, we have gathered many examples of both historical and modern body measures used by the Yup'ik people. Elders have shown us uses for each body measure displayed in Figure 2 within the contexts of sewing, fishing, trapping, building, and other everyday tasks. The Yup'ik body measures and spellings contained within this module were gathered from two regions—Kuskokwim and Bristol Bay—during many meetings and long discussions with elders. There are differences in dialect, and even definitions of terms, for many of the body measures between these two regions and among other Yup'ik regions.

Yup'ik is a traditionally oral language and has only recently begun to be transcribed; thus, the lexicon is flexible and diverse among its speakers. Often times, working together, elders would settle on a term to describe a body measure which made sense within the context of the meeting, but which might not be familiar to all Yup'ik speakers. For example, the *taluyaneq* was used by Henry Alakayak to describe the length of a person's outstretched arm to the middle of his chest. It stems from the term *taluyag* which means blackfish trap since this is the length one can reach to take the fish out of the trap. The body measure *taluyaneq* is also referred to as *yaaltaq* by women when measuring cloth for sewing. The term *yaaltaq* means one yard, since most Yup'ik women approximate a yard using a modification of the *taluyaneq* measure.

Further, over time many body measures have taken on names that represent standard measures. For example, the *malruk naparnerek* is often also called *it'ganeq* which is the word for foot. Frederick George explained that for him, this body measure is approximately 12 inches or one foot. Many of the male elders use the same measure and also call it a foot, as shared by Wassilie Evan and Dora Andrew-Ihrke, who referred to her grandfather's use of the term.

Thus, the body measure information contained within the module was gathered, synthesized, and refined through a process of continuous collaboration with Yup'ik elders, translators, and consultants from both regions. However, when it became necessary, Evelyn Yanez, a long-term Yup'ik consultant from the Bristol Bay region, has been the final editor of these terms. We feel this is an accurate representation of the knowledge shared with us by the elders working with this project, but it is not perfect. These measures can also be found on the CD-ROM, *Yup'ik Glossary*, in an interactive format including pronunciations and on the Body Measurements poster that accompanies the module.

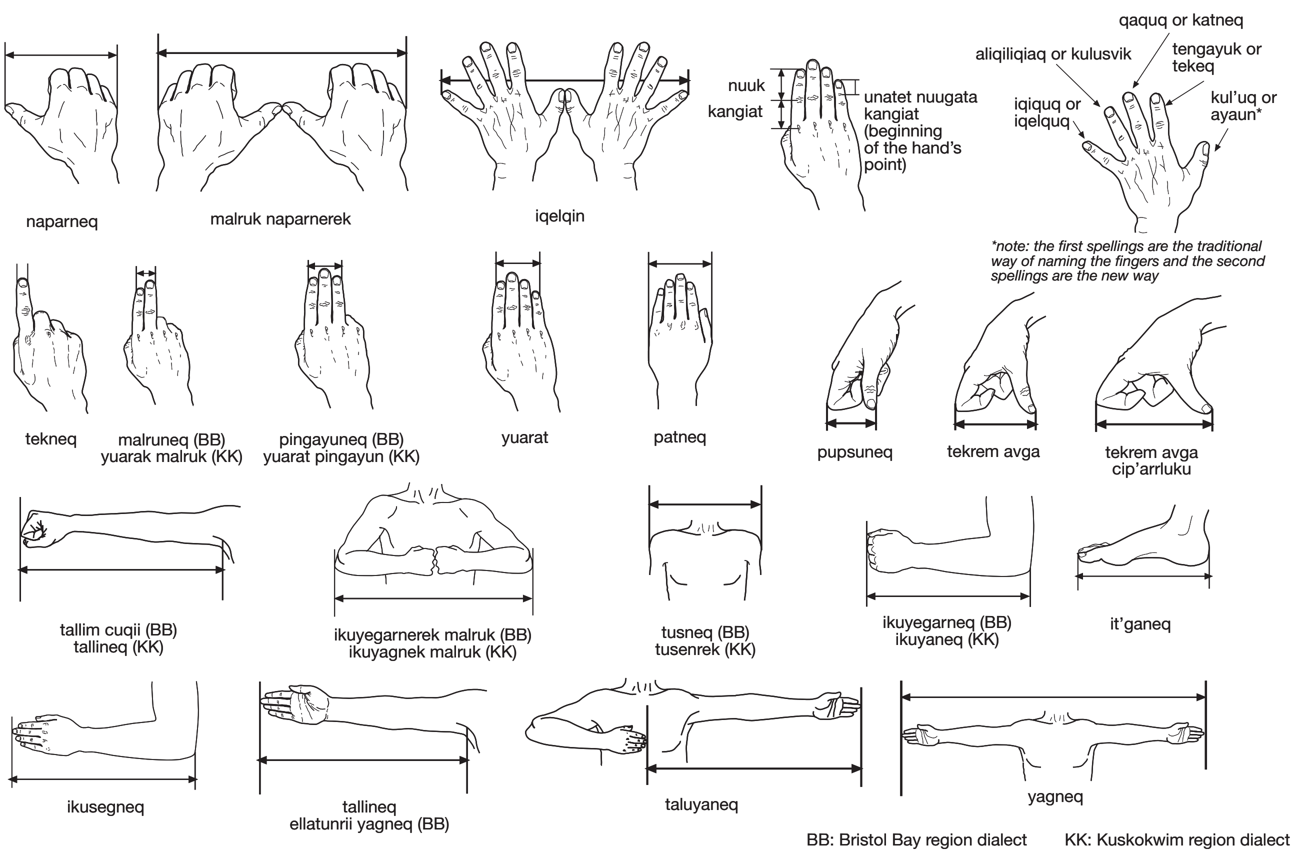

Fig. 2: Examples of some Yup'ik body measures.

Eskimo People and Southwest Alaska

The unpredictable and wind-scoured environment of southwest Alaska may appear featureless and barren to "outsiders," yet the Yup'ik have learned to excel here, in part by visualizing and estimating distances and by developing a rich vocabulary for describing spatial relationships. In this module, we translate some of the richly textured systems of Yup'ik measurement into lessons using variables, proportions, and the beginning levels of algebraic thinking.

Traditional Yup'ik culture uses measuring, estimating, and proportional thinking for many tasks ranging from building structures and making clothing, to hunting, gathering, and traveling across the land. The measures used by the Yup'ik people are often related to the human body. For example, in Figure 3, Henry Alakayak of Manokotak, Alaska, shows how he uses his arm and fist like a ruler to construct this ptarmigan snare. For practical tasks such as these, body measures are far more convenient than yardsticks.

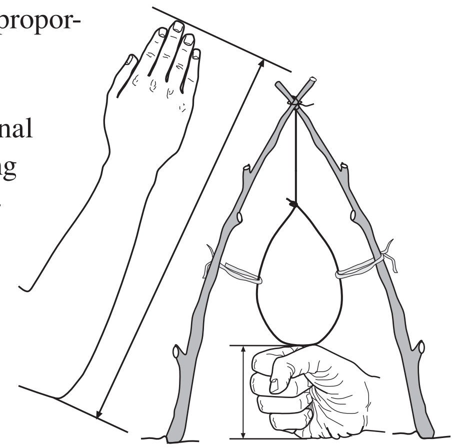

Fig. 3: Ptarmigan snare with measures.

During the short Alaskan summer, most coastal people are busy fishing. Salmon, in particular, constitutes a large part of the year's food supply, and therefore a successful salmon-fishing season is critical. The need to harvest a year's supply of salmon in a short time necessitates efficiency. People build their fish racks using locally available materials and their own body measures.

When building a fish rack, a person must consider the amount of salmon the rack needs to hold, the environment in which the rack is to be built, and the heights of the people using the rack. For example, a rack built in a windy area needs to meet different criteria than one in a calm area, while a rack built in a wet and rainy environment may differ from one used in a dry and sunny area. In addition, Mary George used her own body measures to gauge the length and height of her fish rack and poles. Since she was keenly aware of how the fish sizes related to her own body measures, this custom-made fish rack was efficient and easy to use.

The Structure of the Module

By depicting ordinary Yup'ik life, this module adds real-world significance to standard math problems and shows how math is a part of everyday life, wherever we are. As the students progress through the lessons in the module, they move from exercises that require them to use their intuitive and concrete understandings of mathematical relationships to those that require more abstract and formal understandings.

Yup'ik body measures differ from standard measures in that each measurement is specific to the person doing the measuring. It is a personal exercise. For example, when building kayaks, two or more kayak makers will use the same "standard measure" (as shown in Figure 4), but each person's measures differ according to his or her body length. This ensures that each kayak is tailor-made for each person. The custom fit that results creates a balance between the person, the kayak, and the sea.

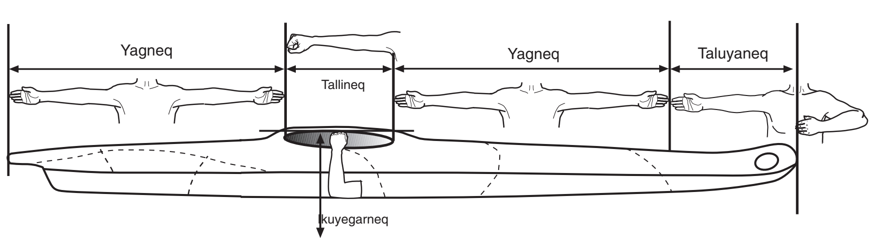

Fig. 4: Yup'ik body measures used when building a kayak.

Because body measures are proportional (a person's height generally equals his arm span, for example), they are the perfect conduit to introduce students to algebraic thinking. To assist students in making the transition from concrete body measures to the more abstract expressions used in algebraic thinking, we have developed a set of cultural math tools, including manipulatives that depict scaled-down versions of the body measures and fraction bars that use symbols to denote the Yup'ik body measures. In Figure 5, T represents the length of one *taluyaneq* and Y the length of one *yagneq*. As shown in Figure 2, these measures are generally related in a 2:1 proportion, no matter the length of a specific person. See Activities 6 and 7 for definitions of the other body measures shown in Figure 5.

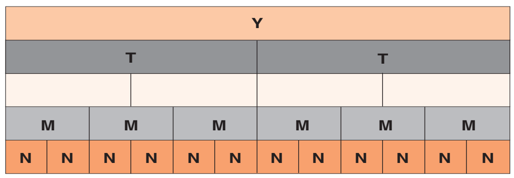

Fig. 5: Body measure manipulative from Activity 6.

To further assist students with learning to think in algebraic terms, we have developed some problem-solving strategies based on the ideas of three educators: George Polya (1945), a noted mathematician who emphasizes asking questions such as "what is the unknown?" when tackling a problem; Philip Brady (1990), an Alaskan educator who emphasizes working in groups and having each student role-play the teacher's role; and Robert Sternberg (1997, 1998), who emphasizes the need to incorporate creativity, practical knowledge, and analytical thinking into the learning process. The section titled "Procedures for Teaching Problem-Solving" is located at the end of this Introduction. We encourage you to use it as often as is appropriate.

Mathematics of the Module

This module is designed to provide a foundation in proportional and pre-algebraic thinking upon which students can build their understanding of algebra. The foundation is created through a very personal tool that students always carry with them—their own body measures. As they relate their body measures to cut salmon within the experiences provided by Mary George, they begin to apply this new knowledge in the forms of graphs, algebraic expressions and equations, proportional equations, and generalized patterns.

This module provides a hands-on, purpose-driven approach to create a conceptual understanding of proportional and pre-algebraic thinking that is often difficult for many students to grasp at the junior high level. The module introduces the proportion notation "Y is to T as 2 is to 1" (Y:T::2:1) and focuses on understanding the relationship between the lengths, which are modeled by body measures, patterns, and graphs. This module differentiates between proportions and ratios in that proportions describe the concrete relationships between variables, whereas, ratios describe abstract relationships, which might be even more difficult for students to grasp. Further, although an understanding of ratios can follow from what the module covers, ratios are not directly addressed. In moving from proportional lengths to building algebraic thinking with general patterns, ratios are tangential to the overall goal of this module.

Concerning graphing, this module focuses on understanding and interpreting graphs. Students gather several types of data such as body measures, mold growth, and weight loss. Once the data is organized, students create bar graphs, line graphs, or scatter plots and analyze the apparent relationships. Students do not work from an equation to a graph as may be typical in a pre-algebra or algebra course, but produce graphs based on actual data they have collected to better display and understand patterns or relationships that may exist in the data.

Ratio and proportion are critical concepts for the K–8 mathematics curriculum, as students need a solid understanding of both to be successful in algebra. Proficiency with algebra and algebraic thinking are, in turn, crucial for all subsequent mathematics. This module introduces students to proportions through physical representations of body measures and extends these models through investigations to help students identify and express patterns and relationships in terms of proportions. By moving from concrete representations and relationships that involve proportions to more abstract reasoning and simple algebraic equations, this module takes an approach to ratios and proportions that is different from the method found in traditional mathematics textbooks, which typically begin with algebraic equations (e.g., $9 = 2x + 1$) and require students to solve them with rote techniques. The traditional approach implicitly assumes an understanding of ratio and proportion that most students have not had the opportunity to fully develop. This module, in contrast, carefully develops the proportions concept so that traditional algebraic skills and reasoning can be built on a solid foundation of understanding rather than on memorized rote techniques, which can be forgotten or confused.

Lastly, this module can be successfully implemented in multi-age classrooms, with the understanding that varying levels of students may achieve or accomplish each goal to different degrees. For example, when describing the proportional relationships between body measure lengths, younger students may only be able to write a sentence stating that two *taluyaneq* are the same length as one *yagneq*. Whereas, more advanced students may be able to write out the proportional notation using variables, such as Y:T::2:1. Each step allows for students to progress towards proportional and pre-algebraic thinking and is appropriate for the level of the students.

Problem-Solving with Algebraic Thinking

Current research in math education encourages the discovery of mathematical principles and the solving of complex problems through hands-on work (NCTM, 2000). In addition, complex problems allow students to develop a rich and flexible approach to problem-solving, as opposed to rote memorization or decontextualized computation problems (Henningsen and Stein, 1997). More specifically, research suggests that students need to move from arithmetic to algebraic thinking in order to engage with proportional thinking. Theories about why students tend to struggle with this transition in thinking range from pedagogical strategies used by teachers to the developmental levels of students (Swafford and Langrall, 2000; Stacey, 1989). Researchers working in the Cognitively Guided Instruction (CGI) group argue that modifying how we teach arithmetic to students may help the transition to proportional thinking in the middle school grades (Carpenter and Levi, 2000; Carpenter and Romberg, 2004). Many of those strategies have been incorporated in this module. Case studies on student thinking and teaching strategies around these topics have been plentiful (Lo and Watanabe, 1997; Carpenter and Romberg, 2004; Edwards, 2000; Lannin, 2003) and can provide further useful insight.

The key components of proportional and pre-algebraic thinking we feel are needed for teachers to understand before taking their students into explorations within this module follow. Afterwards, procedures for teaching problem-solving are provided as well as a protocol to use with students throughout this module.

Arithmetic and Algebraic Thinking

Algebraic thinking differs from arithmetic thinking in its focus and is typically a middle school or junior high topic. Arithmetic thinking involves basic operations and is a one-step-oriented approach to learning mathematics. For example, when teaching principles of addition, the teacher might give a few problems such as $3 + 2 = ?$, $1 + 4 = ?$, $5 + 0 = ?$. These types of problems focus on drilling the concept of addition and are very specific to the numbers used. They do not encourage students to think of the big picture or the relationships that might exist between the problems. They can also lead to students believing that the equal sign only works one way (3 + 2 = 5 but not that 5 = 3 + 2). In contrast, algebraic thinking focuses on the solving process. In short, "Algebraic reasoning involves representing, generalizing, and formalizing patterns and regularity in all aspects of mathematics" (Van de Walle, 2001). The following problem fosters algebraic thinking rather than arithmetic thinking.

Problem: List all whole number sums of 5.

Solution: 5 = 5 + 0 = 4 + 1 = 3 + 2 = 2 + 3 = 1 + 4 = 0 + 5 = 5

This pattern shows a systematic approach to finding all the whole number sums of 5 and incorporates the idea of the commutative property (4 + 1 = 1 + 4). It also shows the reflexive property, or that the equal sign works "both ways," since 5 = 5 + 0 and 5 + 0 = 5. This type of thinking deepens the students' understanding of mathematical reasoning and relationships.

Three Key Concepts of Algebraic Thinking

Patterns: Students often think that mathematics merely means working with numbers, but this is a misconception. Mathematics can be thought of as the study of patterns. Patterns exist all around us—within our bodies, as discussed throughout the module; in nature, such as in the growth of sunflowers and seashells; and in human-made things, such as buildings, artwork, or music. Formal, modern mathematics stemmed from discovering these patterns in everyday situations as opposed to creating abstract mathematical concepts. It is helpful when constructing patterns from real-world examples to imagine, think about, or watch what is physically happening. For example, in Activity 13: Increasing Surface Area of a Fixed Volume, students explore the surface area of a number of 1 cm cubes, as separate entities and connected together to form rectangular solids. Two 1 cm cubes have a collective surface area of 12 cm, while the two cubes connected together have a surface area of 10 cm. Students can more easily appreciate this change in surface area by getting their hands on some 1 cm cubes, as opposed to looking at a picture of the problem. They will soon see that when they stick two cubes together, thereby creating a rectangular solid, two faces, one on each cube, disappear. Put another way, the number of visible faces goes from 12 to 10.

place	value	pattern
1	5	3(1)+2
2	8	3(2)+2
3	11	3(3)+2
4	14	3(4)+2
5	17	3(5)+2
6	20	3(6)+2
n	?	3(n)+2 ← general pattern

Fig. 6: Table showing pattern discovery of the sequence of terms.

When identifying a pattern, it is also helpful to have at least three terms in a series. There must be enough terms to fully identify a pattern, see that it can continue, and determine whether the pattern works for the first and second terms. Encourage students to write out several terms before they jump to conclusions about patterns.

Consider the following sequence of numbers as an example: 5, 8, 11, 14, 17, 20, etc. Students will generally recognize that the next term in the sequence will be 3 more than the previous term. This type of pattern is called a recursive pattern; we need to know the answer from the previous step in order to move forward. Recognizing this pattern is a key step in determining the general pattern, one that allows any number in the sequence to be found without knowing the previous number. The general pattern provides the big picture, or perhaps a summary, of what is happening and opens doors to more abstract reasoning.

To help determine the general pattern, look at the relationship between the numbers in the sequence and their order in the sequence. For the example, the number five comes first in the sequence; the number eight comes second; the number eleven comes third. We have already established that the pattern increases by three at each term, or in other words, it involves multiples of three. Multiples of three can be expressed as 3n [or 3 x n or 3 * n or 3(n)] where n = the term's order in the sequence. So, to determine an equation for the first term, one might write: 5 = 3(1) + ?. Two is the answer here. Let's try an equation for the second term in the sequence: 8 = 3(2) + ?. Again, the answer is two. The general pattern seems to be 3(n) + 2 where n = the term's order in the sequence. Let's see if it holds true for the third term. Does 11 = 3(3) + 2? Yes. See Figure 6 for examples of how to write out the pattern discovery. The general pattern of 3(n) + 2 exemplifies the type of algebraic thinking we want students to achieve. Through hands-on activities, this module provides the foundation students need to transition into discovering that these emerging patterns are stepping stones that lead to algebraic and proportional thinking.

Representation: Mathematics relies on various forms of representation. Abstracting from concrete to symbolic representation provides freedom for mathematical manipulation. For example, when first learning to add, it might be useful to count certain objects such as oranges or pencils. However, as the concept of addition is mastered, the physical objects only provide limitations. We use symbols to represent objects and equations or expressions to represent relationships among these objects. For example, in Activity 6: System of Body Measures, we discuss the relationships of various body measures. In this lesson, students discover that it is easier to use a symbol, such as Y, to represent the measure of outstretched arms *(yagneq)* rather than to write out the word each time. They will also discover that they can write the relationship, "two *taluyaneq* (from the middle of the chest to the end of one arm fully extended) equal one *yagneq*" much more quickly with the equation 2T = 1Y than with a sentence (Fig. 7).

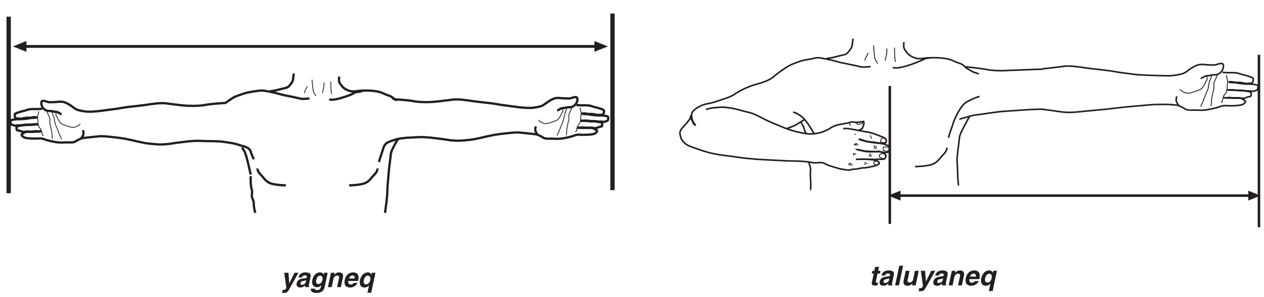

Fig. 7: Two specific body measures, yagneq *and* taluyaneq.

Remember that symbols generally represent a variable. For example, although Y represents a physical measure of a *yagneq*, Y is also a variable as the length of a *yagneq* varies from person to person. As we move deeper into algebra, there is a trend toward using x and y as variables. Students, however, should be encouraged to be more creative in naming variables in hopes that they will keep a stronger connection to the concrete (such as Y for *yagneq*). Alternatively, once we've moved from the concrete to the symbolic, we will have the freedom to manipulate equations and expressions irrelevant to the concrete. We have incorporated a variety of representations throughout the module including pictures, sentences, symbols, graphs, and tables. Often, we will introduce a concept using pictures or hands-on manipulatives, but then we will encourage students to move from those devices to using symbols, graphs, and tables. Writing out sentences to describe what is happening will help the student learn to express what is going on and move from the comfortable to the new. It will also benefit the teacher, as you will gain insight into how your students are thinking.

Equations and Expressions: There are two major categories of algebraic problems: expressions and equations. An expression is any combination of symbols and numbers involving mathematical operations. In Activity 6: System of Body Measures, students investigate adding various body measures and write their findings as an expression, such as T + 2I. An equation shows that two expressions are equal. For example, Y = T + 2I. It is useful to think of an equation as a balance beam. In whatever way we change one side, we must change the other in an identical manner to keep the equation balanced. For example, in the equation Y = T + 2I, if we multiply Y by three, we must also multiply T + 2I by three. Throughout the activities, we have provided extra challenges, often optional to the lessons, that broaden the students' practice in this area.

Tips for Implementation of Problem-Solving Process

The first step in any type of problem-solving situation, whether it is a worksheet full of math problems, a real-world dilemma, or a fun puzzle, is to ask some underlying questions such as (a) What type of problem is this? (b) What is the big picture? or (c) What is the most important thing to take care of first? Often times, students can solve simple equations, but may have more difficulty in determining how to take a more complex problem, determine what needs to be solved, set it up, and solve it.

For example, consider this real-world problem that can be solved mathematically. Suppose someone wants to catch enough salmon to feed a family of four for a year. The two adults each eat thirty salmon per month, while the two children each eat roughly fifteen per month on average. What's the big picture here? The big picture is figuring out the family's yearly consumption of salmon. What type of problem is it? Well, it's a multiplication problem, as each individual's monthly average of salmon consumption must be multiplied by twelve to get the yearly figures: 30(12) = 360, 30(12) = 360, 15(12) = 180, 15(12) = 180. It's also an addition problem, as each individual's yearly consumption habits must be added together: 360 + 360 + 180 + 180 = 1080. What is the most important thing to take care of first? Although there are various ways to approach this problem, in the math conducted above, the first thing to take care of is the multiplication, the second, the addition.

Keep in mind, as you work out a problem, its type may change along the way. For example, the problem above changed from a multiplication to an addition problem.

As you ask the fundamental questions, it is also helpful to understand vocabulary, relationships, and methodology. In the above problem, a student can ask himself if he knows all the words. He can also consider the important relationships involved in the problem, such as the individual's relationship to the family or a month's relationship to a year. Methodology in this case is knowing the correct operations to use (addition and multiplication) and the proper order in which to use them.

The final step is to check the answer by carefully rereading the story problem or by reworking the problem another way. For example, instead of working the problem as we did above, the student could first add the family's monthly averages together to get 90. He could then multiply 90 by 12 to get the family's yearly average of 1080. Since both the first and second method produce the same answer, it's a safe bet that 1080 is correct. If the two methods do not produce the same answer, the student needs to start over, perhaps by looking at his multiplication or by rethinking the relationships (for example, is month:year::1:12?).

Procedures for Teaching Problem-Solving

Outlined in the following two sections entitled "Teacher Modeling" and "Students Take Leadership Roles," are some problem-solving strategies. We devised these strategies based on the work of three educators. Polya (1945) formulated such problem-solving questions as "What is the unknown?"; "What is the data?"; "What is the condition?"; "Do you know a related problem?". Sternberg (1998) gave us insights into student learning by suggesting that students learn more about any subject if they use creative, analytic and practical intelligences in the learning process. Brady (1990) introduced us to an instructional strategy: reciprocal teaching, which is used in reading instruction but is easily adapted to mathematics. Reciprocal teaching allows students to work in small groups, taking turns assuming the role of teacher. The ideas presented by Polya, Sternberg, and Brady are consistent with our work in collaboration with Yup'ik teachers (Lipka, 1998) which also supports group-oriented, expert-apprentice modeling in which students learn to utilize the skills and knowledge of more expert learners.

As you follow the suggested problem-solving techniques throughout the course of the module, you will want to encourage and aid students in establishing problem-solving norms in their groups, generating multiple solutions through diverse approaches, justifying results and correctness mathematically both within groups and amongst groups, and reconciling differences in approaches and solutions. While students work in groups, you will want to encourage students to reach conclusions within their own group and talk among groups. Then, challenge them to create an entire class conclusion by discussing, sharing, showing, and presenting group results to the entire class.

An assessment tool grows naturally out of the suggested problem-solving techniques. At first, students may say, "I don't know" when asked to solve a problem. Note such comments, the date, the problem, and other relevant information. As the course proceeds, note what students continue to say and do as they solve novel problems. As students move from "I don't know" and use of "trial and error" to develop more effective problem-solving skills, your documentation will help you to assist the students in becoming more independent learners throughout the remainder of the module.

Sample Problem

This problem is adapted from George Lenchner's (1983) *Creative Problem-Solving in School Mathematics.*

Example: Two king salmon weigh the same as two chum and six red salmon. A chum weighs the same as three red salmon. How much does a king weigh in terms of a red salmon?

Solution: Two king weigh the same as 2 chum and 6 red salmon, so $2k = 2c + 6r$.

A chum weighs the same as 3 red salmon, so $c = 3r$.

Therefore, $2k = 2(3r) + 6r$, $2k = 6r + 6r$, $2k = 12r$, $k = 6r$.

A king weighs the same as 6 red salmon.

Modeling and Talking Aloud: A Sample Script

It may be difficult for students to explain their thinking aloud when approaching a new problem to solve. We feel this is an important step in understanding and refining students' problem-solving skills. Lenchner explains that there is a difference between solving exercises, where the procedure is already known, and problems, where the strategy is not always apparent and may require some creativity (1983, p. 8). The ultimate goal of math education should be to help students solve problems. The following two approaches capture the thinking of MCC writers while working through the sample problem above. The first approach may seem confusing at times, but it shows how the problem-solver was able to think through her confusion and find the important information and approach needed to work towards the solution. The second approach shows a different method of creative thinking and how finding other representations may be helpful in problem-solving.

First approach: This approach is similar to the "Read Aloud—Think Aloud" approach used in content reading (Vacca & Vacca, 1999). Here is a verbatim account of how one of the MCC writers solved this problem:

> I am trying to understand what the problem is saying. I was confused by the language—"2 king weigh the same as 2 chum and 6 red." So then I said to myself that two king equal 2 chum and a chum weighs the same as 3 red salmon. Then, logic tells me that two king can't be the same as 2 chum. Something is wrong here.
>
> Drawing a picture looks like a lot of work and it might not be a good strategy here. My first reaction to this problem is to write it out as a symbol. I realized that king, chum, and red are the important things. So, I used symbols for them instead: k = king, c = chum, and r = red. This looks very straightforward when you work it out; the chum weighs the same as 3 red salmon. Instead of using chum, I will use 3 red. The reason for this is that a chum is the same as 3 red. Why did I solve the problem this way? There is too much information—because the first sentence has three different types of salmon.
>
> Three variables make it too hard to compare. So, I use the other information to make that comparison easier. That is where we get 2k = 12r. From here, the problem is easier to understand and solve. We can see that 1k has to be equal to 6r.

Second approach: Another approach is to model the problem using manipulatives. Be sure to talk through the problem-solving as you did in the first approach.

For example, use a banana to represent a king, an apple for a chum, and a cherry for the red (Fig. 8). Put two bananas in a bowl to show the first group. Put two apples with six cherries in another bowl to show the second group. Since the other sentence in the problem says a chum is the same as three reds, replace each apple with a set of three cherries. Now, the problem is simplified because there are only bananas and cherries (or king and red) left.

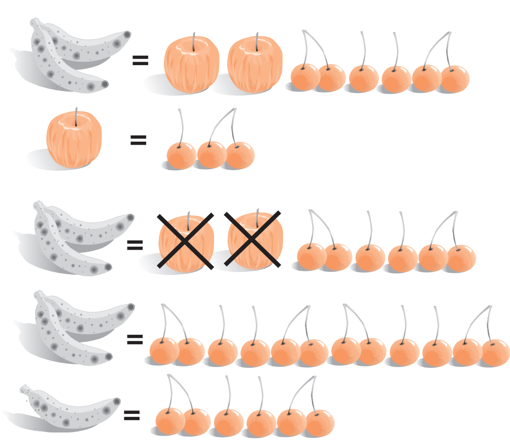

Fig. 8: Using everyday items to help solve a complex math problem.

Problem-Solving Protocol

I. Teacher Modeling

For the first phase of the problem-solving strategies, the teacher models for the students what to do as in the sample problem/solution strategies previously shown. The teacher explains the pedagogical technique at the outset of the demonstration so the students know what's happening in the first "definition" phase. Choosing a problem in the module (or constructing his/her own problem), the teacher should follow these steps.

Problem Definition

- Read the problem aloud.
- Go through the vocabulary. Do the students know what the words mean?
- If not, can students infer the meaning of the unknown word(s) from the rest of the problem?
- Ask: Do we understand the problem?
- Ask: What type of problem is this (i.e., addition, subtraction with fractions, etc.)?
- Ask: What information is given?
- Ask: What information is missing? What do we need to find out?

Pedagogical Technique

The teacher should explain that sometimes explaining the problem to someone else may help to understand the problem. Also, often times it may be helpful for students to rewrite a problem in their own words.

Devise a Plan

The teacher demonstrates how to devise a plan by asking, "What can we do to solve this problem?" We can:

- make a table,
- find a pattern,
- solve a simpler problem,
- use logic,
- apply trial and error,
- create an experiment,
- draw a picture, or
- ask students for other possibilities.

Carry out the Plan

The teacher demonstrates how to carry out the plan. This step may include using paper and pencil and making the necessary calculations. The teacher should emphasize that if a problem-solver (including the teacher) gets stuck before finding an answer, he/she may need to throw away the first plan and devise a new one.

Looking back

The teacher emphasizes that the last step in problem-solving is to review the question and decide if it has been answered correctly. The teacher models methods of reviewing the problem and solution.

II. Students Take Leadership Roles

The second phase involves students working in small groups (two to five students per group) and taking on the role of the teacher. It is important for students to shift roles, so everyone can have a turn taking the lead in prompting the other members of the group to use the problem-solving strategies. Have students take on the following roles:

- Leader—this person leads the discussion within the group to help identify the type of problem with which they are working.
- Planner—once the problem is established, a student other than the leader in the group now takes on the role of facilitating the planning.
- Evaluator—after the students have carried out the plan, this student checks or facilitates the process of checking whether the result satisfies the requirements of the problem.

Students change and rotate roles for each new problem-solving session. Encourage groups of students to visit other groups to view and discuss different approaches.

The Literacy Connection: Using Literacy Counts! in the Module

Literacy Counts! is an integrated approach that provides points of entry to math and literacy that are both engaging and accessible to students of all levels. Students will keep a math notebook to track their discoveries, record any new material, draw tables and graphs, and organize any other information gained throughout the module. In general, students will want to record not only the data found, but also the thinking involved in the process and any ideas about their results. This recording of information should be done in the best way each student knows how to express his/her ideas, although suggestions are often given in the module to guide students in the direction of algebraic thinking.

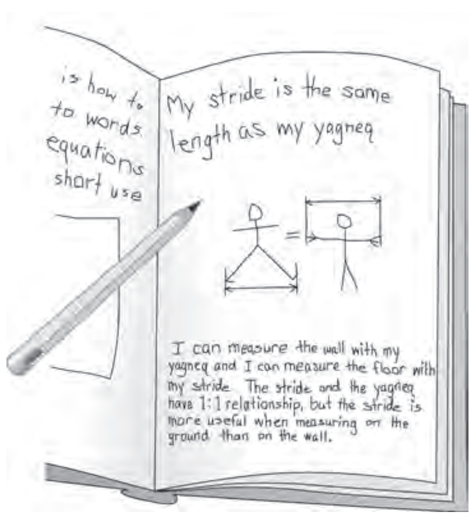

Fig. 9: Example of a math notebook entry.

Progression of the Module

The first section of the module provides the introductory material needed for the entire module—cultural, scientific, and mathematical. Activity 1 introduces students to the Yup'ik people and ways of preserving food. Activity 2 is a science experiment to investigate factors that create food spoilage, continuing the idea of food preservation. Activity 3 asks students to begin exploring their own body measures and how they may relate to each other within the context of hanging fish on a fish rack pole, one method used to preserve fish in the Yup'ik culture. Further, students start to learn that for the individual, each body measure is a constant, but when compared to other people, that same body measure becomes a variable. To deepen students' experiences with the ideas of constants and variables, comparisons are made by converting nonstandard measures to standard measures (inches or centimeters) and through representing the lengths in tables and graphs. Students construct both bar graphs (single-variable) and scatter plots (two-variable) of their measurements in Activity 4, and continue to practice with various graphs to further understand patterns in the data. From the experiment in Activity 2, students gather qualitative and quantitative data in an ongoing manner while working on the other activities in this section. After they have worked on graphing in Activity 4, they are also to return to their data from the experiment and graphically show those results as well. Through the investigations of body measures and various representations of those measures, students start to build the proportional thinking needed to continue throughout the module.

In Section 2, students build their knowledge of body measures by exploring proportional body measures and creating a system where each measure is related to the others. This idea provides the transition from arithmetic to algebraic thinking with a focus on proportions. Students are encouraged to start abstracting their body measures by working with a cultural math tool similar to Cuisenaire Rods that represents the proportional lengths of their body measures. Further, students abstract their body measures into variables. Recognizing body measures differ between people, students write sentences describing the relationships between variables and build algebraic expressions and equations. Once students feel comfortable manipulating their tools and working with the variables, they are encouraged to evaluate the variables using lengths other than those originally discovered. This step in abstraction helps students begin to work algebraically. Lastly, in Activity 7 students learn the notation for proportions as they continue working with the same relationships between body measures discovered earlier.

Section 3 connects body measures with the notion of food preservation introduced in Section 1. First, students set up and conduct another science experiment, with the focus on understanding how increased surface area decreases drying time. To do so, they use quarter sections of an apple with each of the four sections cut into various numbers of pieces and analyze the changes in weight over time while drying. As students work with their drying apples, they learn more about how Mary George used proportional measures to efficiently hang cut salmon to dry on her fish rack poles. They learn which body measures are equivalent to the length of the cut salmon on a pole and more details about how to hang salmon to dry. The actual process of cutting a king salmon to dry is explained in detail in an appendix. Students work with another cultural math tool, model salmon cut-outs (proportional pieces representing the body measures and cut fish), to build equations of the number of drying salmon, varying in cut and species, that fit on a fish rack pole. This pole is the same length with which students worked extensively in Activity 3. The section culminates with students playing a fishing game in which they algebraically model catching and hanging salmon to dry. Although students work to catch enough to feed their families for the winter, they must adhere to the cultural guidelines that counsel against any waste. At this stage, you can either conclude the module or continue with Section 4, which contains challenging mathematics for discovering general patterns and writing the results using formal algebraic notation.

Lastly, Section 4 provides a series of activities created to help students fully understand why the Yup'ik people cut salmon before hanging it to dry. Cutting salmon is a required skill for survival and many Yup'ik women are known as expert fish cutters. Yet, there is also an art to cutting salmon, so that the pieces are not too thin or too thick and do not spoil, all of which affects the taste. Students investigate the surface area and volume of a rectangular solid first, to be sure they understand these concepts. If your students are already comfortable with these ideas, they can skip Activities 11 and 12. In Activity 13, students analyze patterns in the data found by considering the surface area and volume of an increasing number of centimeter cubes both separated and connected. Students are provided with multiple ways of viewing the problem so that they can create general patterns describing the changes in surface area. Mathematically, this activity epitomizes the algebraic thinking of this module by encouraging students to develop formulas such as $4n + 2$ where n is the number of cubes. The final exploration in this activity leads students to analyze the pattern between separated and connected cubes showing that surface area increases, yet volume remains the same. In conclusion, they piece this information together with the results of the science experiments in order to explain the specifics of how cutting salmon to dry is an effective method for food preservation.

Throughout the module, students are encouraged to use the problem-solving skills developed and refined as they work. A problem-solving protocol is provided in this Introduction that explains how to set up groups and provide roles to each student so that together they are able to engage problems and use the discovery process to deepen their understanding of the mathematics and science concepts contained within the module. As the teacher, you will want to consistently reinforce and encourage students to follow their assigned roles so that they begin to feel comfortable with the approach.

We have also included three assessment sections following Activities 4, 7, and 10 that contain practice problems that integrate the lessons from the previous activities with real-world situations and problem-solving tactics. The assessments seek to measure a range of knowledge, from the students' understanding of vocabulary to their ability to solve multi-step problems. Further, the assessments are designed so you can evaluate the progress of students' mathematical thinking and their use of the problem-solving protocol.

We hope you and your students enjoy this approach to developing proportional and pre-algebraic thinking. By using students' body measures as the foundation and the Yup'ik practice of cutting and hanging salmon to dry as a cultural guide, this module deepens students' mathematical comprehension of relationships, providing the foundation for transitioning their thinking from arithmetic to algebraic, needed for success in higher levels of mathematics.

Master Vocabulary List

Algebraic equation—a mathematical sentence showing equivalent combinations of operations, numerals, and/or variables using an equal sign.

Algebraic expression—a mathematical phrase that combines operations, numerals, and/or variables.

Approximate measures—measurements which are estimated or close, but not exact.

Bar graph—a graph that uses separate bars (rectangles) of different lengths to display and compare data.

Brine—salt water solution.

Constant—a value that does not change.

Controlling a variable (in a scientific experiment)—focusing on a variable by defining its values and/or limits. For example, focusing on moisture by defining the values of moist and dry.

Coordinate—an ordered pair representing the independent and dependent variables (x,y).

Coordinate system—the x- and y-axes attributed to Rene Descartes (the Cartesian coordinate system).

Dependent variable—the variable in a relationship that relies on the independent variable, usually on the y-axis.

Dimension—the fewest number of independent coordinates required to specify uniquely the points in a space; the range of such a coordinate.

Edge—a one-dimensional intersection (line segment) of two faces in a three-dimensional solid.

Equivalent—having the same value, measure, or meaning.

Estimate—to guess or calculate approximately; the value of a guess or approximation.

Face—a two-dimensional polygonal surface (flat side) of a three-dimensional solid.

Fish camp—a temporary place usually established along a river in order to more easily catch and process fish for storage for the long winter.

Fish rack—a three-dimensional structure used for drying fish.

Food preservation—to prepare food for future use, as by canning or salting, so as to prevent it from decaying or spoiling.

Formula—a rule that is expressed using symbols.

Graph—a display of information through pictures or symbols.

Horizontal axis (or x-axis)—parallel to the plane of the horizon; not vertical.

Independent variable—the variable in a relationship that can change freely, usually on the x-axis.

Line graph—a graph in which data points are represented by dots and connected by line segments.

Migrate—to move from one region to another with the changes in seasons, as many birds and some fish do.

Model—(n.) a small object, usually built to scale, that represents in detail another, often larger, object.

Model—(v.) to make a physical or mathemathical model.

Mold—fungus that often forms on food as part of the decomposition process (spoilage).

Nonstandard measure—measurement found using units such as body measures or objects like paper clips.

Ordered pair—a pair of numbers used to locate a point on a graph with the first number showing the value on the horizontal axis and the second number showing the value on the vertical axis.

Origin—the specific ordered pair on the coordinate system representing (0,0).

Pattern—recognizable regularities in situations such as in nature, shapes, events, sets of numbers.

Plot—to graph.

Practical measures—measurements that have practical meaning or develop within or out of a context with purpose.

Proportion—an equation which states that two ratios are equivalent.

Proportional—having the same or a constant ratio.

Qualitative—of, relating to, or concerning quality.

Quantitative—expressed or expressible as a quantity; of, relating to, or susceptible to measurement; of or relating to number or quantity.

Ratio—a comparison between two values written in fraction form.

Rectangular solid—a three-dimensional shape having six rectangular sides, such as a shoebox.

Recursive pattern—a number pattern in which one or more previous terms must be used to determine the next term.

Relationship—the quality or state of being related; a connection.

Salmon run—the movement of salmon upstream to spawn.

Scale—ratio between the dimensions of a representation and those of an object.

Scatter plot—a graph made by plotting points on a coordinate plane to show the relationship between two variables in a data set. The points are not connected by line segments.

Scientific experiment—an experiment in which certain properties are controlled and results are recorded.

Side—another term for the face within a three-dimensional solid.

Smokehouse—a covered structure used to smoke fish and meat. It includes a fire pit in the floor and racks to hang the objects. The sides are vented so the smoke can move freely through the structure.

Standard measure—measurement in standard units such as feet, inches, meters, etc.

Subsistence—means of support or livelihood; often the barest means of food, clothing, and shelter needed to sustain life.

Surface—the outside of a three-dimensional solid; each surface can be called a face or side.

Surface area—the sum of the areas of all the faces, or surfaces, of a solid figure, expressed in square units.

Table—a compact, systematic list of related details, facts, figures, etc.

Three-dimensional—having length, width, and height.

Units—the smallest whole numbers of any quantity, amount, distance, or measure.

Variable—a letter or symbol used to represent one or more numbers in an expression or equation.

Vertical axis (or *y*-axis)—that which is perpendicular to the horizontal axis.

Volume—the measure of the amount of space inside of a solid figure (cube, ball, cylinder or pyramid), expressed in cubic units.

Iqelqin—the measure of both hands with outstretched fingers, between the tips of one's little fingers.

Ikusegneq—the measure from one's elbow to the end of one's fingers.

Kiarneq (*kiarneret*, plural)—fillet of fresh fish without skin.

Malruk naparnerek—the measure of both fists connected by outstretched thumbs.

Naparneq—the measure of one's fist with outstretched thumb.

Patneq—the measure of one's hand width, from the outside of one's pinky finger to the outside of one's thumb.

Pingayuneq—the measure of the width of one's first three fingers.

Segta—person who cuts fish for drying.

Segvik—place to cut fish.

Tallineq—the measure of one's outstretched arm from the underarm to the fist or to the fingertips.

Taluyaneq—the measure from the middle of one's chest to the end of one's outstretched arm.

Taryaqvak—king salmon.

Tusneq—the measure of the width between one's shoulders.

Uluaq (*uluak* for 2; *uluat* for 3 or more)—specialized knife with a broad, curved blade.

Yagneq—the measure between fingertips of one's outstretched arms.

Master Materials List

Teacher Provides:

Apples
Blank paper
Blank transparencies
Bread with and without preservatives—four pieces for each group
Butcher paper (optional)
Calculators (optional)
Cardboard
Centimeter cubes
Colored pencils
Colored pens (optional)
Compass for drawing circles
Crayons
Dried and/or preserved foods for examples
Eyedroppers
Four pieces of bread for each group; two with preservatives, two without
Knives for cutting apples
Labels for Petri dishes or other containers
Markers
Masking tape
Material such as screens, oven grates, etc. for laying out apple slices to dry
Math notebooks
Pencils
Petri dishes or other suitable containers
Protractors
Rulers
Scales for weighing in grams
Scissors
Standard 6-sided dice
String, plain (optional—different colored)
Tape—clear
Transparency pens
Yardsticks or meter sticks

Package Includes:

CD-ROM: *Yup'ik Glossary*
Poster, Hanging Salmon to Dry
Poster, Body Measurements
Poster, Fish Cuts

Blackline Masters for Transparencies

Alaska's Southwest Region
Bar Graph Example
Body Measure Chart
Centimeter Graph Paper
Design Matrix
Fish Camp
Fish Rack
Fish to Body Measures Chart
George's Yearly Salmon Catch
Graphing on the Coordinate System
Map of Alaska
Salmon Fishing
Salmon Life Cycle
Smokehouses
Temporary and Permanent Fish Racks
The Five Salmon Species

Blackline Masters for Worksheets

Arithmetical Progressions (optional)
Assessment Activity A
Assessment Activity B
Assessment Activity C
Body Measure Chart
Extra Challenges (optional)
Family Cards
Fish Cards: King
Fish Cards: Red
Fish Cards: Chum
Fish Distribution Table
Game Board
Game Instructions
Graphing Practice
How Do the Sizes Relate?
Model-Sized Pieces
Scenario Cards
Surface Area and Volume Patterns
Surface Area Grid
Surface Area Worksheet (optional)

NCTM Standards and This Module

The skills and knowledge emphasized in these activities relate directly to the NCTM standards (1989, 2000) as listed here. Aspects of grades 6–8 math content included in this module focus on problem-solving, reasoning and proof, communication, connections, and representation. Further descriptions and examples of each standard can be found at the NCTM website: http://standards.nctm.org/document/chapter6/index.htm.

Number & Operations

Partially:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates

Algebra

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

Geometry

Partially:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems

Measurement

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

Reasoning & Proof

- Select and use various types of reasoning and methods of proof

Problem-solving

- Build new mathematical knowledge through problem-solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem-solving

Communication

- Organize and consolidate mathematical thinking through communications
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

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Photography Credits

Anchorage Museum of History and Art: *Fish drying rack and fish camp scenes;* Ward Wells Collection B83–91, WW5-1277-008 (31, 100).

Internet Resources

ETA Cuisenaire: http://www.etacuisenaire.com/cuisenairerods

National Council of Teachers of Mathematics: http://www.nctm.org/

Section 1: Becoming Familiar with Body Measures

In this section, students are introduced to two important aspects of Yup'ik life: the fishing season and food preservation. Through this section, students learn about measuring using their own bodies. This section provides the foundation needed for further journeys into proportional and pre-algebraic thinking. Throughout the remainder of the module, students will look at mathematical concepts such as variables and constants, units, graphing, and various methods of representing the same relationships based on their results from this section.

Mary George modeling the tumarneq *measure.*

Activity 1
Fishing in Southwest Alaska

Along Alaska's coasts and rivers, Alaskans prepare to catch, dry, and store the bountiful run of the five species of Pacific salmon. The Yup'ik along the southwest coast of Alaska have particularly large runs of salmon. To take advantage of this influx of salmon, entire villages may move to more advantageous locations and set up temporary residence. For example, the village of New Stuyahok, located on the Nushagak River, moves to a place called Lewis Point where the king and red salmon run in good numbers. Here people prepare existing fish drying racks or build new ones and smoke their salmon to preserve food to last through the long winter.

The activities in this module follow the social and cultural storyline of catching and preserving salmon. Food preservation occurs in all societies. In modern Western society, with the preponderance of store-bought food, most elementary-aged students may not be aware of the various ways foods are prepared and preserved. However, all students in their daily lives come into contact with preserved foods, such as store-bought bread which has preservatives to ensure a longer shelf life. This activity, therefore, will alert students to both their own society's and the Yup'ik people's food preservation techniques.

Please see the module *Building a Fish Rack* for extensive cultural and historical information about the Yup'ik people as well as geographical and biological information on Alaska. If you have already taught *Building a Fish Rack,* then you can either review some of its information as you prepare your students for this module or you can teach the brief introductory material in this activity. Further cultural information related to the various aspects of the fishing season can be found in the following modules: *Building a Smokehouse* and *Salmon Fishing.*

Goals

- To gain a basic understanding of subsistence fishing in Yup'ik culture
- To review basics of Alaska geography
- To learn about Pacific salmon species and migratory behavior
- To estimate and compare yearly meat consumption of a student's own family to a Yup'ik family
- To learn about food preservation in Yup'ik culture and in the students' own culture

Materials

- Poster, Hanging Salmon to Dry
- Transparency, Fish Camp

- Transparency, Map of Alaska
- Transparency, Alaska's Southwest Region
- Transparency, Salmon Fishing
- Transparency, Fish Rack
- Transparency, Smokehouses
- Transparency, Salmon Life Cycle
- Transparency, The Five Salmon Species
- Transparency, George's Yearly Salmon Catch
- Math notebooks

Preparation

Prepare the nine transparencies listed above in order. Post the Hanging Salmon to Dry poster and, if possible, leave posted for the entire duration of the module.

Duration

One class period.

Vocabulary

Estimate—to guess or calculate approximately; the value of a guess or approximation.

Fish rack—a three-dimensional structure used for drying fish.

Fish camp—a temporary place usually established along a river in order to more easily catch and process fish for storage for the long winter.

Migrate—to move from one region to another with the changes in seasons, as many birds and some fish do.

Salmon run—the movement of salmon upstream to spawn.

Smokehouse—a covered structure used to smoke fish and meat. It includes a fire pit in the floor and racks to hang the objects. The sides are vented so the smoke can move freely through the structure.

Subsistence—means of support or livelihood; often the barest means of food, clothing, and shelter needed to sustain life.

Instructions

1. To introduce the module to students, explain that the class will learn how the Yup'ik people in southwest Alaska catch and process salmon as part of their winter food-gathering. Students will learn a system taught by Mary George from Akiachak, in the Lower Kuskokwim region. Through the activities, as students apprentice under Mary, they will encounter several math ideas that lead to pre-algebraic thinking, including body measures, proportions, and pattern recognition.

2. Ask students where their food comes from. Discuss the various responses including grocery stores or supermarkets, home gardens, farms, food banks, restaurants, hunting, fishing, gathering, etc.

3. Show the Fish Camp transparency and elicit connections between this scene and similar activities in which your students may have been involved, such as any activity that includes gathering food (berry picking), growing vegetables, or fishing. Discuss how the camp is set up each summer to catch, cut, and dry salmon for the winter.

4. Show the Map of Alaska transparency. Point out the major cities (Anchorage, Fairbanks, and Juneau) where life is similar to most cities in the U.S. Note the vast tracts of land not reached by highways (including the capital). Show the Alaska's Southwest Region transparency, and focus on southwest Alaska, the area in which the materials for this module were developed. Point out Akiachak, as this is the home of Frederick George and his late wife, Mary George.

5. Point out the Hanging Salmon to Dry poster and discuss how the species of fish are cut differently and how they fit on the drying rack using body measures. Show the series of transparencies Salmon Fishing, Fish Rack, and Smokehouses.

6. Explain that many people in Alaska live a subsistence lifestyle. Subsistence means depending on food people can hunt, fish, or gather. The major salmon runs occur from May to September, and it is during these months of the year that the Yup'ik people build their fish camps to catch and process their yearly salmon supply. Ask your students to imagine that they and their families are going to purchase all the chicken or beef needed for one year at one time. How much would they need? How would they store it all? How would they make sure their food didn't spoil during the year?

7. Ask the students to name some common food items that have been preserved, such as pickles, bacon, fruit cocktail, beef jerky. Ask them to think of some preservation techniques, such as canning, salting, drying, or adding certain food-preserving chemicals.

8. Show the Salmon Life Cycle and The Five Salmon Species transparencies. Discuss the various stages of a salmon's life and then the various types of salmon. Explain that the amount of each type of salmon passing through an area may be very different from that in other areas. Since migrating salmon return to their original birth places, various species travel at different times of the year and in different waterways to get home.

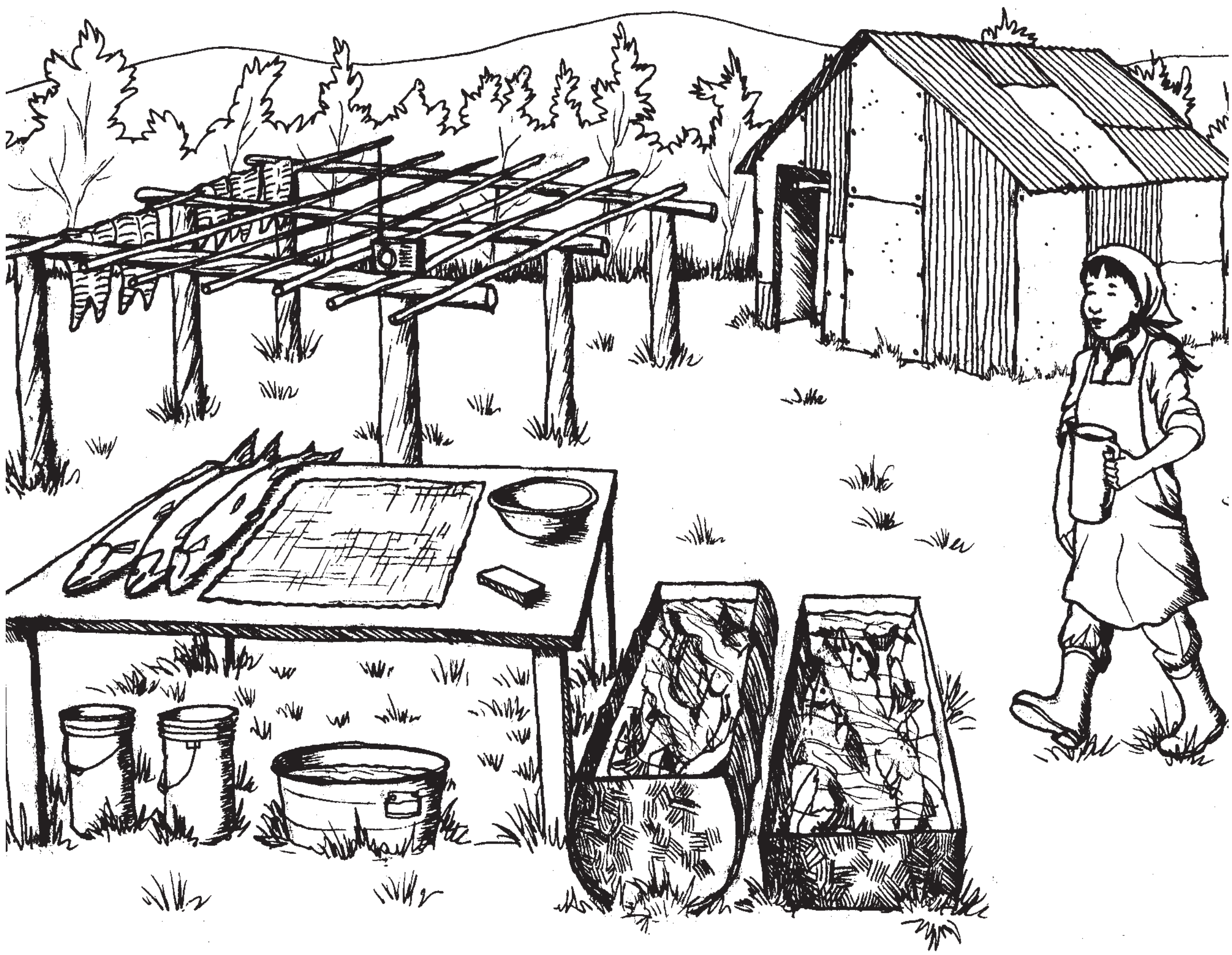
Cultural Note

Salmon are very important to the livelihood of most Yup'ik people. The king salmon *(taryaqvak)* are the largest of the fish and often times the preferred species. The sockeye or red salmon *(sayak)* are also among the favorites. The chum salmon (called *kangitneq* in Bristol Bay, *iqalluk* in the Bethel area, and *aluyaq* in the village of New Stuyahok since they become colorful by the time they reach that place) are also referred to as dog salmon as they were often times used for dog food instead of human consumption. Lastly, there are also the coho or silver salmon *(qakiiyaq)* and the pink salmon *(amaqaayak)*. Depending on the size of the fish, each one is processed differently to allow for the best use of the meat. Later, in Section 3, students will learn which cuts are used for each species and the relative size each requires on a fish rack pole.

9. Share with the class how much salmon the George family catches in a season by showing the transparency, George's Yearly Salmon Catch. Explain that these numbers are based on an estimate for one year's catch, but can vary from year to year. The weight of the salmon caught also varies. This catch is for the George's immediate family as well as for their extended family.

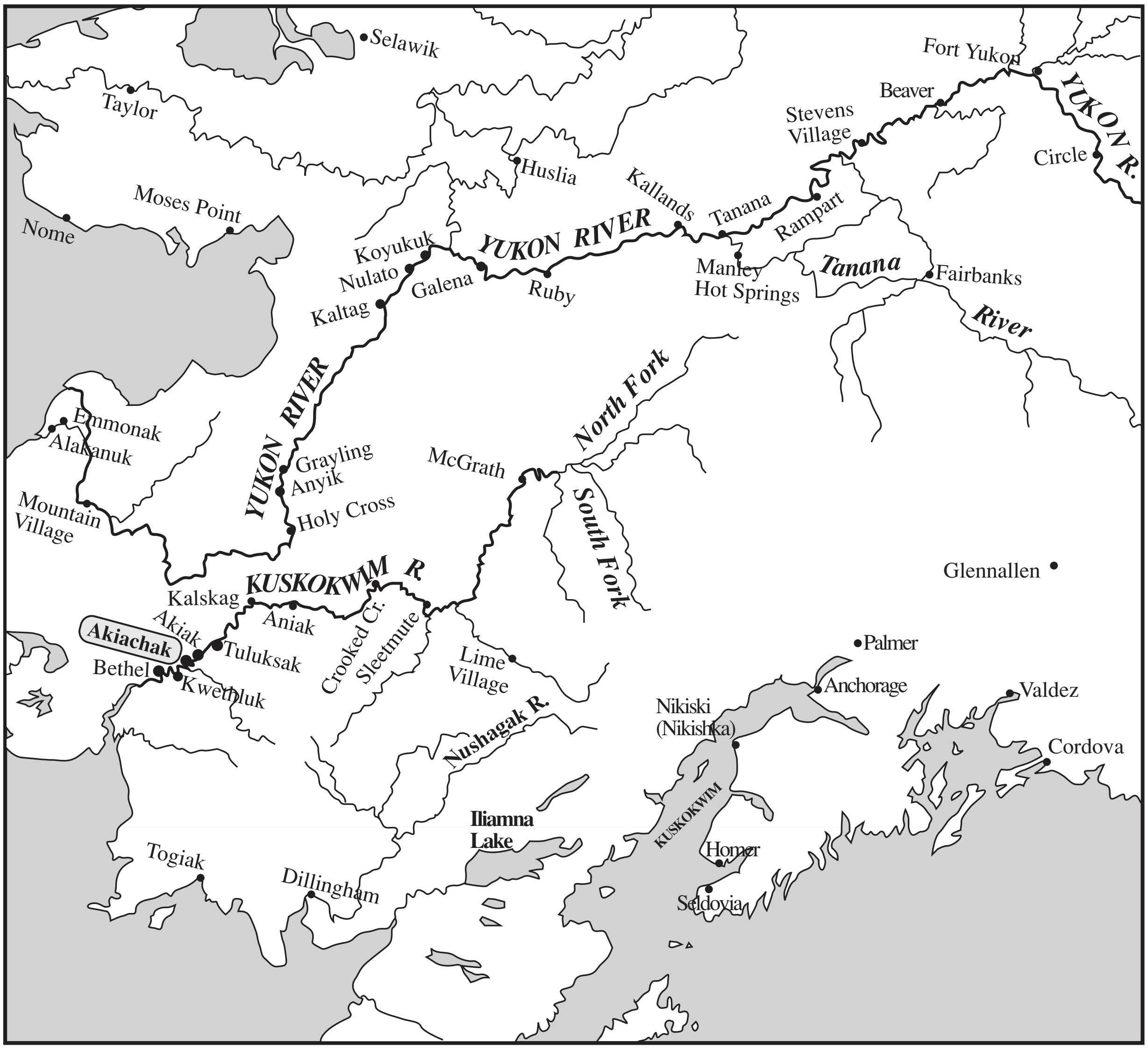
10. Summarize the day's discussion by handing out math notebooks and asking students to write about the importance of salmon for the Yup'ik people and the necessity of preserving fish for an extended period of time.

Fish Camp



Map of Alaska

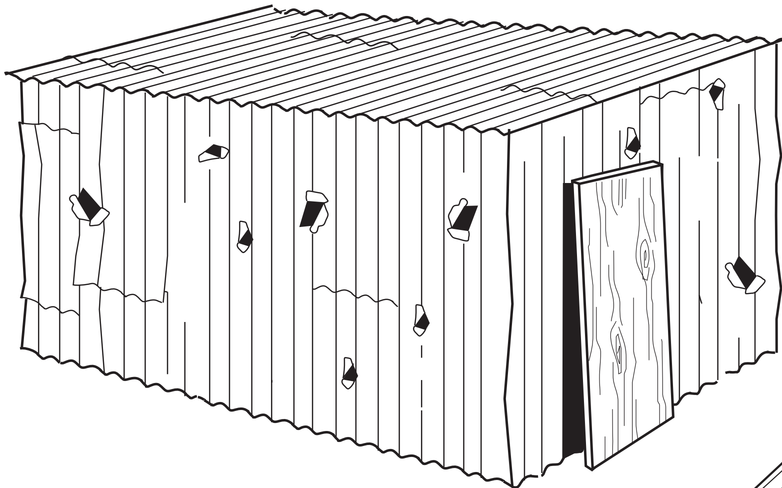
Alaska's Southwest Region

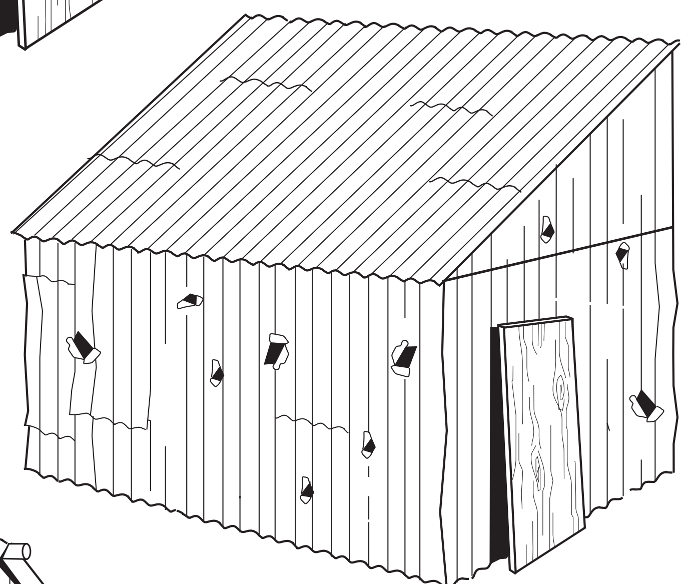


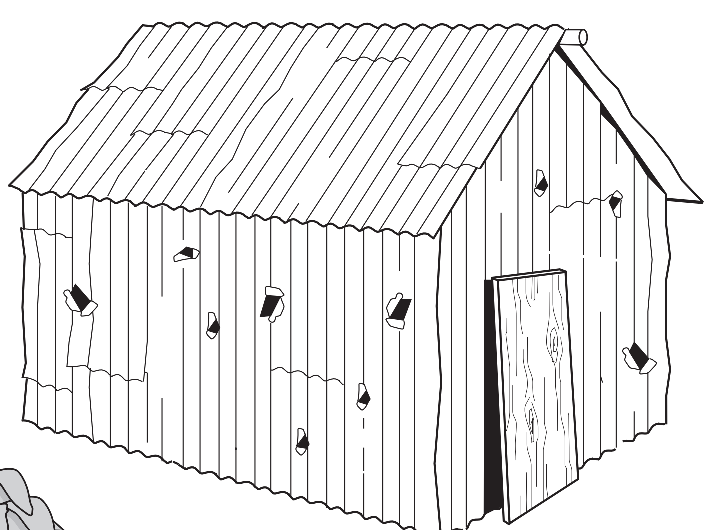
Salmon Fishing

Fish Rack

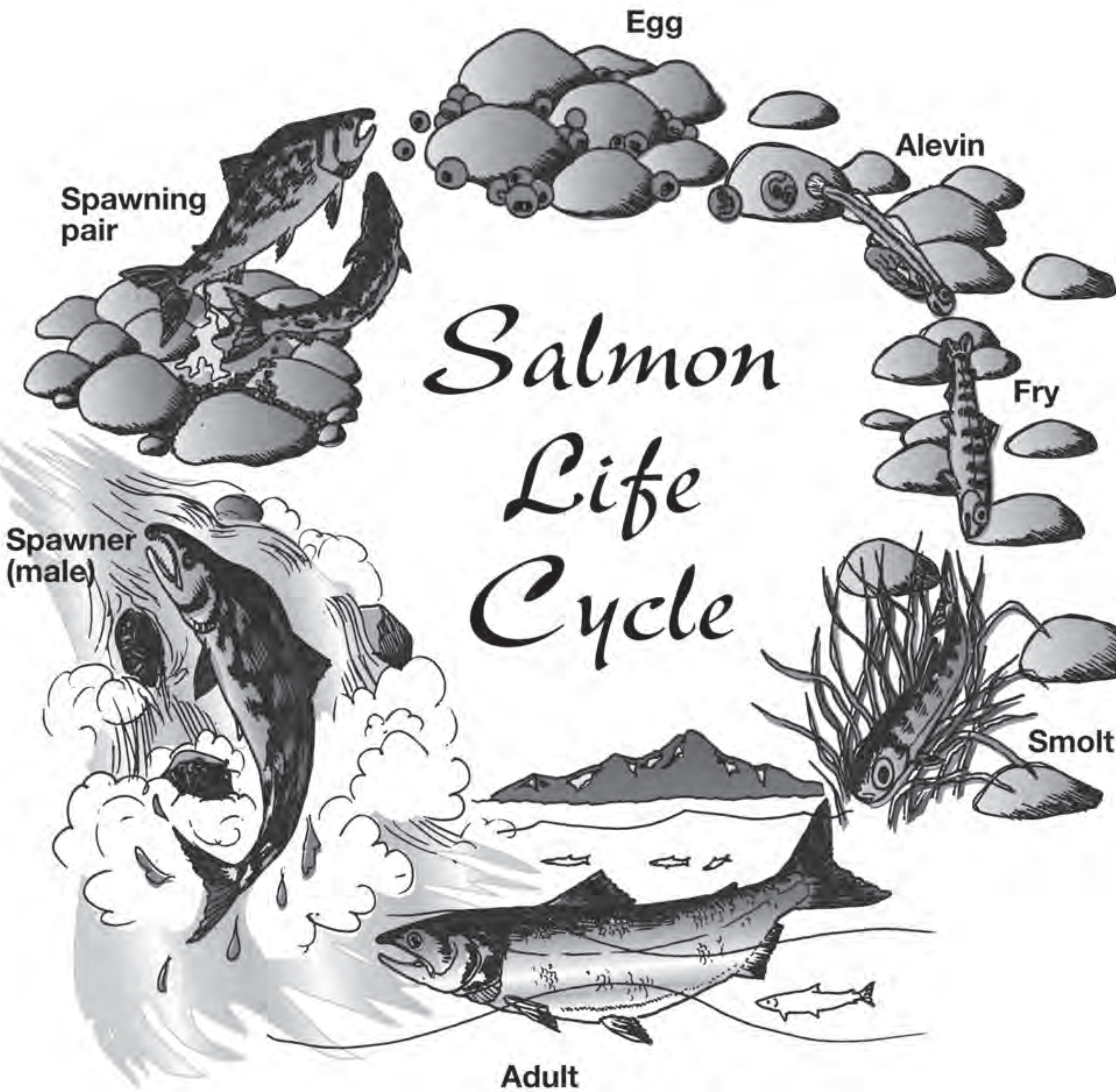
Smokehouses



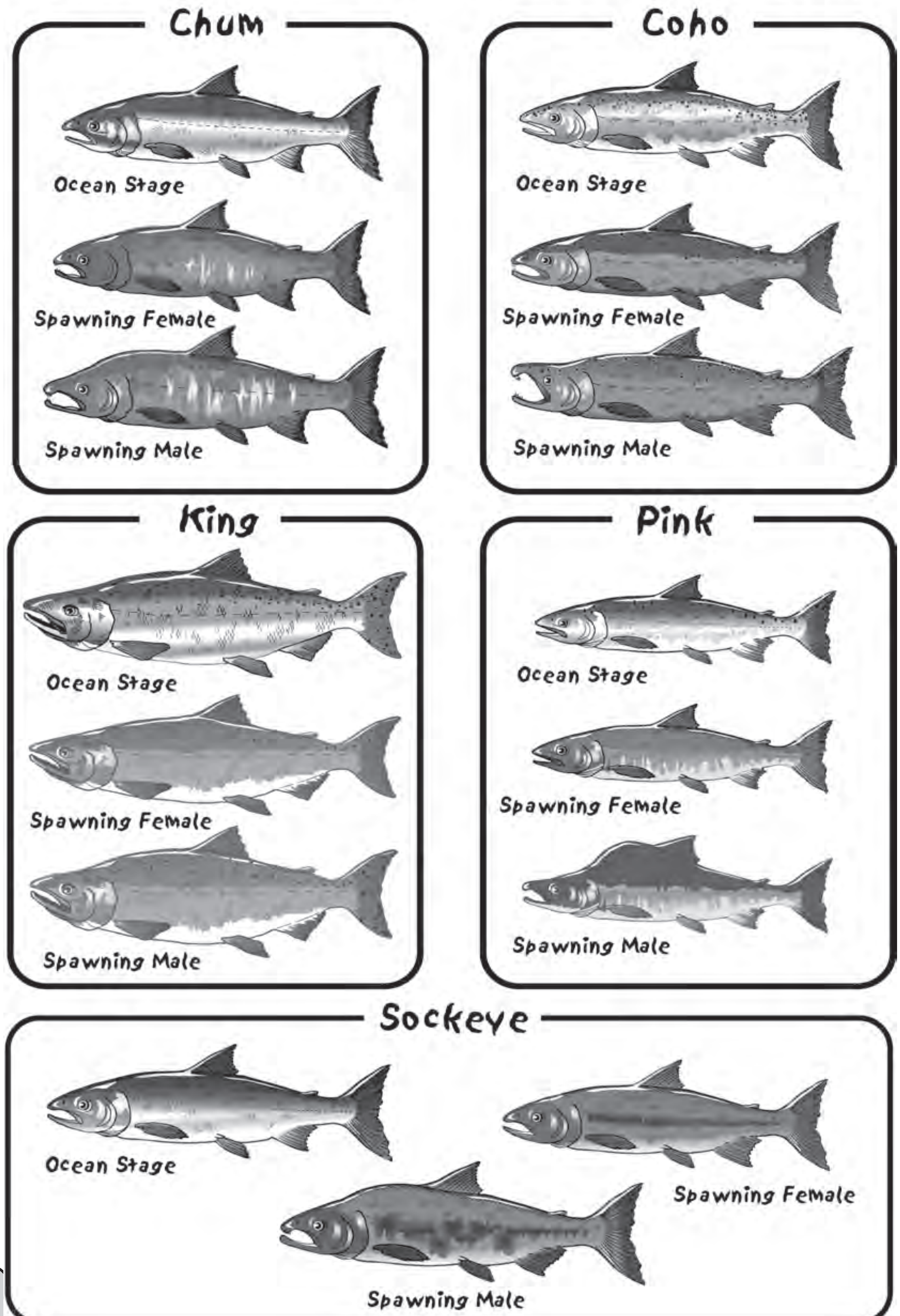




Salmon Life Cycle



The Five Salmon Species



George's Yearly Salmon Catch

Salmon Species	Number Caught	Approximate Weight of One Fish (lbs.)	Total Approximate Weight (lbs.)
Chum	55	7	385
King	220	15	3300
Red	22	5	110
Silver	15	9	135

Activity 2
Food Preservation Experiment

Why do people all over the world use drying as a technique to preserve food? This method has existed for centuries as it is easy, effective, and does not require modern technology, such as refrigeration. In this module, your class will follow the lead of Yup'ik elders as they catch and prepare salmon for the winter season. Hanging salmon on a fish rack to dry is one of the most common methods used to preserve salmon.

In this activity, students experiment with factors that enhance or hinder food preservation. They use slices of bread, both preserved and unpreserved, that are exposed to the different variables of light and dark and moisture and dryness. At the end of the experiment, students record their results both quantitatively, measuring how much mold has grown on each slice of bread, and qualitatively, noting color, smell, and other features of the mold process. They also use tables and graphs to note the results of their work.

The experiment is set up for students to follow directly. Alternatively, depending on your class, students may be encouraged to create or design as much of the experiment as they can. This can range from designing the data tables to determining the experimental design. In the latter case, the activity as written provides a model that can help you facilitate student-designed experiments.

Goals

- To establish an experimental design to address a specific question
- To conduct an experiment
- To determine factors essential for food preservation
- To collect and analyze qualitative data
- To collect and analyze quantitative data through graphing

Materials

- Transparency, Design Matrix
- Transparency, Centimeter Graph Paper, for optional activity (several for each group)
- Four pieces of bread for each group; two with preservatives, two without
- Petri dishes or other suitable containers (eight for each group)
- Labels for Petri dishes or other containers
- Eyedropper for each group
- Colored pencils or markers for optional activity
- Dried and/or preserved foods for examples
- Math notebooks

Preparation

Bring to class two loaves of bread, one with preservatives and the other without. Also, if possible, bring a few of the following items—dried apples, freeze-dried food, and a package of soup to which water must be added. If available, also bring some examples of preserved foods, such as Twinkies, pickles, bacon, etc.

Duration

One class period to set up experiment. Two weeks total duration: checking daily or every other day for 10 minutes at a time.

Vocabulary

Controlling a variable (in a scientific experiment)—focusing on a variable by defining its values and/or limits. For example, focusing on moisture by defining values of moist and dry.

Food preservation—to prepare food for future use, as by canning or salting, so as to prevent it from decaying or spoiling.

Mold—fungus that often forms on food as part of the decomposition process (spoilage).

Qualitative—of, relating to, or concerning quality.

Quantitative—expressed or expressible as a quantity; of, relating to, or susceptible to measurement; of or relating to number or quantity.

Scientific experiment—an experiment in which certain properties are controlled and results are recorded.

Variable—a letter or symbol used to represent one or more numbers in an expression or equation.

Instructions

1. Relating back to the idea of food preservation in Activity 1, ask your students to think of some conditions that might spoil food. If they need help, ask them if they have ever seen mold grow on bread. Explain that in this activity students will investigate factors that promote or hinder food spoilage: moisture, heat, and preservatives.

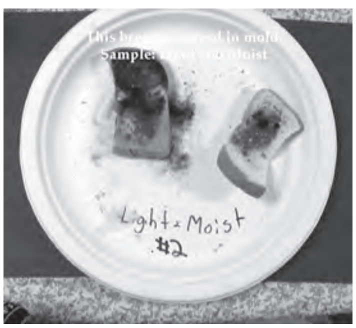
2. If available, show your students the dried apples, freeze-dried food, or package of soup to which water must be added. Ask them, what do these foods have in common. (Water has been removed.)

3. Discuss some basic food preservation methods with your students: drying, salting, canning. If available, show some of the preserved foods you brought to class. Ask why they think preservatives are added to bread and other foods.

4. Explain to your students that they will be conducting an experiment with their bread to determine some of the conditions that promote and inhibit mold growth. Ask them which conditions they think will be important in their experiment, i.e. preservatives vs. no preservatives, environmental conditions, and light.

5. Work with your class to determine an experimental design to observe mold growth on bread. If needed, show the Design Matrix transparency. For example, they will need to control the variables of light and dark and moisture and dryness for both types of bread (with and without preservatives).

6. Have the students work in groups of two to four. Give each group four slices of bread, two with preservatives, two without. Explain that they need to break each slice in half so that each group has eight pieces, four pieces of unpreserved bread and four pieces of preserved bread.



Light and moist

Light and dry

No light and moist

No light and dry

Fig. 2.1: Examples of mold growth on bread under various conditions of light, moisture, and preservatives. (Photos courtesy of Leah Lipka.)

7. Have groups place each piece into a Petri dish or other suitable container. Label each Petri dish according to the variables that will be tested: light, dry, dark, moist, preserved, or unpreserved. Also label with students' names or initials. There should be four Petri dishes with preserved bread: one subject to moisture and light, one subject to moisture and darkness,

Teacher Note

This is an example of a qualitative study—observing the changes in certain qualities (appearance, texture, mold growth) each day. A typical qualitative question might be, "Is there more mold today than yesterday?" An answer could be, "There's a little bit more today than yesterday." Optional instructions approaching the observation process quantitatively are given at the end of this activity. In other words, students use a method to actually measure a specific quantity of mold and assign some type of number to it. A typical quantitative question might be, "How much more mold is on the bread today than there was yesterday?" The answer to this question would be in the form of an actual number.

one kept dry and in the light, and one kept dry and in the dark. There should also be four Petri dishes with unpreserved bread: one subject to moisture and light, one subject to moisture and dark, one kept dry and in the light, and one kept dry and in the dark (Fig. 2.1).

8. Provide students with an eyedropper to use to moisten the bread that will be tested under moist conditions; five drops of water should be added to each piece. Place the containers in light and dark places, depending on the specifications under which they are to be tested.

9. Each day, have your students add five more drops of water to the moist bread and place the lid back on each dish.

10. Have your students check each piece of the bread daily or every other day and carefully observe any changes in their math notebooks. They should note both qualitative and quantitative characteristic (see Teacher Note). For example, on the first day mold is observed they should write about their observations and draw the mold's color, location, and size. Remind students to observe the bread from every possible vantage point so that they do not miss any mold growth. They should continue to observe, make notes, and illustrate during the next two weeks.

11. **Discuss.** After two weeks, your students should observe major differences between the various bread pieces—have them share their findings. To facilitate the discussion, ask some of the following questions:

    - Which type of bread had the most explosive growth in mold?
    - Why do you think the mold grew so rapidly?
    - Do you have evidence?
    - What is the difference between the preserved and unpreserved bread?
    - What is the difference between the bread pieces subject to light and those subject to darkness?
    - How does this experiment relate to drying salmon?
    - How does dryness relate to inhibiting mold growth on bread and on salmon?

Optional: Quantitative Approach to Observation

(a) Photocopy the Centimeter Graph Paper onto transparencies for the students. Hand out several transparencies and a pair of scissors to each group. Have the students cut apart the transparencies to fit over each Petri dish of bread. If possible, have students secure the transparency to the dish with tape or trace the bread on the transparency for future reference. After each day, have your students trace the amount of mold observed onto the grid and label it. This can be done using different colored markers each day, being careful to place the grid in the same spot from day to day. Students should estimate the amount of mold for each condition as a fraction of the number of squares colored to the total number of squares that the bread covers and record this data along with the qualitative data in their math notebooks.

(b) Every two days, have students represent the mold growth in their math notebooks in whatever method they find natural. Working together in their groups, have them illustrate the pattern and the amount of growth under each condition for both types of bread.

(c) After two weeks, have each group graph the mold growth over time as shown in the example in Figure 2.2. Encourage them to label the axes and use a different color for each piece of bread. Have them describe the relationships found.

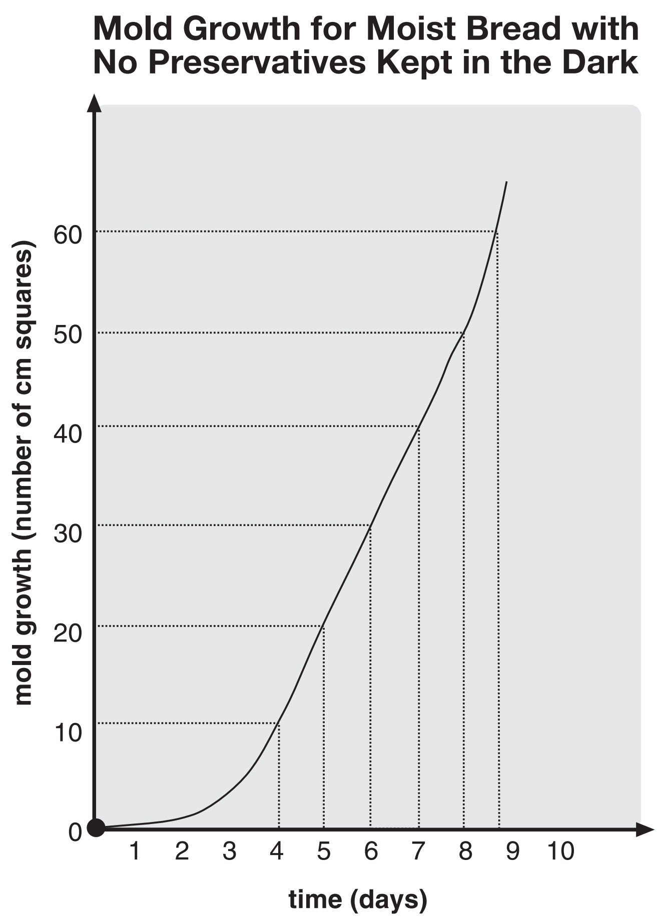
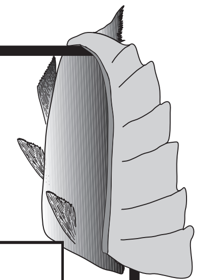
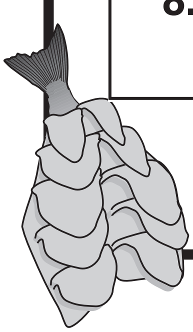


Fig. 2.2: Example of graph of mold growth for one condition.



Design Matrix

Trials	Light	Dark	Moist	Dry
1. Preservatives	X		X	
2. No Preservatives	X		X	
3. Preservatives		X		X
4. No Preservatives		X		X
5. Preservatives	X			X
6. No Preservatives	X			X
7. Preservatives		X	X	
8. No Preservatives		X	X	



Centimeter Graph Paper

Activity 3
Using Body Measures

Students will now begin to apprentice under Mary George in hanging salmon to dry by learning the body measures she relied on to create and use her fish rack. Students are introduced to nonstandard body measures and their distinct benefits and uses. This lesson is fundamental to the module, as it establishes experiences and a framework for much of the mathematics that follow. For example, as students use body measures to estimate the length of a fish rack pole (represented by a string), the lesson introduces students to the concept of representation. Students will represent the data derived here in the following activity by graphing. Further, comparing body measures to each other and to fish rack poles provides examples of proportional thinking, which is the topic of Activities 5, 6, and 7, and aid in students' transition to algebraic thinking.

To become familiar with body measures, students will begin by estimating a length of twelve feet and comparing it to string "fish rack poles" made using Yup'ik body measures provided by Mary George. Students will compare their own body measures to standard measures, using the fish rack pole as a comparison. Lastly, students will collect and organize all the data into a table in preparation for graphing and analyzing relationships in Activity 4.

Goals

- To estimate a 144-inch (12-foot) length
- To convert nonstandard measures to standard measures
- To construct a model of a fish rack pole using body measures
- To learn representation by using string to symbolize a fish rack pole
- To use tables for data collection and organization

Materials

- Poster, Body Measurements
- Different colored string—each student gets two 15' strings that are the same color (Alternative: string with masking tape labeled with students' initials)
- Blank transparencies (one per group)
- Scissors (one pair per group)
- Masking tape
- Ruler or yardstick (one per group)
- CD-ROM: *Yup'ik Glossary*
- Math notebooks

Preparation

Hang up the Body Measurements poster and, if possible, leave hanging for the entire duration of the module. Keep the Hanging Salmon to Dry poster from Activity 1 hanging.

Duration

Two to three class periods.

Teacher Note

An alternate approach for keeping the students' strings organized is to give all students one color of string for String #1 and a different color for String #2.

Fig. 3.1: String marked with tape labeled with student initials.

Vocabulary

Nonstandard measure—measurement found using units such as body measures or objects like paper clips.

Standard measure—measurement in standard units such as feet, inches, meters, etc.

Table—a compact, systematic list of related details, facts, figures, etc.

Taluyaneq—the measure from the middle of one's chest to the end of one's outstretched arm.

Yagneq—a measure between the fingertips of one's outstretched arms.

Instructions

1. Explain to students that in Activity 1 we discussed the importance of catching and processing salmon in southwest Alaska. Today we will simulate one aspect of this process in the classroom—estimating the length of a fish rack pole.

2. Have the students get into groups of three to four; distribute colored string—a different color for each member of the group, and tape. If colored string is not available, use plain string and tape marked with student's initials (see Figure 3.1). In addition, all students should label this string, "String #1."

3. Without rulers or other standard instruments of measure, have students estimate a length of 12 feet (4 meters) or 144 inches on the string, to represent a 12-foot (4-meter) fish rack pole. Allow them to use other objects readily available in the classroom (body measures, tiles on the floor, textbooks, etc.) Have students cut the string at the estimated length and hang it in a location where it can stay throughout this activity and the next (Fig. 3.2). Have them write in their math notebooks how they estimated the 144 inches (4 meters); for example, they may have used body measures (3 strides) or object measures (10 book lengths). Note: Using inches may be easier for graphing purposes.

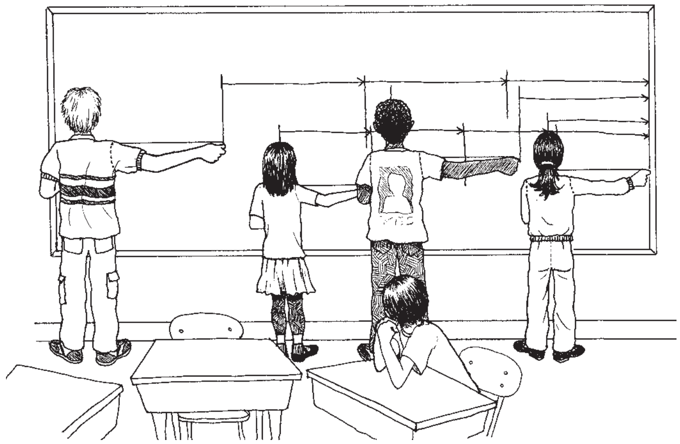


Fig. 3.2: Students hanging strings on the board.

4. To aid the students in estimating, hand out a ruler or yardstick to each group for them to check their 144-inch (4-meter) approximations. Have students measure their strings using the appropriate units, and then record these values in their math notebooks.

5. Encourage students to think about their data and to organize it in ways that make sense to them. For example, they might draw pictures to enhance their values and written explanations. Note that each group should have two values per person representing the length of the string (a value in nonstandard measures and a value in standard units). Encourage students to record their ideas and data in their math notebooks.

6. Distribute one blank transparency to each group. Once the groups feel comfortable with their organized data, have them compile their data on a single group transparency.

7. Have each group share its methods with the class by showing the transparency and demonstrating its methods.

8. After each group has shared, decide on one method to use to gather all the data together for the entire class. Introduce or suggest using a table, if no one shared this idea.

This may be a good place to end for the day. If so, begin the next lesson with a review by asking students to share what they discussed yesterday about organizing data.

Teacher Note

Since Mary George commonly converted her body measures into American Standard units, the module uses the relationship of her body measures to American Standard units. Metric equivalents are provided in parentheses and can easily be substituted. Choose what is best for your classroom.

9. Introduce your students to Yup'ik body measures by sharing the following story about Mary George, a Yup'ik teacher from Akiachak, Alaska.

   Mary was told to attach ribbons to balloons so that they could be hung from the school ceiling for a special event. The school principal told her to make the ribbons 5 feet (1.52 meters) long, and she knew that her *yagneq* body measure (between the fingertips of the outstretched arms) was 5 feet. (Point to the measure on the Body Measurements poster, see top of Figure 3.3.) She measured out the ribbon very quickly, using two *taluyaneq* body measures (the distance from the middle of the chest to the end of an outstretched hand) for each piece. (Point to the measure on the Body Measurements poster, see bottom of Figure 3.3.) When the principal saw her doing this, he was upset and told her that he had wanted her to measure the ribbon accurately. Much to the principal's surprise, when the ribbons were checked against a standard ruler, they were indeed accurate!

Teacher Note

Pronunciation of the names of the Yup'ik body measures can be found on the accompanying CD-ROM: *Yup'ik Glossary*, by clicking on Glossary and then Measurements. Clicking on any of the names of the measurements will show a picture and pronounce the Yup'ik term used.

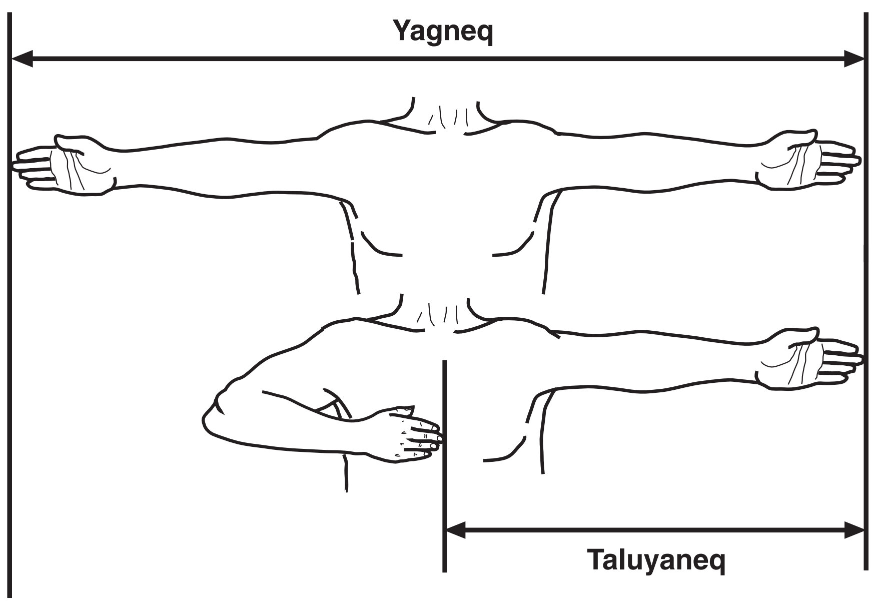


Fig. 3.3: Yagneq *and* taluyaneq *measures.*

10. Explain that when her husband, Frederick, builds a fish rack for Mary, he uses Mary's body measurements allowing her ease and efficiency when hanging the fish. Mary uses five *taluyaneq* measures to determine the length of her fish rack pole (see Figure 3.4). For Mary, this length is approximately 12 feet or 144 inches (4 meters). Although we realize that in strict mathematics the length would be 12.5 feet or 150 inches (3.81 meters), for simplicity's sake we chose to use an approximation of 12 feet.

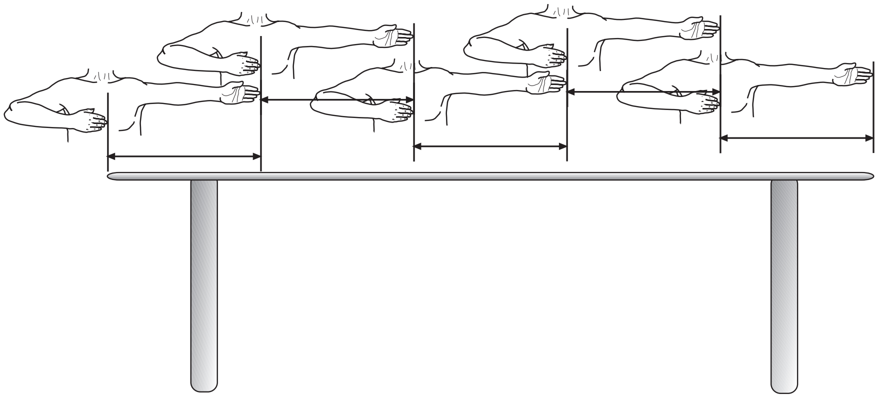


Fig. 3.4: Fish rack pole measured in five taluyaneq *measures.*

11. Pass out a second string to each student. If colored string was used, make sure that each student receives the same color as before. Have the students measure along the string five *taluyaneq* body measures to approximate Mary's length of a fish rack pole. They should mark with the tape each unit of *taluyaneq* and cut the string at the end of five *taluyaneq* measures. Students should use a piece of tape to label this string with their initials and "String #2."

Fig. 3.5: String #2 with tape marking the end of each of the five taluyaneq *measurements.*

12. Have students measure the second string using a ruler and record all data in their math notebooks. Encourage them to use a table to organize this new data along with the previous data.

13. Within their groups, encourage students to compare the lengths of their strings in multiple ways. Each student should compare the lengths of both of his or her strings to each other and write their observations in their math notebooks. Further, all students within a group should compare the lengths of their first strings to each other and write their

Teacher Note

Caution your students as they discuss this method of measuring that they are not judging the accuracy of body measures. Once you know the approximate length of a particular body measure on you, then it becomes a constant measure and you can use it as accurately as a ruler, unless, of course, you are still growing. Here students are using the *taluyaneq* to measure 144 inches (4 meters), but their actual lengths may well differ from Mary's and, thus, not produce the exact length.

Teacher Note

In mass production for the general population, standardized sizes help to make things easier to manufacture, speeding the process and making it easier to replace certain items. For example, shoes and shirts are made to specific sizes. Light bulbs are made to fit standard sockets. However, there are reasons that some objects need to be tailor-made. It is important for a kayak to fit the kayaker as perfectly as possible so that he/she can comfortably paddle it. Therefore, applying the kayaker's own body measures when designing the craft results in a unique and very personal craft. Ask your students for examples of other items for which tailor-made fits would be important.

observations in their math notebooks. Allow enough time for comparison among all students in a group with their second strings as well.

14. **Discuss.** As students discuss their findings with the whole class, prompt them to discuss how body measures differ according to each person. For example, why did Mary's measure work accurately for some students and not others? Why did it work for Mary?

15. Conclude with students comparing and contrasting the use of standard measures and body measures. Discuss the importance of using standard measures for certain applications.

16. Share with students that what they have started in this activity will continue throughout the module, so they will continue to come back to their strings, body measures, and comparisons.

Activity 4
Graphing Information

Mary learned a variety of body measures and found her own way of remembering and applying them, so she didn't have a need to represent them on paper. However, the need to write them down and draw pictures became necessary through the course of putting this module together. As a teacher, Mary wanted to convey her system of proportional body measures easily to her students. She found drawing all the relationships, representing them in an organized fashion, and keeping all the material proportional and accurate to be of the utmost importance.

In this activity, students will graph (draw a picture of) the data that was collected and recorded in the previous activity. They will move from creating and interpreting single-variable graphs to creating and interpreting two-variable graphs such as scatter plots. They will also compare the difference between the two types of graphs and learn when and why to use each.

Goals

- To convert data from table format into graph format
- To create and interpret a bar graph
- To compare scaled versions of a length in different units
- To read *(x,y)* coordinates and plot points on a two-variable graph
- To recognize simple relationships on a scatter plot

Materials

- Transparency, Bar Graph Example
- Transparency, Graphing on the Coordinate System
- Worksheet, Graphing Practice (three per student)
- Organized data from Activity 3 for each group
- String #2 for each student from Activity 3
- Transparency pens
- Markers or colored pens—several for each group
- Spare transparencies (for Step 4)
- Math notebooks

Preparation

Prepare two transparencies for this activity as listed above. Copy Graphing Practice worksheet, three for each student.

Duration

Two to three class periods.

Vocabulary

Bar graph—a graph that uses separate bars (rectangles) of different lengths to display and compare data.

Constant—a value that does not change.

Coordinate—an ordered pair representing the independent and dependent variables (x,y).

Coordinate system—the x- and y-axes attributed to Rene Descartes (the Cartesian coordinate system).

Graph—a display of information through pictures or symbols.

Horizontal axis (or x-axis)—parallel to the plane of the horizon; not vertical.

Line graph—a graph in which data points are represented by dots and connected by line segments.

Ordered pair—a pair of numbers used to locate a point on a graph with the first number showing the value on the horizontal axis and the second number showing the value on the vertical axis.

Origin—the specific ordered pair on the coordinate system representing (0,0).

Plot—to graph.

Scale—ratio between the dimensions of a representation and those of an object.

Scatter plot—a graph made by plotting points on a coordinate plane to show the relationship between two variables in a data set. The points are not connected by line segments.

Units—the smallest whole numbers of any quantity, amount, distance, or measure.

Variable—a letter or symbol used to represent one or more numbers in an expression or equation.

Vertical axis (or y-axis)—that which is perpendicular to the horizontal axis.

Instructions

1. Review yesterday's work with the whole class by discussing the data collected in the table on the worksheet. Have students share what was measured and how. Have them discuss whether they found any patterns or relationships in the data when forming the table.

2. Explain to the class that today we want to draw a graph (a picture) of the data collected in the previous activity and organized in the table. In particular, we will focus on each group member's *taluyaneq* measure,

the measure Mary used to determine the length of her fish rack poles. Have each student get out String #2 from Activity 3.

3. In their math notebooks, have students draw several bar graphs (pictures) to represent all the measurement data for their group's second strings. Have one bar graph (picture) showing the group members' results in *taluyaneq* units, another showing the results using standard units of inches or meters. Make sure they label each bar on the graph with the appropriate group member's name. They should use their notes from the previous activity to help with this exercise.

4. **Discuss**. Have each group present its procedures and outcome to the rest of the class by drawing its graph on a blank transparency.

5. If students did not create a bar graph, then show the transparency, Bar Graph Example. Explain that this is one example of a single-variable (length) graph, also called a bar graph. Connect this type of graph to the students' representations. Use this time to introduce related vocabulary words and to demonstrate how to read a bar graph. Explain that we use the term single-variable for two reasons: (1) we are only concerned with one measurement, the length of the string, and (2) measurements differ for different people, and so it is called a variable. Refer to Teacher Note on page 56 for more detailed explanation.

6. If needed, allow students time to redo their graphs in bar graph form, if they had not done so previously, or to modify their original bar graphs.

7. **Discuss**. Encourage groups to present their graphs and discuss similarities and differences by comparing and contrasting their data in different units of *taluyaneq* and inches. Have them practice reading and asking questions about their graphs. For example, encourage them to discuss why there are different results for different students. Use this discussion to review the difference between a standard and nonstandard measure as discussed in the previous activity.

This may be a good place to end for the day. If so, begin the next lesson with a review by asking students to remind the class of what they discussed yesterday about graphs.

8. Explain to students that Mary not only needs to know how long she wants her fish rack poles, but she also needs to decide how high to place them off the ground. To determine the height at which to hang the pole, she chooses a point midway between her wrist and elbow when the arm is stretched overhead (see Figure 4.3). In this way the drying rack is tailor-made to Mary's size, and reflects a height that is comfortable for her reach. Demonstrate this with a few student volunteers.

Teacher Note

If students need some help creating graphs, here are some questions to ask and some appropriate responses: (Q1) What are the important aspects you want your picture to show? (A1) I want my picture to show how long each string was for each person. I want it to show that mine was a little longer than Jill's, and that Dolly's was the longest. (Q2) Which words would be used for axis labels? (A2) The units measured will be on one axis. Also, next to each line (drawing of the string), we'll write the name of the student. (Q3) How will you fit a 12-foot string on a paper that is only 8.5" by 11"? (A3) We have to draw it smaller; maybe make a centimeter on the paper represent a foot of string.

Math Note

Students should notice that drawing each string in *taluyaneq* units and in standard units of inches produced different-sized lines on paper, but that the string is the same length in real life. They should also notice that none of the lines drawn on paper are the same length as the actual string. Encourage discussion on the concepts of units, variable vs. constant, and scaling. Refer to the math note on the following page for further explanation of these terms.

Math Note

There are three points to bring out during this activity: units, variable vs. constant, and scaling. The two graphs depicted here should aid in understanding the importance of units. In the previous activity, the students measured their second string in five of the *taluyaneq* units. When graphing this measure, each student will have the same result, since they all used the same number of units, five; however, when graphing that same length of string using standard units such as inches, students will most likely have different results. A taller person with a wider arm span will have a length of string that is longer in inches than a shorter person's string.

The graphs also show the difference between a variable and a constant. In the previous activity, when students measured their second string in five *taluyaneq* units, the number of *taluyaneq* was a constant since all students used five of them as seen in Figure 4.1. But when the *taluyaneq* is calculated in, or converted into, standard units, we see the total length varies from person to person. In mathematical terms, we call this a variable. The variable idea is made concrete in Figure 4.2.

As students draw or graph this information, they are also considering scaling as they reduce the measure to fit the graph paper. Graphing in a variety of units continues the practice of scaling. When students reconcile the different lengths, using different units on paper, they discover the scaling factor, or the number, that allows them to convert from one representation to another. Note that all your students may not be at a developmental level to understand scaling yet. This activity provides an opportunity for you to begin introducing the concept.

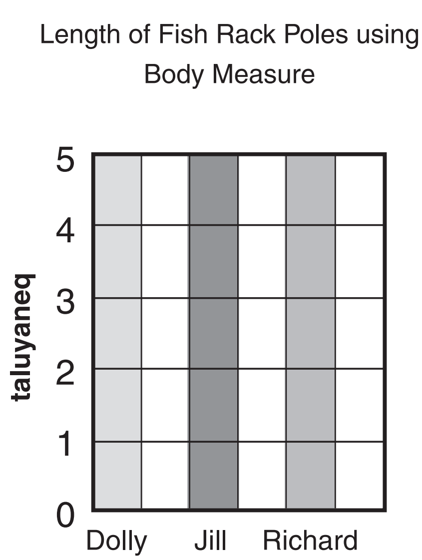


Fig. 4.1: Example of bar graph showing each student measuring a fish rack pole using five taluyaneq *measures.*

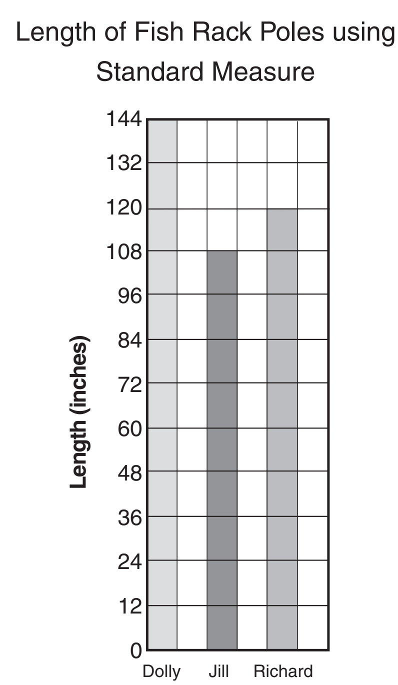


Fig. 4.2: Example of a bar graph showing each student measuring a fish rack pole using standard units.

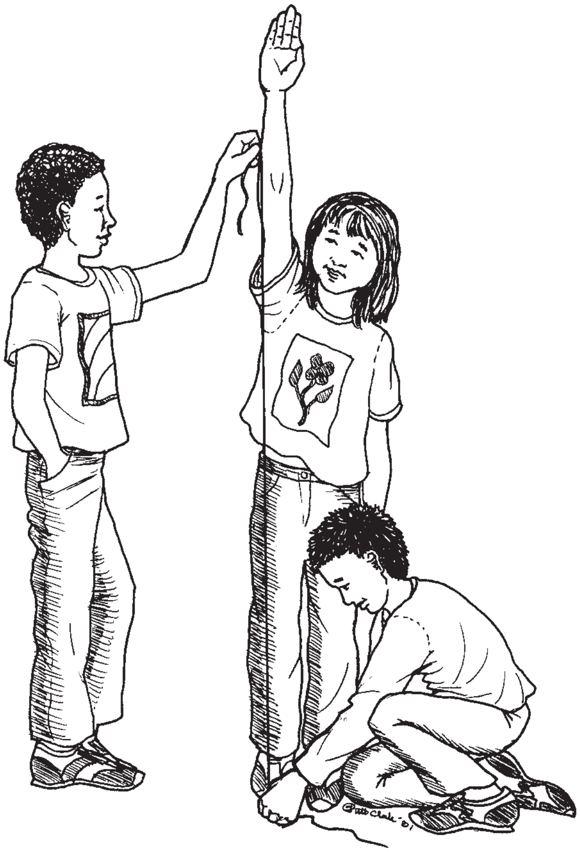


Fig. 4.3: Students demonstrating Mary's method of how to determine how high to hang a fish rack pole.

9. Have students work in their groups to determine how high each students' fish rack pole needs to be using a ruler and string. Have students record the data in inches or feet in their math notebooks.

10. Have each group create a new bar graph on a transparency to depict the height at which each student would hang his or her fish rack poles. Make sure the graph depicts the height data for each member of the group. Encourage students to label the axes and provide a title for their bar graph. Note that you can use this challenge as a short assessment of what students learned previously about creating bar graphs.

11. Have students share their results on the overhead with the class. Use this as a time to begin collecting ordered pair data in the following manner. As students share, ask them to tell you both the height and length data for each group member by referring to both the second and first bar graphs. So, for example, as Jill's height data is given, ask for the length of her string as well. On the board, write the given information in ordered pairs with the height given first, the length second, like so: (H, L). Richard's data might look like this: (60 inches, 120 inches).

Teacher Note

Bar graphs can be oriented either vertically or horizontally. The bar graphs shown in this activity are vertical; however, it is just as correct to draw them horizontally. In fact, the horizontal orientation may be more natural as students hung their strings on the wall in Activity 3 in that way. We chose to draw them vertically so that the variable (length) was on the *y*-axis, setting up the transition into a two-variable graph. When moving into the two-variable graph, one must choose the independent and dependent variables falling on the *x*- and *y*-axes respectively. The height of a person would be the independent variable and thus graphed on the *x*-axis. The length of a person's arms is typically dependent on her height, and so the length is graphed on the *y*-axis. You may want to encourage students to draw their bar graphs in both directions or have various groups share examples showing both orientations.

12. **Demonstrate.** Explain to the class that sometimes it's easier to understand the two measures together by using a two-variable graph. Use the transparency, Graphing on the Coordinate System, to demonstrate a two-variable graph. First, provide the essential vocabulary, such as coordinate system, *x*-axis, *y*-axis, and origin. Explain that in order to plot two-variable data, we organize our measurements into ordered pairs, *(x,y)*. Here we will use the ordered pairs (height, length) written on the board. To plot an ordered pair we begin at the origin (0,0) and move in the correct direction. Explain that each axis is just a number line with the direction to the right for height and the direction up for length. Demonstrate by graphing the point (60,120) on the overhead and labeling it (see Figure 4.4). Now, plot some of the ordered pairs that you jotted down on the board in the last step. Label each pair with the appropriate person's name and values.

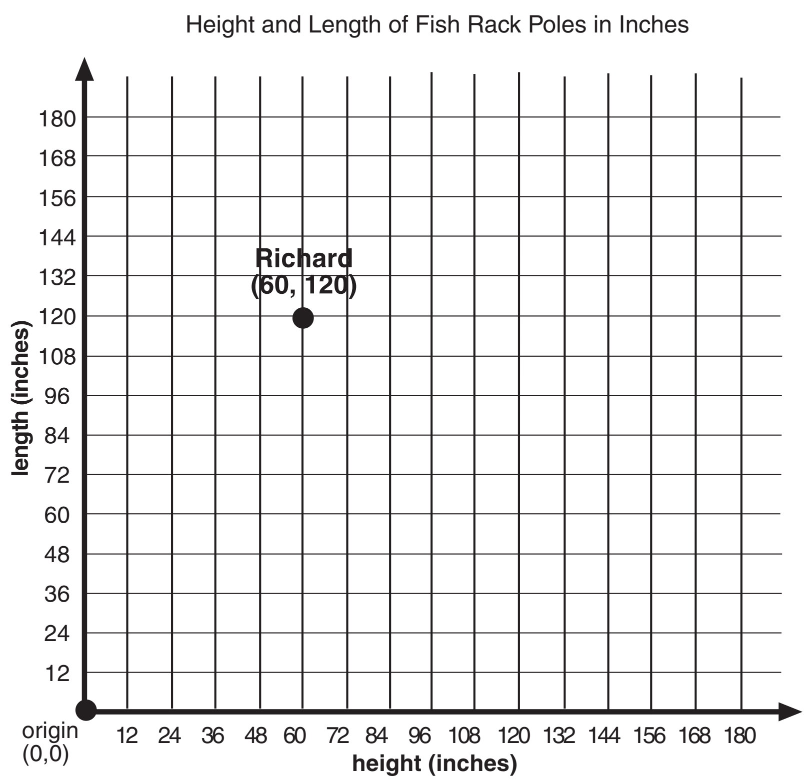


Fig. 4.4: Example of a two-variable graph.

13. Pass out three copies of the Graphing Practice worksheet to each student. Have the students label the axes with a measurement name (height on the horizontal axis and length on the vertical axis). Using their height bar graphs and their length bar graphs, have them write in their math notebooks the information for each group member as an ordered pair. Be sure to check each group's work for consistency in their ordered

pairs. Then have them plot those values on the graph, labeling each ordered pair with the appropriate group member's name.

14. Clean off the transparency, Graphing on the Coordinate System, and have students share their results using the overhead. Encourage students to label the axes and then plot on the graph their group's ordered pairs. Perhaps have different groups use different colors to keep the data organized.

15. Have students discuss the type of relationships seen in the class data. (For example, taller people generally have a longer arm span than shorter people, so the taller the person, the longer the string.) Discuss how the bar graph and the two-variable graph each have a distinct purpose. Bar graphs allow for comparisons of a single measure across a number of people or instances. The coordinate system is used to show relationships between two variables sometimes shown as scatter plots.

It is essential that students practice plotting on a coordinate system. If there is enough time to continue, then follow with Step 16, otherwise you may want to spend the next class period graphing in different ways and discussing methods of constructing, reading, and interpreting the graphs.

16. **Practice**. Continue to plot the data in various ways. For example, plot all the boys' data in one color and the girls' in another color. You can have students use their length measurements for the first string in Activity 3 with their heights as well. Spending time on this will allow students ample graphing practice, aid their interpretation of graphs, and help them recognize different types of relationships.

Teacher Note

It is important for students to practice reading graphs and looking for relationships. Have students make conjectures or guesses about their graphs and provide proof for their conjectures. Ask students to look at a completed graph and explain what it is showing in words. Have them practice interpreting the picture. For example, suppose Dennis is 71 inches tall and his pole is 157 inches long, but Luke is 70 inches tall with a pole 165 inches long. Have students explain what is going on between Dennis and Luke. Although Dennis is taller, Luke has longer arms than Dennis, so his pole is longer. This doesn't fit the general pattern, but still describes an important relationship.

Math Note

Most likely, your first plot (from Step 14) showed a positive relationship; as the height increased, the length increased too. You may find other relationships. See Figure 4.5 for an example. Depending on the number of students, the pattern may be more or less obvious.

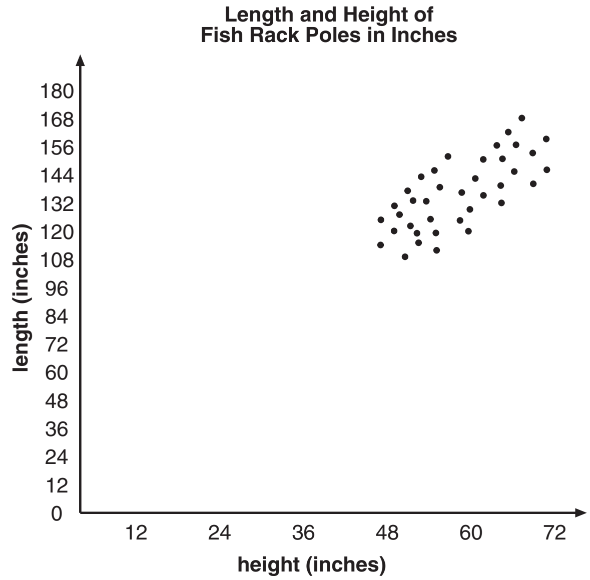


Fig. 4.5: Example of a positive relationship between height and length.

A constant relationship would occur when plotting the height in inches and the length of the string in *taluyaneq* units. See Figure 4.6 as an example of a constant relationship. Even though the height (*x*-axis) changes, the length (*y*-axis) does not.

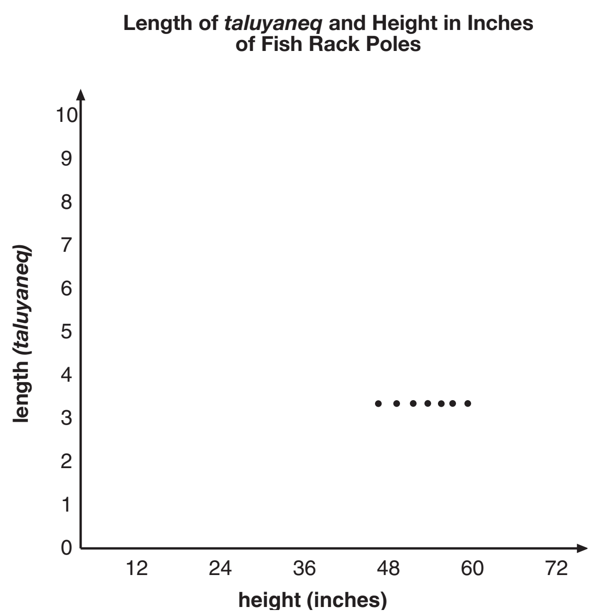


Fig. 4.6: Example of a constant relationship between height and length.

Most likely, if a student plotted the group's data for the first string used in Activity 3, when students were asked to estimate a 144-inch (4-meter) length in any manner they chose, then no relationship would result (Fig. 4.7). Students will also encounter exponential, negative, and leveled-off relationships when plotting results from the two science experiments in the module.

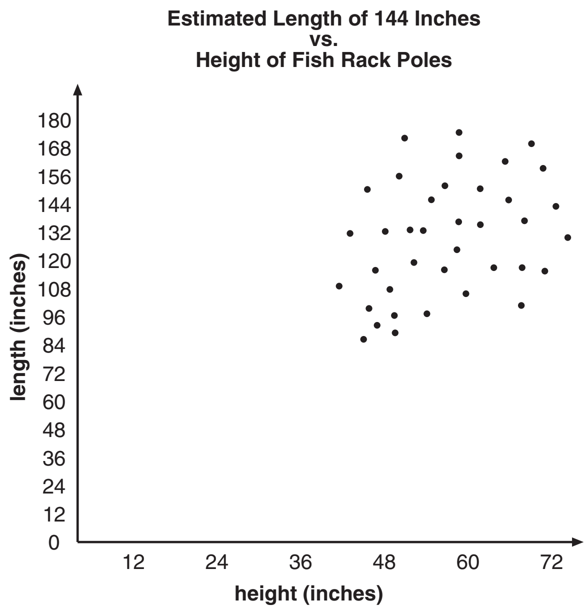
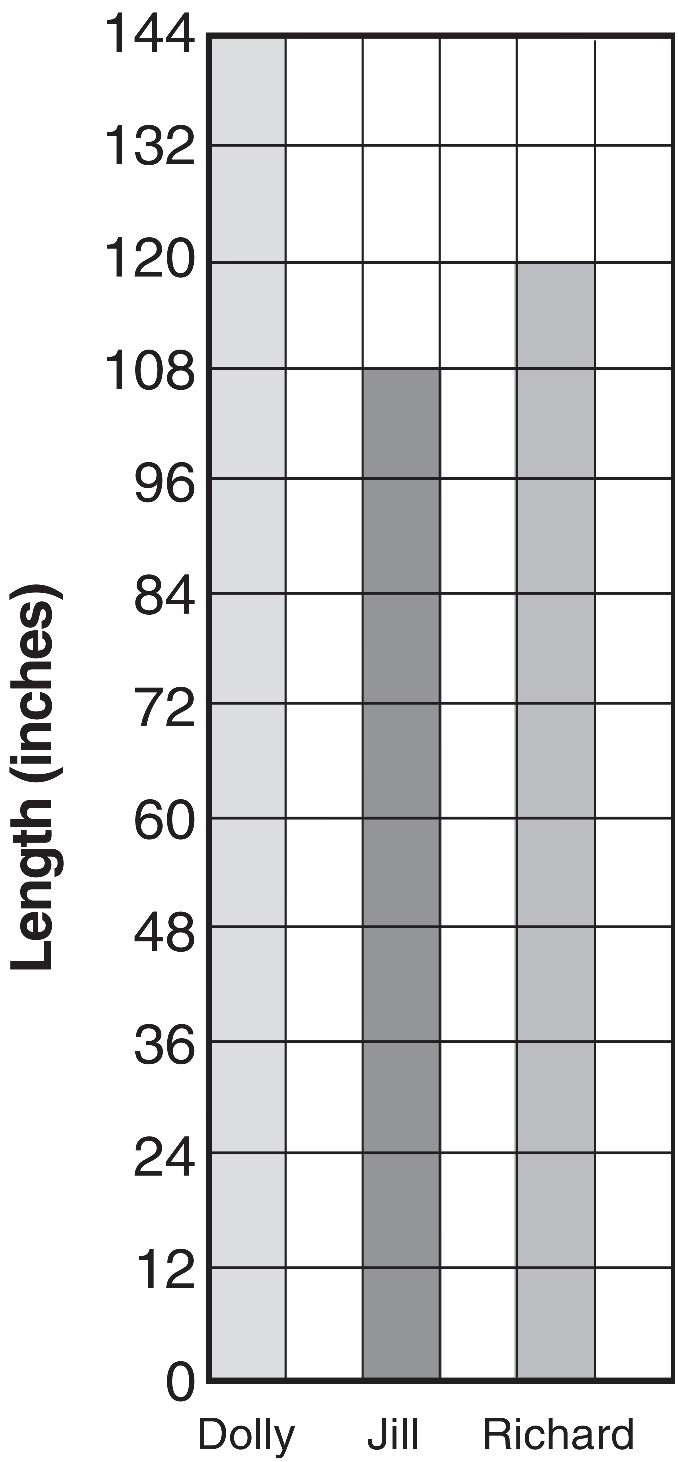


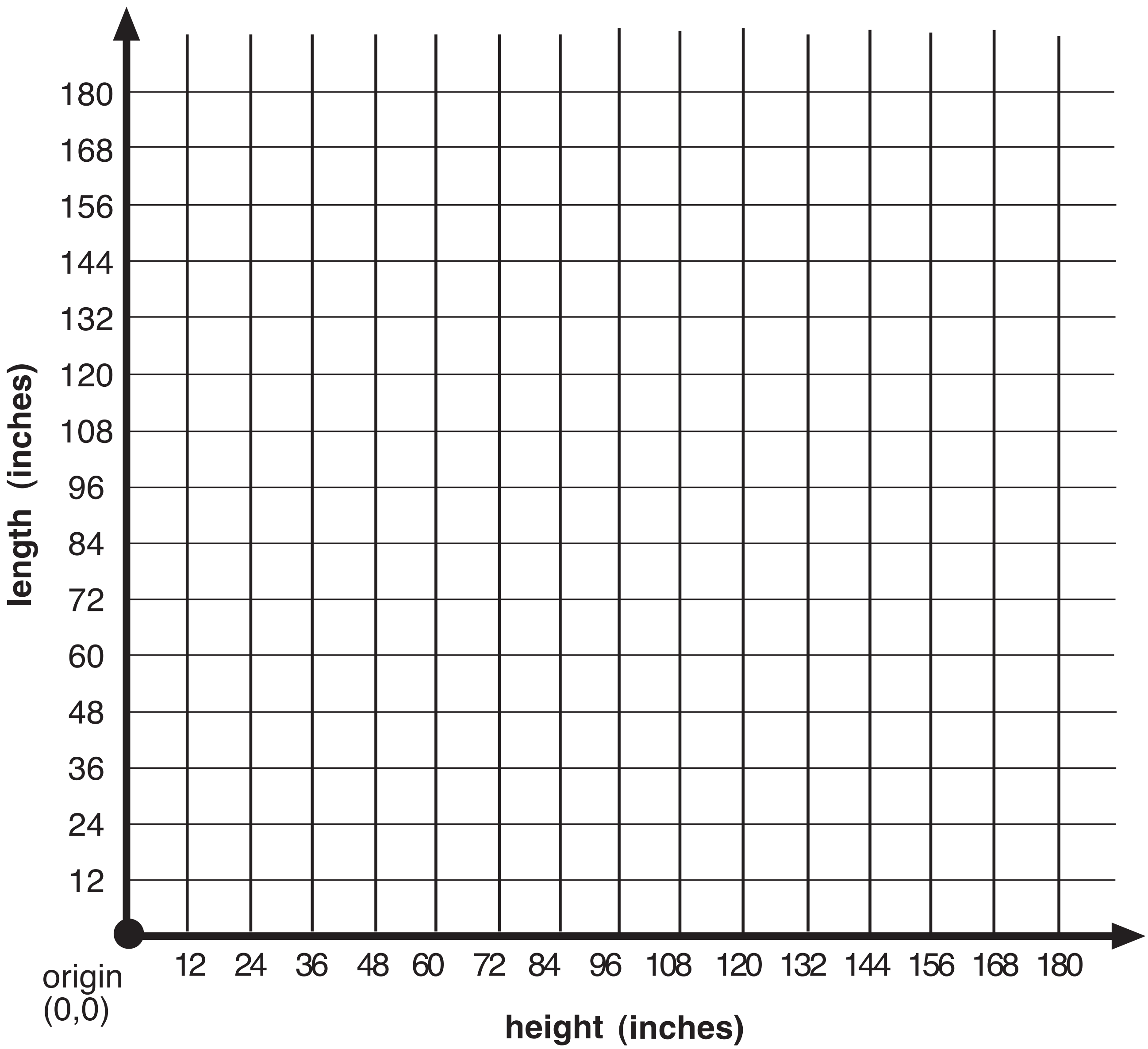
Fig. 4.7: Example of no relationship between height and length.

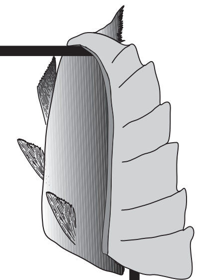
Bar Graph Example

Length of Fish Rack Poles Using Standard Measure

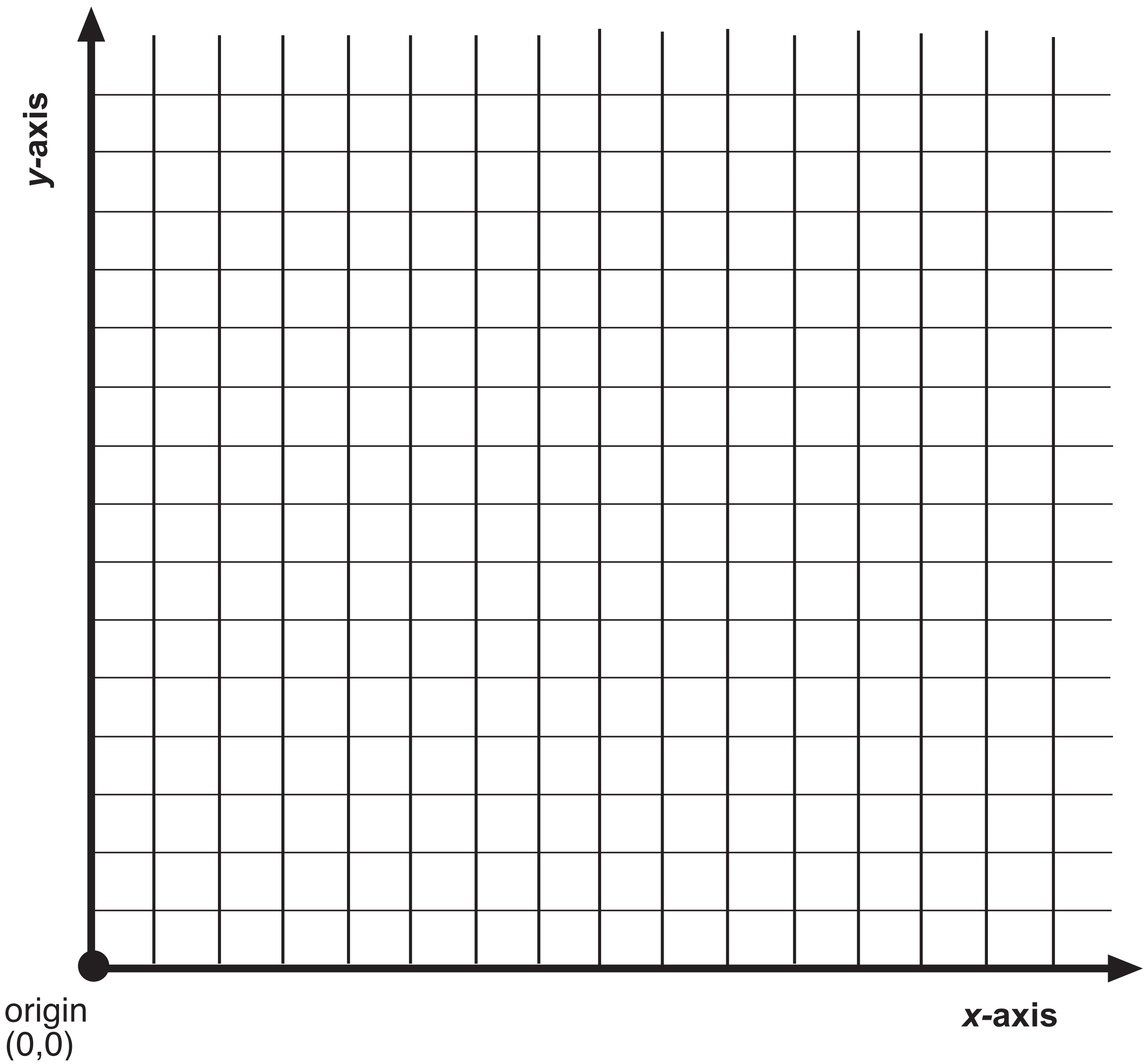


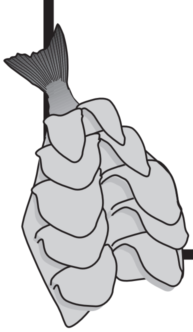
Graphing on the Coordinate System





Graphing Practice





Assessment Activity A

This is the first of three assessment activities. It is designed to focus on the thinking developed in Activities 1–4 as well as the process of problem-solving. Students have seen these types of problems in previous activities, but they are now posed with a problem-solving approach. This is the first time they'll try to solve problems on their own in a group, but more opportunities like this will follow.

Have the students work in groups of three to four and use the problem-solving approach (outlined in the Introduction to the Module) to solve the following challenges. Remind them of how the group problem-solving strategy works—one student takes the leadership role to facilitate discussion on solving the problem, asking such questions as: What is the unknown? What type of problem is it? What information is missing? What do we need to find out? Another student facilitates the planning, asking such questions as: Should we draw pictures to help with the problem? Should we use symbols? Should we make up a similar but easier problem to solve to help us with this problem? After students have worked individually on their answers, another student takes on the role of the evaluator and compares everyone's answers, leading the students to rework the problem if they think the answer(s) are not correct. Students should switch roles for each question. It may be necessary to remind them to switch.

For your convenience, the solutions follow the problems. On page 67, a blackline master with only the problems is included for you to copy and use as a worksheet for your students. As part of your assessment you may want to consider the following: Can students establish a problem-solving norm within their groups and amongst the classroom? Do they generate multiple solutions to the problem? Are they accepting of divergent approaches? Do they justify whether or not results are equivalent?

After groups have had the chance to work long enough in their problem-solving roles, lead a discussion about the important ideas of constants and variables, the cognitive processes of working together, and graphing.

Problem-Solving Challenge 1

Jerry and Janet were building a smokehouse using the *malruk naparnerek* measure (Fig. A.1). After laying out one side of the smokehouse, Jerry found the length to be 10 *malruk naparnerek* long. When Janet measured the same side, she found it be to 12 *malruk naparnerek* long. When they used a tape measure instead, the side of the smokehouse was 10 feet long. Complete the following steps to help explain why the numbers are different.

Fig. A.1: Malruk naparnerek *measure.*

a. Draw a bar graph to show the length of the side of the smokehouse in *malruk naparnerek* measures for both Jerry and Janet. Label the graph with a title and units (Fig. A.2).

Solution:

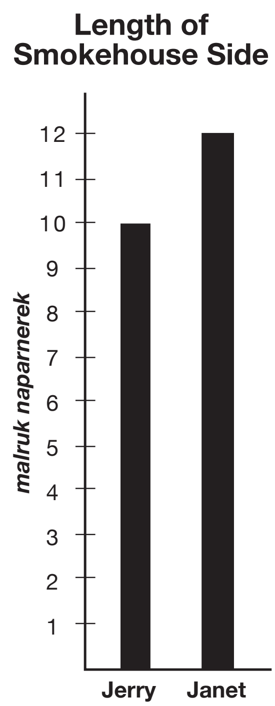


Fig. A.2: Bar graph example in nonstandard measures.

b. Draw a bar graph showing the length of the side of the smokehouse in inches for both Jerry and Janet. Label the graph with a title and units (Fig. A.3).

Solution:

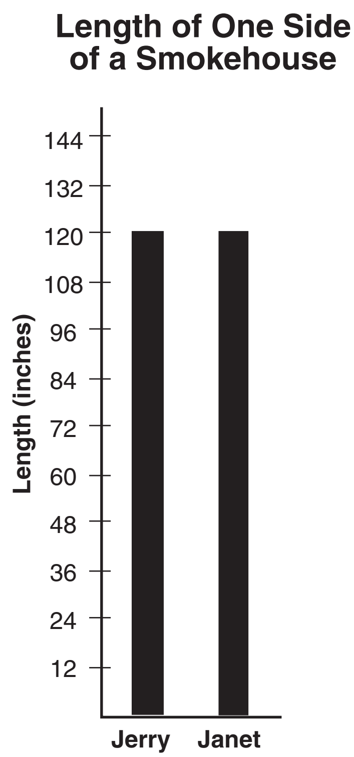


Fig. A.3: Bar graph example in inches.

c. Compare the two graphs and explain why the numbers are different.

Solution:

Students should talk about how the *malruk naparnerek* measure is specific to each person, so Janet's is smaller than Jerry's and it takes more of them to make the same length. If the total length was really needed to buy building materials, then using the standard length would help. Students should also talk about the difference between a variable and a constant.

d. How long is Jerry's *malruk naparnerek* in inches? Janet's?

Solution:

It's possible that students may realize that Jerry's *malruk naparnerek* measures 12 inches but Janet's *malruk naparnerek* is 10 only inches long. Janet's hands are smaller so she needs more of the *malruk naparnerek* to fit into 10 feet. To calculate these values, divide 120 inches by 10 for Jerry and 120 inches by 12 for Janet.

Problem-Solving Challenge 2

a. When Jerry holds up his *malruk naparnerek* measure, his hands are at a height of about 60 inches. Janet holds her hands at 48 inches above the ground. Draw a graph showing the *malruk naparnerek* measure found in Challenge 1 above and the height for both Jerry and Janet, using the coordinate plane (two-variable graph). Label the graph with a title and units (Fig. A.4).

Solution:

Note that students had enough information to calculate the length of the *malruk naparnerek* in inches for both Janet and Jerry, so a graph in inches vs. inches is appropriate.

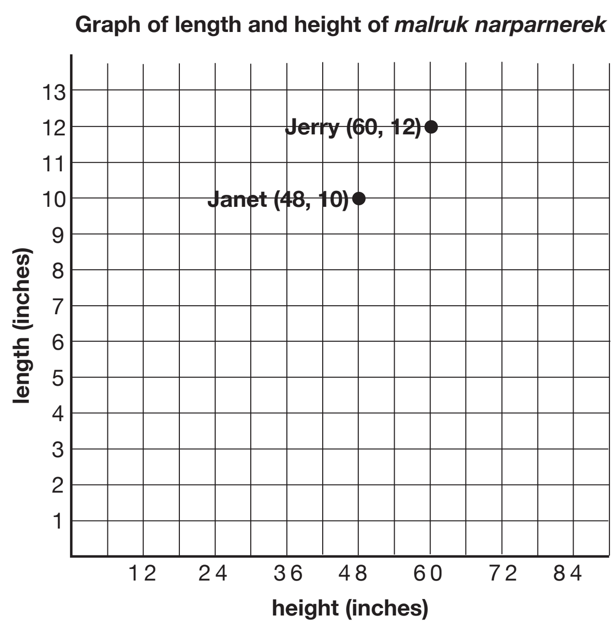


Fig. A.4: Graph of length and height of malruk narparnerek.

b. Imagine that you were showing someone the *malruk naparnerek* measure. At what height (in inches) would you hold your hands? How many inches is your *malruk naparnerek* measure? Plot the length and height of your measure on the same graph as Jerry's and Janet's.

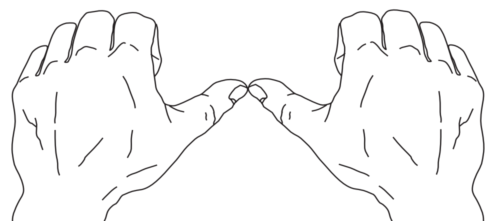
Solution:

Students' own measurements will vary. This should just be the comfortable height at which to hold their hands and measure against a wall.

Assessment Activity A

Problem-Solving Challenge 1

Jerry and Janet were building a smokehouse using the *malruk naparnerek* measure as shown below. After laying out one side of the smokehouse, Jerry found the length to be 10 *malruk naparnerek*. When Janet measured the same side, she found it be to 12 *malruk naparnerek* long. When they used a tape measure instead, the side of the smokehouse was 10 feet long. Complete the following steps to help explain why the numbers are different.



Malruk Naparnerek

a. Draw a bar graph to show the length of the side of the smokehouse in *malruk naparnerek* measures for both Jerry and Janet. Label the graph with a title and units.

b. Draw a bar graph showing the length of the side of the smokehouse in inches for both Jerry and Janet. Label the graph with a title and units.

c. Compare the two graphs and explain why the numbers are different.

d. How long is Jerry's *malruk naparnerek* in inches? Janet's?

Problem-Solving Challenge 2

a. When Jerry holds up his *malruk naparnerek* measure, his hands are at a height of about 60 inches. Janet holds her hands at 48 inches above the ground. Draw a graph showing the *malruk naparnerek* measure found in Challenge 1 above and the height for both Jerry and Janet, using the coordinate plane (two-variable graph). Label the graph with a title and units.

b. Imagine that you were showing someone the *malruk naparnerek* measure. At what height (in inches) would you hold your hands? How many inches is your *malruk naparnerek* measure? Plot the length and height of your measure on the same graph as Jerry's and Janet's.

Section 2: Transitioning into Variables and Proportions

In this section, students will continue working with body measures. They will consider how the measures relate to each other and use equations, expressions, and proportions to describe these relationships. They will use a variety of representations throughout the activities including manipulatives, mathematical sentences, word sentences, pictures, variables, and numbers. Lastly, students are introduced to proportional notation.

Annie Blue modeling the taluyaneq *measure.*

Activity 5
Finding Equivalent Body Measures

Now that students have investigated a couple of Yup'ik body measures, experienced the measures needed for designing their fish rack poles, and explored relationships among those measures, they are ready to learn more about Mary's system. Mary used different body measures to measure objects by finding the one that worked perfectly. Depending on the situation, she used several body measures to measure the same object. For example, she used the shoulder-to-shoulder *tusneq* to measure a king salmon after it had been cut for drying to approximate the space it would take on the pole. She also used her arm length, *tallineq*, for the same width of the fish because when she draped the cut fish over her arm to carry it to the pole it took up that amount of space (Fig. 5.1). She understood that the *tusneq* was equivalent to the *tallineq*.

Fig. 5.1: King salmon fake blanket draped over Mary's tallineq.

In this activity, students will discover equivalent body measures on their own and then be introduced to the whole system of body measures used by Mary for hanging salmon. Finally, students will begin to create abstract representations of these measures by writing sentences and drawing pictures. In Activity 6, they will abstract the measures further by moving from sentences to algebraic equations and expressions and then into proportions in Activity 7. These abstractions will help move the students into algebraic thinking, readying them for the subsequent activities.

Goals

- To discover body measures used in the Yup'ik culture
- To discover applications of body measures
- To investigate proportional relationships among body measures
- To discover equivalent body measures

Materials

- Poster, Hanging Salmon to Dry
- Poster, Body Measurements
- String
- Scissors
- Butcher paper or blank transparency (optional)
- Rulers (one per group of four)
- CD-ROM, *Yup'ik Glossary*
- Math notebooks

Math Note

Examples: The *yagneq*-to-*taluyaneq* relationship is a two-to-one relationship since the *yagneq* is twice as big as the *taluyaneq*. The *taluyaneq* is half the size of the *yagneq*. You need two of the *taluyaneq* to make the same length as one of the *yagneq*.

Duration

One to two class periods.

Vocabulary

Equivalent—having the same value, measure, or meaning.
Relationship—the quality or state of being related; a connection.

Instructions

1. Ask the class to explain what they have learned so far about relationships (between two types of body measures or between two types of graphs). If needed, remind the students of Activity 3, during which they used the Yup'ik body measure *taluyaneq* to estimate a 144-inch-long piece of string (representing a fish rack pole). They hung the string on the wall, and then converted the data collected in the exercise into a table. Remind them that in Activity 4 they learned to convert their string data into several types of graphs. Write their answers on the board, on butcher paper, or on a blank transparency.

2. Show students your *yagneq* and *taluyaneq* measures again. Ask them to describe the relationship between these two measures. Allow them to express the relationship in a variety of ways. Write their comments on the board, on butcher paper, or on the transparency. (If needed, you may want to discuss the vocabulary of relationships or being connected. Using the example of the relationships in a family or being connected to your family may be helpful.)

3. If you did not discuss the various uses of Mary George's body measures in Activity 3, this is a good time to do so. Point out the Body Measurements poster and explain the various measures on it. Have the

Figure 5.2: Sam Ivan's blackfish trap.

Cultural Note

When using body measures, we don't need to refer to a standard measure or assign numerical values to our measurements. This is important from a cultural standpoint. In Yup'ik culture a "number" is always a property of something else, i.e. four fish, four people. In conventional mathematics, we view a number as an independent entity with properties of its own.

In the Yup'ik culture, body measures are usually used for specific tasks. For example, the name of the *taluyaneq* measure stems from the word used for blackfish trap *(taluyaq)*. This is because a blackfish trap usually is the length of a person's *taluyaneq*, allowing him to be able to reach the fish caught in the trap. In Figure 5.1, Sam Ivan shows how the width of a blackfish trap is the length of his *malruk naparnerek*. On the left, Nick Gumlickpuk from New Stuyahok, Alaska, watches.

students practice the measures and guess at their applications as you explain them. After the students guess, reference the Hanging Salmon to Dry poster to explain the Yup'ik use for each measure. You may want to again refer to the CD-ROM for help in pronunciation here. Further, a full page Cultural Note in Activity 6 provides first-hand accounts from elders of alternate body measure uses.

4. To demonstrate finding equivalent body measures, have a group of three students show the class the *yagneq* of one student compared to his or her height. Use a string to make the comparison easier. Two students measure a third student's *yagneq* with a string (Fig. 5.3). They should cut the string at the appropriate length, and then use the cut string to measure the student's height. The string should roughly be equal to the height (see Figure 5.4). Ask the students to describe the relationship between these two measures. (This is a one-to-one relationship.) Encourage them to express this relationship in a variety of ways. Write their suggestions on the board, on butcher paper, or on a transparency. Note that since children are growing and their bodies are changing at this age, the relationships may not be exact.

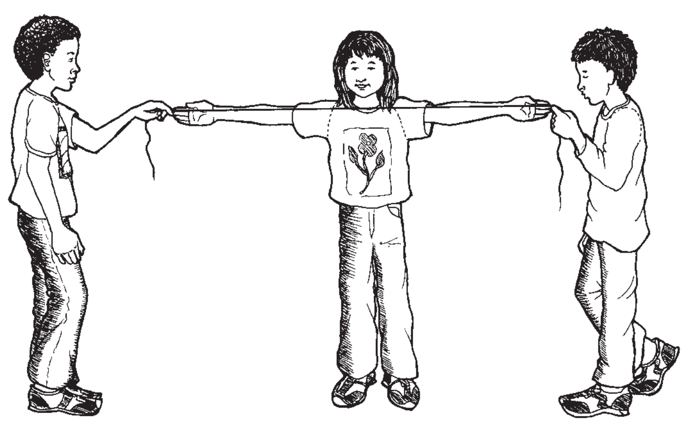


Fig. 5.3: Two students measuring another's yagneq.

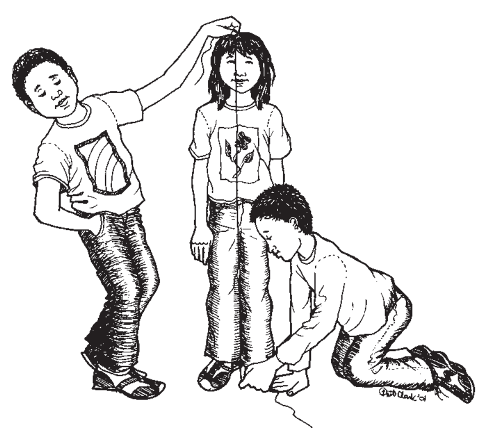


Fig. 5.4: One's height is equal to one's yagneq.

5. **Challenge**. Have students explore equivalent body measures by working in groups of three to four. Hand out plenty of string to each group. Explain that they should look for examples of body measures with relationships of one-to-one, two-to-one, three-to-one, four-to-one, and any other relationships.

6. After students have had enough time to discover equivalent body measures, have each group write one relationship. Have them record in their math notebooks:

   a. A drawing of each body measure in the relationship.
   b. A sentence describing the relationship.
   c. A possible application for each measure (for example, you might use your stride to measure the width of the floor of the classroom).
   d. Which of the two measures would be best if you wanted to measure the length of your fish rack pole? Why? Why would the body measure you chose be better than the others you did not? (See Figure 5.5.)

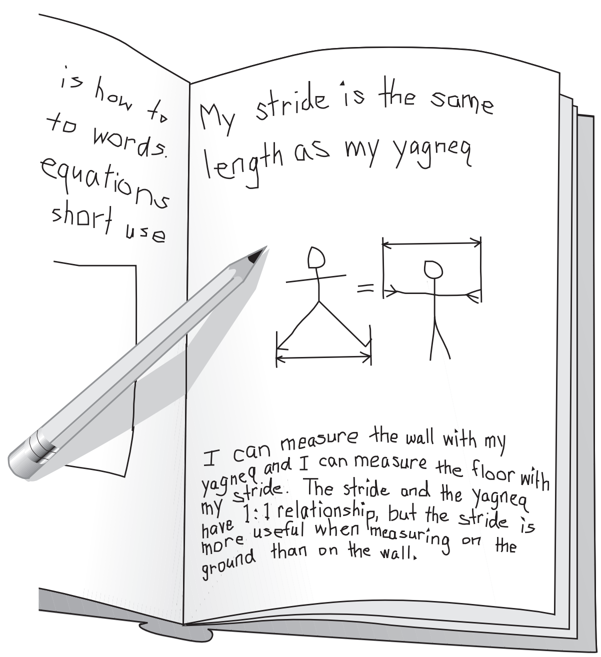


Fig. 5.5: Example of a math notebook entry.

7. Have each group present their equivalent body measures to the class by physically demonstrating the body measures and sharing their discoveries.

8. Provide time for groups to write down all the relationships they found in their math notebooks as they will need them for Assessment Activity B.

Teacher Note

Encourage students to discover many different body measures when looking for relationships. They do not need to be limited to the relationships in the challenge and may be able to find other relationships such as five-to-one, twelve-to-one, or three-to-two. For some measures, it may be helpful to use lengths of string to make comparisons. Note that body proportions may not necessarily be exact, and students should decide what levels of accuracy and approximation are acceptable. Lastly, try to use the terminology "equivalent body measures" while students are exploring. For example, if it takes 12 *naparneq* measures to make one *yagneq* measure, then they are equivalent body measures and have a 12-to-1 relationship. Later, we will describe this relationship as a proportion, Y:N::12:1. Although this can be thought of as a ratio, we feel focusing on the proportional relationship is more important.

Activity 6
System of Body Measures

We have seen that certain body measures are used for specific tasks; however, there are times when you want to use a measure that you cannot physically apply. For example, a basket weaver may want to place designs at equal intervals around a basket. A skilled weaver would have techniques, perhaps visualizing, to measure her materials and to accurately place the designs. This example shows a practical use for abstraction away from the physical measure.

In this activity, students will expand their development of proportional thinking to work with algebraic symbols. They will build upon the experiences from previous activities by using variables to represent body measures, which, as we have learned, vary from person to person. They will explore groups of body measures that interrelate as a system and calculate them in standard measures, bringing out the concepts of variables, constants, and units. They will move from writing descriptive sentences to writing algebraic expressions and equations, and finally, to manipulating those expressions and equations.

Teacher Note

Students will use cultural math tools based on their body measures starting in this activity. They will develop similar tools on their own using other body measures. Encourage students to continue to use these tools after the module is over whenever they help in other problem-solving situations.

Goals

- To implement the use of symbols for variables
- To identify variables, constants, and units within the system of body measures
- To create algebraic expressions and equations from sentences and pictures
- To practice manipulating algebraic expressions and equations using addition and subtraction

Materials

- Poster, Hanging Salmon to Dry
- Poster, Body Measurements
- Transparency, Body Measure Chart
- Worksheet, Body Measure Chart, one per student
- Scissors
- Crayons or colored pens
- Transparency pens
- Math notebooks

Math Note

Throughout this activity you may want to reinforce mathematical vocabulary by asking questions such as, “Which one is the variable here?” or “Which one is the constant?” You may also want to differentiate between an equation and an expression as the students share.

For example, suppose a student shares that the measure of his *yagneq* is 50 inches. This can be written as Y = 50 inches. The Y is a variable because it is a symbol representing a measure that we know changes from person to person. The entire mathematical sentence is an algebraic equation, since it contains a variable and an equal sign. The constant measure of 50 inches shows that for one student, the *yagneq* is always the same if we specifically refer to its length in the units of measure called inches. Later in this activity, students will look at the combination of a *yagneq* and two *taluyaneq* and convert it to the expression Y + 2T. This is an algebraic expression since it does not contain an equal sign. There is a constant of 2 involved, and both Y and T are variables. Refer back to the Introduction to the Module, specifically the section “Three Key Concepts of Algebraic Thinking” (page 6), for more detail on these definitions.

Preparation

Hang the Hanging Salmon to Dry and the Body Measurements posters from Activities 1 and 3 on the wall. Prepare the Body Measure Chart transparency and color each bar a unique color with transparency pens. Also copy the Body Measure Chart, one per student, to use as a worksheet.

Duration

Two to three class periods.

Vocabulary

Algebraic equation—a mathematical sentence showing equivalent combinations of operations, numerals, and/or variables using an equal sign.
Algebraic expression—a mathematical phrase that combines operations, numerals, and/or variables.
Constant—a value that does not change.
Variable—a letter or symbol used to represent one or more numbers in an expression or equation.
Ikusegneq—the measure from one’s elbow to the end of one’s fingers.
Naparneq—the measure of one’s fist with outstretched thumb.

Instructions

1. Review some of the students’ sentences of equivalent body measures from Activity 5 and briefly discuss the relationships of the body measures.

2. Using their math notebooks and results from Activity 5, challenge students with one of the following questions:

   - How can we change the relationship we wrote in word sentences into a number sentence?
   - How can we change our word sentence into a math sentence?
   - How can we write this relationship using math symbols?
   - How can we write a number sentence showing how the body measures connect to each other?

   You may want to have students work either as a whole group, within small groups, or as individuals for about five minutes on this task.

3. Share with the class the Body Measure Chart transparency that you have colored. You may want to refer to the *Yup’ik Glossary* CD-ROM for pronunciation here. Ask the students to explain the chart the best that they can. For example, students should see that the symbols stand

for various body measures: Y stands for *yagneq,* T for *taluyaneq,* I for *ikusegneq,* and N for *naparneq.* They should also see that the chart shows equivalent body measures. For example, 4I = 2T. If needed, describe one of these relationships to help students see. You do not need to say much as students will explore using this cultural math tool in the next step. Refer to the Body Measurements poster when necessary.

4. **Challenge.** Have students work in groups of two to four. Pass out the Body Measure Chart worksheet, one to each student. Hand out the crayons (or pens) to each group and have the students color in the chart, making sure that each row is only one color, similar to the transparency. In their math notebooks, have the students create five equations (involving addition or subtraction). Each equation must use at least three of the measures in the chart. Pass out scissors to each student, and encourage students to cut up the charts and use the individual pieces as manipulatives (see Figure 6.1).

Fig. 6.1: Students working with proportional body measure strips.

5. As students begin to discover equations, have them select one and prove the equation is true by measuring those body measures using standard units (inches or meters) and using the values in the equation. For example, if a student has the equation Y = 2T + I have them measure their own *yagneq, taluyaneq,* and *ikusegneq* and show that they are related in the way that the equation describes.

6. Have students write their proofs and discoveries in their math notebooks.

7. **Discuss.** Have each group write a different equation on the board or overhead and show how the group found it and proved it was true.

8. Ask students within their groups to write equations using only multiplication, for example, Y = 2T. They may have to share pieces to do this.

9. **Discuss**. Have groups discuss the meaning of their multiplication equations.

Math Note

An example of an equation using addition of variables is Y = T + 2I. Using the manipulatives, place one of the T bars and two of the I bars together underneath the Y bar, since T + 2I is the same length as Y, we say they are equivalent measures. Note that 2I is also an addition operation since it's shorthand for I + I.

Since 12N can mean 12 lengths of N units long or focusing on the twelfth length of one *naparneq*, the equation Y = 12N provides students with their first step from arithmetic thinking to algebraic thinking. In Section 4 of the module, this idea extends to creating general patterns such as $4n + 2$ where n represents the number of cubes instead of the length of a *naparneq*.

Teacher Note

Two extension activities are provided below. You can use them to provide students with additional practice on the concepts of proportional thinking and related vocabulary.

A. Now is a good time to spend one class period working on the vocabulary that has been presented. Either use a vocabulary game you have handy, or write the words on 3 x 5 cards to play an act-it-out game. You may want to consult your school reading experts for other ideas.

B. If students need additional practice, pass out Cuisenaire Rods and use them in a similar manner to the proportional body measure strips. Encourage students to write equations of equivalent measures using variable names for each rod. Since the rods are designed in an arithmetic sense (each rod is one unit longer than the previous one), not all lengths will easily fit into proportions. However, students will be able to add two rods together of unequal length to find the same length as a longer rod. Other activities can be found at http://www.etacuisenaire.com/cuisenairerods

Cultural Note

Henry Alakayak, an elder from Manokotak, Alaska, and Frederick George, an elder from Akiachak, Alaska, shared their knowledge of body measures and their various purposes.

Henry first explained that the body was used because it was all people had at the time. Measures were found that fit the length that needed to be measured. He explained,

> In those days, we didn't have any measuring tools. So what was readily available at that time was used. The smaller units of body measurements, such as the hand, can be used if they are appropriate or would fit to make the final measurement.

Henry further explained how men used their own heights with some additional length to estimate how high to build their ceilings. He connected this height to the height of a fish rack, since the men built the fish racks for the women to use when cutting and hanging the fish to dry.

> I mentioned the height of the body as a unit of measure [demonstrates it by standing against the wall. Henry marks his height by placing his right hand at the top of his head. He places his left hand where the mark of his height is, then uses his right hand to estimate an additional foot or half of a foot more in determining the final height.] Notice I mentioned, '*naparyaq*,' the post where the height of my body with an additional foot or half a foot determines how high I want to make it. The estimate of an additional foot or half a foot of the height is to prevent my head from constantly bumping into it. How high the post is would be for the height of a dwelling place or it can be less than the height of the person who is to hang fish on a fish rack.

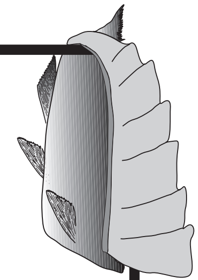
When asked about the measures used for larger structures, Henry mentioned using his feet *(it'ganeq)* or strides *(amllineq)*. Frederick talked about how his father taught him one particular body measure within the context of cutting logs to fit in a wood stove.

> Over in the Yukon River, my father lived during the steamboat-with-a-wheel era. Even if they were around, jobs were scarce at that time. Whatever they gathered was cut and put into cords and was ready whenever the steamboat came ashore. They [the steamboat operators] bought the cords from them at maybe $4/cord. At that time it was okay and he didn't seem to mind, for it was a lot to them. They had units of a 2-foot log [gestures the length] which was fed into the stove of the steamboat which gave it energy to move.

Frederick explained the measure he learned was the *malruk naparnerek*, but in English he calls it a foot. For Frederick, that body measure is equivalent to a 12-inch foot and is the measure he continues to use in most circumstances today.

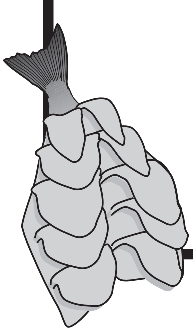
> The only unit of measure I know is 'one foot' [demonstrates by using his fists and extending the thumb out when the two are connected they are a foot]. I will measure four feet, "one, two, three and four". Maybe it will be a foot, two feet, or four feet for a unit of a cord or half of a cord. Maybe a full cord is 24 inches. It is a square. That is a full cord.

Information gathered during a meeting in Anchorage, October 2003. Transcription and translation by Ferdinand Sharp from Manokotak, Alaska.



Body Measure Chart

Y											
T						T					
I			I			I			I		
N	N	N	N	N	N	N	N	N	N	N	N



Activity 7
Proportional Thinking

Your students have had the opportunity to investigate body measures, discover equivalent body measures, and explore a specific system of related body measures. Now they will practice calculating various body measures using proportional thinking. Now that students have been introduced to the concept of proportions, they will learn the related terminology and notation, and practice calculations. Further, students will be introduced to the notation of proportions using colons similar to the way they are used in word analogies. For example, big is to small as lion is to kitten can be written as big:small::lion:kitten. In mathematics, the proportion *yagneq* to *taluyaneq* is 2 to 1 can be written as Y:T::2:1. This notation is read "Y is to T as 2 is to 1", which also means that Y is twice as big as T or that two Ts are needed to make one Y.

Goals

- To compare lengths of body measures using tables
- To use proportional notation for body measure proportions
- To measure body measures in standard units

Materials

- Poster, Body Measurements
- Transparency, Body Measure Chart from Activity 6
- Worksheet, How Do the Sizes Relate? (one per student)
- Strings from Activity 3
- Body Measure Chart manipulatives from Activity 6
- String
- Yardsticks, rulers, or meter sticks
- Cardboard for cutting out circles of 4-inch and 2-inch radii (one per pair of students)
- Compass for drawing circles (one per pair of students)
- Protractor or string attached to a pencil
- Math notebooks

Preparation

Copy the How Do the Sizes Relate? worksheet, one per student. Keep out the Body Measure Chart transparency from Activity 6.

Teacher Note

Students should recognize the worksheet is similar to the proportional chart. Review more specifically the equations they developed in Activity 6, if needed. Further, remind students of the *malruk naparnerek* measure referring to the Body Measurements poster if necessary.

When completing the worksheet, have students divide the blank row in the chart to show how the *malruk naparnerek* relates to the other measures (see example in Figure 7.2). Then, given a length for one variable, have them calculate the length of the remaining variables. Allow them to use any materials from previous lessons such as their personal strings from Activity 3 or the cut-up manipulatives from Activity 6.

Duration

Two to three class periods.

Vocabulary

Proportion—an equation which states that two ratios are equivalent.
Proportional—having the same or a constant ratio.
Ratio—a comparison between two values written in fraction form.

Instructions

1. Explain to students that they will continue working with the body measures they discovered in the previous activity by describing the relationships in a new way.

2. Facilitate how to use values for the lengths of the body measures on the chart by writing on the board or on the Body Measure Chart transparency. After the Y write = 12 feet as shown in Figure 7.1. Ask the class, "If the length of Y is 12 feet, then how long is T?" Be sure students include units in their responses. Check their guesses verbally. For example, if a student guesses 5 feet, then say to the class, "If T is 5 feet then T + T would be 5 + 5 or 2 times T would be 2 times 5—that only gives me 10 feet and we said Y is 12 feet." Allow them to continue providing answers until the correct value of 6 feet is shared. Continue with the other measures (I should be 3 feet and N should be 1 foot).

<table>
<tr><td colspan="12">Y = 12 feet</td></tr>
<tr><td colspan="6">T</td><td colspan="6">T</td></tr>
<tr><td colspan="3">I</td><td colspan="3">I</td><td colspan="3">I</td><td colspan="3">I</td></tr>
<tr><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td></tr>
</table>

Fig. 7.1: Example of picking a value for Y.

3. Pass out the How Do the Sizes Relate? worksheet, one per student, and have students describe how this sheet relates to what they were doing in Activity 6.

4. Have students complete the worksheet in groups of two to four.

RESULTS:
PROPORTIONAL CHART REPRESENTING BODY MEASURES

<table>
<tr><td colspan="12">Y</td></tr>
<tr><td colspan="6">T</td><td colspan="6">T</td></tr>
<tr><td colspan="3">I</td><td colspan="3">I</td><td colspan="3">I</td><td colspan="3">I</td></tr>
<tr><td colspan="2">m</td><td colspan="2">m</td><td colspan="2">m</td><td colspan="2">m</td><td colspan="2">m</td><td colspan="2">m</td></tr>
<tr><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td><td>N</td></tr>
</table>

Fig. 7.2: Example of M *measure* (malruk naparnerek) *with respect to other body measures.*

5. If needed, after students complete the worksheet, have them continue practicing calculations of the various measures in their groups and have them write the results in their math notebooks. To do this, provide students with a constant value for one of the variables and have them calculate the length of the remaining variables. For example, suppose N = 1 inch, have students calculate the length of Y, T, I, and M. Or let Y = 1 yard. What are the lengths of the remaining measures? After students begin to feel comfortable calculating from small to large and large to small, continue by choosing (or having them choose) a value for one of the variables in the middle, for example, let I = 36 cm.

Math Note

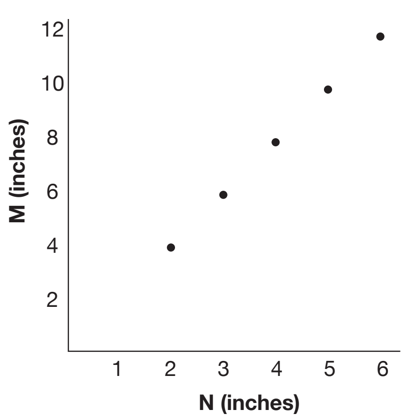


Fig. 7.3: Example of a scatter plot comparing M lengths to N lengths showing the two-to-one proportional relationship.

Now that students are abstracting from their original body measures to using any value for the variables (even those that may no longer represent a valid measure, such as N being 1 inch), you may want to encourage them to draw tables and use two-variable scatter plots similar to those in Activity 4 to see that the relationships continue to remain the same. For example, in Figure 7.3, no matter what length is assigned to N, the M length is always twice as big. This provides a scatter plot that shows a stronger relationship than those in Activity 4 as it demonstrates the actual proportion M:N::2:1.

Additionally, you may want to ask students to work on problems such as the following.

Assume Mary and Alice have the same body measures. If Mary uses 5Y to measure a length but her friend Alice uses T to measure the same length, then how many Ts will Alice use?

6. Discuss with students the concept of proportion, using the colon ":" notation and values they provided on the worksheet. For example, when Y = 12 feet and students determine that T = 6 feet, then write on the board *yagneq*:*taluyaneq*::12 feet:6 feet. This describes the proportion "*yagneq* is to *taluyaneq* as 12 feet is to 6 feet". Another example could be Y:N::12:1. Explain that we use this notation when we want to compare quantities. In other words, this shows us how big Y is in terms of N or how big Y is compared to N. Continue practicing this notation using the other examples on the worksheet as a whole class or in small groups until students feel more comfortable.

Math Note

Proportions are a special type of relationship showing how two ratios are equivalent. However, the concept of proportions can be considered without investigating the mathematics of ratios themselves. In looking at the relationship between lengths, simplifying the meaning provides a proportion, such as saying that T is twice as big as Y or that Y:T::2:1.

We do not pursue ratios in this module; however, these explorations do provide the foundation on which to build should you decide to do so. Keep in mind, focusing on ratios brings up several other ideas. A ratio can either represent a comparison between objects, a rate, or a scaling factor.

If we think about comparing values with the same units, then the units will divide out and a constant value will result. For example, if we compare the height of two students, one 45 inches tall, the other 50 inches tall, the ratio would look like: 45 inches/50 inches or 45/50, which is also 9/10. We see that when dividing, the inches cancel and we get the fraction 9/10. To interpret this result, we would say that the first student's height is 9/10th of the second student's height. Using this relationship excludes the actual height measurement—we cannot determine the students' heights anymore—only the relationship between their heights.

Examples of everyday ratios include 60 miles per hour or 12 rotations per minute. Lastly, ratios are frequently used as scaling factors, providing a different concept than the comparison or rate concepts already described. For example, when transferring information from a map into real-life calculation, the ratio of 1 inch to 40 miles may be used.

Extension Exercise

Proportions of Circumferences

1. Have students work in pairs and draw two circles on cardboard or paper using a compass or a string attached to a pencil. One circle should have a radius of two inches; the other, a radius of four inches as in Figure 7.4.

2. Have students wrap a string around the circumference of each circle and cut.

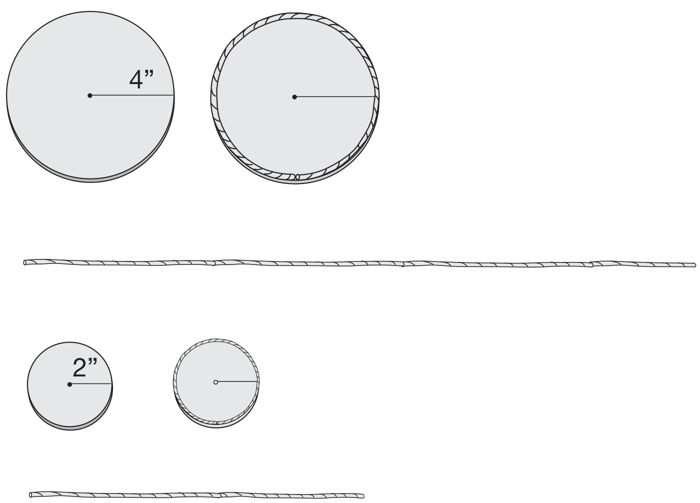


Fig. 7.4: The top circle has a radius of four inches and the bottom circle has a radius of two inches.

3. Have students compare the circumferences of the two circles measured using the string.

4. Have them write their results using proportional notation in their notebooks. Ask them to also write a sentence describing in words the relationship they found.

Teacher Note

Since the circumference of a circle is 2πr, students should find the two strings have a 2:1 proportion. The string from the larger circle (C = 8π) should be twice as long as the string surrounding the smaller circle (C = 4π). In proportional notation, the results should look something like

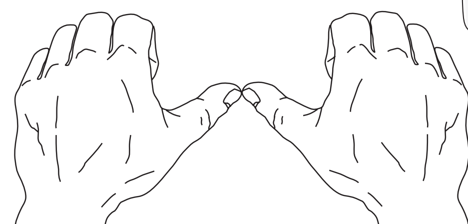
C1:C2::2:1
R1:R2::4:2

where C1 is the circumference from the first circle. Other names could be C or R, etc.

How Do the Sizes Relate?

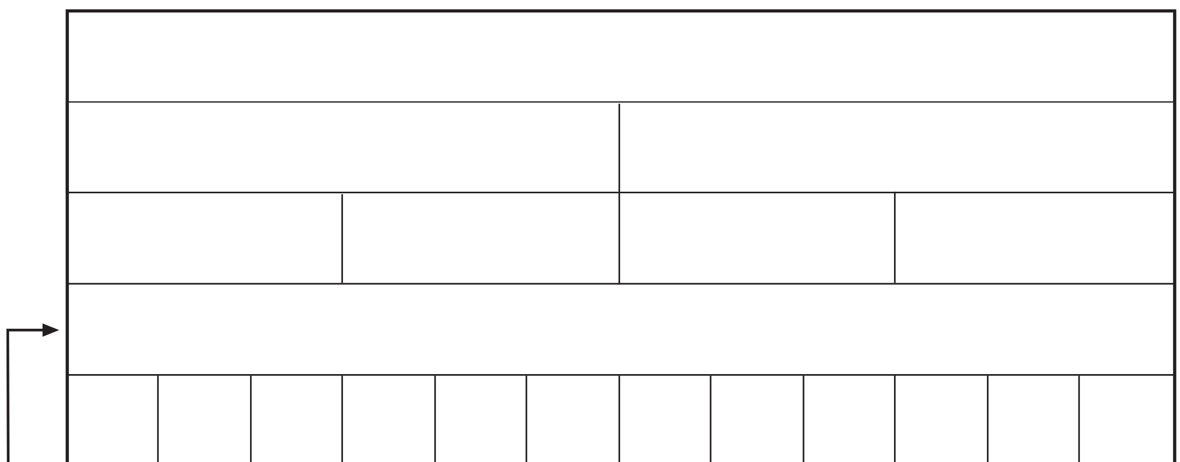
Label the chart below to show which bar can represent which body measure length.

Break apart the open row to show how the *malruk naparnerek* length relates to the other lengths.



Malruk Naparnerek

Proportional Chart Representing Body Measures



Divide and label this row for *malruk naparnerek*.

Using the chart as a guide, find the values of the following measures if Y is 36 inches long.

T = ___________ I = ___________ M = __________ N = ___________

Using the chart as a guide, find the values of the following measures if N = 2 inches.

M = ___________ I = ___________ T = ___________ Y = ___________

Assessment Activity B

As in the first assessment activity after Activity 4, have the students work in groups of three or four and use the problem-solving approach (outlined in the Introduction to the Module) to work out the following challenges. Remind students how the group problem-solving strategy works—one student takes the leadership role to facilitate discussion on solving the problem, asking such questions as: What is the unknown? What type of problem is it? What information is missing? What do we need to find out? Another student facilitates the planning, asking such questions as: Should we draw pictures to help with the problem? Should we use symbols? To help us with this problem, should we make up a similar but easier problem to solve? After students have worked individually on their answers, another student takes on the role of the evaluator and compares everyone's answers, leading the students to rework the problem if he/she thinks the answer(s) are not correct. Students should switch roles for each question. It may be necessary to remind them to switch.

For your convenience, the solutions follow each problem listed here. On page 88 a blackline master with only the problems is included for you to copy and use as a worksheet for your students.

Problem-Solving Challenge 1

Create your own set of proportional body measure strips using new measures. Find proportional measures from your explorations in Activity 6. Select three to six related measures and create your own set of tools, one for each member of your group.

Solution:

Be sure that the measures students choose create a proportional system of lengths. For example, they could use a foot, their stride which is usually three feet (body measure not standard) long, and their *taluyaneq* to create a new system.

Problem-Solving Challenge 2

Create at least five equations using variables to represent your lengths from your new set of proportional body measure strips.

Solution:

Be sure that the equations are accurate and students use a variety of methods such as adding, subtracting, and multiplying as a shortcut to repeated addition.

Assessment Activity B

Problem-Solving Challenge 1

Create your own set of proportional body measure strips using new measures. Find proportional measures from your explorations in Activity 6. Select three to six related measures and create your own set of tools, one for each member of your group.

Problem-Solving Challenge 2

Create at least five equations using variables to represent your lengths from your new set of proportional body measure strips.

Section 3: Hanging Salmon to Dry

In this section students conduct another science experiment, with the focus on understanding how increased surface area decreases drying time. Students learn Mary George's system, in which body measures are equivalent to the length of the cut salmon on a pole, and more specifics about how to hang salmon to dry. The section culminates with students playing a fishing game in which they algebraically model catching and hanging salmon to dry working within the cultural guidelines of not wasting, yet needing enough to feed their families for the winter. Depending on the developmental level of your students, you may want to conclude the module with the Fishing Game of Activity 10. If you feel your students are ready to investigate patterns and determine equations of general patterns, then the module continues with Section 4 and culminates in the quintessential math discovery of how cutting increases surface area but allows volume to remain constant as is the case with the practice of drying salmon.

Henry Alakayak modeling the tekrem avga cip'arrluku *measure.*

Activity 8
Apple Drying Experiment

In this activity, students learn that increasing surface area speeds drying time. Students will slice apples and observe the difference in drying time and weight loss as the surface area changes. This connects quite directly to the process of drying salmon, as students will discover in Section 4. Note that you may want to start this activity, conduct Activities 9–10 while the apples are drying, and then finish up with the remainder of this activity.

As in the previous experiment, Activity 2, this experiment is set up for students to follow directly. Alternatively, depending on your class, students may be encouraged to create or design as much of the experiment as they can. This can range from designing the data tables to determining the experimental design. In the latter case, the activity as written provides a model that can help you facilitate the students' experiment designs.

Goals

- To determine the importance of moisture in the drying process
- To experience the importance of increased surface area in the drying process
- To graphically represent the effect of drying over time

Materials

- Apples (one per group of four)
- Knives (one per group of four)
- Scales for weighing in grams (one per group of four)
- Material (screens, oven grates, etc) for laying out apple slices to dry
- Math notebooks

Preparation

Have a quarter section of apple ready for each group to weigh before cutting. Determine the most appropriate method for cutting sections into slices for your classroom.

Duration

One class period to set up the experiment. One week total duration checking daily or twice a day.

Instructions

1. Introduce the activity by explaining that we want to see how cutting an apple in different ways will affect drying time. Ask the students how they think this experiment relates to drying salmon.

2. Have the students work in groups of four. Pass out an apple and knife, if appropriate, to each group. In their groups have the students:

   a. Cut the apple into four equal sections.
   b. Set aside one quarter section, leaving it as one slice.
   c. Take a second quarter and slice it in half (creating two slices) and set aside.
   d. Cut the third quarter section into four equal slices and set aside.
   e. Take the fourth quarter (last piece) and cut it into eight slices (see Figure 8.1).

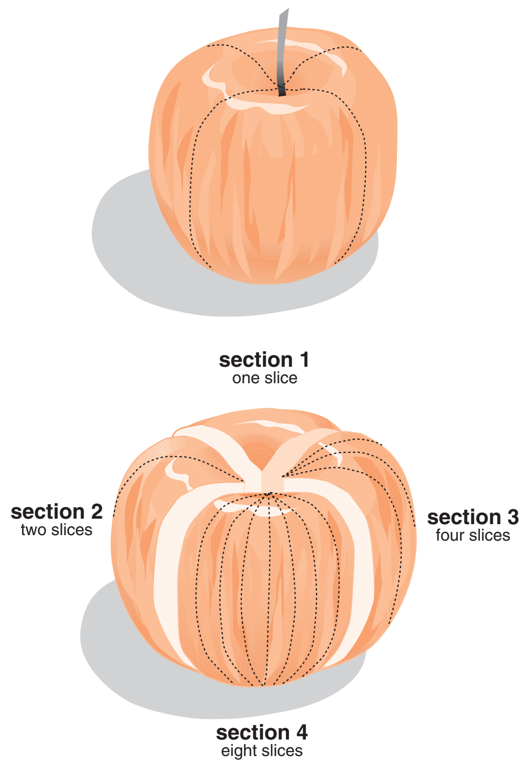


Fig. 8.1: Diagram of how to cut the apple for this experiment.

3. Each student will be assigned one of the original sections. So, for example (Fig. 8.1), student A gets Section 1. Student B gets Section 2, etc. Students weigh the apple slices and record in their math notebooks the weight of their assigned sections keeping all the slices together to obtain a total weight for that section. So, for example, for Section 2, weigh both pieces together to obtain the weight of that section. Be sure to weigh the sections after they have been cut, since the act of cutting may slightly change the total weight from the orginal.

4. Have students place a label with their initials near their slices. Have students lay out the slices of apple on their drying rack, and place them in a warm, dry place (a sunny window, by a heater, etc).

5. Have students check on their section and weigh the apples two times per day over for the next five days. Students should write their findings in their math notebooks using both qualitative and quantitative data and also noting the date and time of day.

At this point, you may wish to teach Activities 9–10, as students gather their data for the experiment.

6. After five days have elapsed, have students compare their data within their groups. Have them discuss: Which quarter section lost the most weight? How do they know? Why use weight as a way to detect the dryness of an apple? Which quarter section has the greatest surface area? How does surface area play a role in the drying process? Ask students if they have other questions to add to this list for the other groups to consider. Students should record their thoughts from the discussion in their math notebooks.

7. Have groups share their results with the rest of the class.

8. If students haven't already done so, in their math notebooks, have them graph the data, comparing the weight to the time (in hours or days) (see Figure 8.2). Note that time may be a bit confusing as they may have checked their apple sections at different times of the day. Allow student discussion when necessary to find a solution to this problem. Make sure they use different colored lines to represent each of the four quarter sections.

Teacher Note

As the apples dry, their weight will give us an indication of how the dryness is progressing. Initially, Sections 3 and 4 (the sections cut into the most pieces thereby having the greatest surface area) should lose the most weight, but later, when these pieces are drier, they may begin to lose less moisture per day than Sections 1 and 2. Although each section contains the same size of apple (some sliced more than others), it may be useful to use the proportion of the weight loss to the actual weight in order to gain further insight.

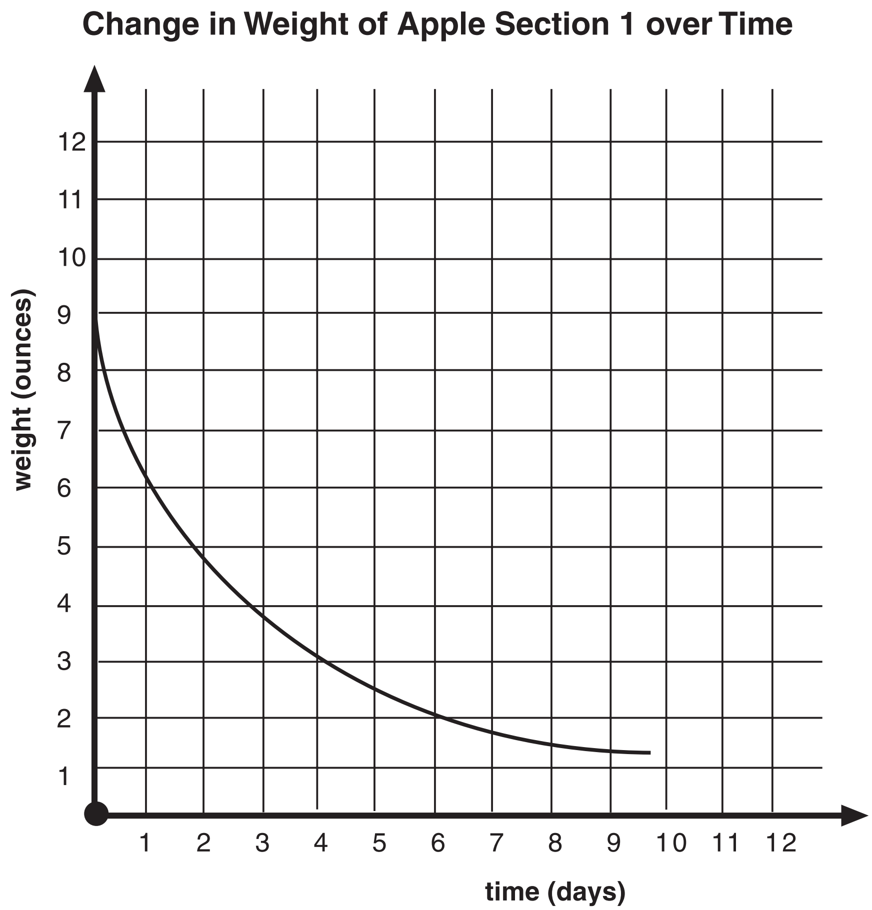


Fig. 8.2: Example of graph showing loss of weight over time for a section of an apple.

9. Within their groups, have students discuss the relationship of weight to time and other information found in their graphs. Have them consider how this relationship relates to other relationships they've encountered, such as in the bread experiment.

10. **Discuss.** Have groups share their findings with the class. Encourage discussion around the ideas of weight loss and drying, connections to food preservation, and the relationship between increased surfaces and drying.

Activity 9
Preparing for the Fishing Season

Many Yup'ik elders are experts at estimating how many salmon they need to catch in a fishing season to sustain their family until the next season. They know how many salmon will fit on their fish rack poles at one time, and how many fish rack poles are required to hold the catch of one fishing season. Elders attain this knowledge through repeated observations, practice, and by using their spatial senses. They are also familiar with the timing of each fish run and the proper nets to use in order to catch the fish species they desire.

Your students have explored how the different body measures relate to one another and how the drying process is essential for food preservation. Next, they will model temporary fish rack poles and use model salmon cut-outs to gain an intuitive sense of the sizes of different salmon and how much space each species needs when hung on a fish rack pole for drying.

In this activity, your students will estimate the number of salmon lengths that can fit on a pole. They will check the accuracy of their estimates against the actual number of salmon that fit on one pole. Their bodies and their sense of space will become important tools for measuring. As a reinforcement of the algebraic process, they will move from visual estimation to scaled model representations, and then to symbolic representation and manipulation. This is the first of a two-part activity that culminates in a game. Today's lesson prepares for that game.

Goals

- To estimate how many fish of each species will fit on one pole
- To compare estimates by using model pieces
- To create algebraic expressions and equations for the number of fish hanging on a pole

Materials

- Poster, Hanging Salmon to Dry
- Poster, Fish Cuts
- Transparency, Temporary and Permanent Fish Racks
- Transparency, Fish to Body Measures Chart
- Worksheet, Model-Sized Pieces (one set per student, two pages)
- Markers
- Scissors
- Math notebooks

Preparation

Hang up Fish Cuts and Hanging Salmon to Dry posters (if not already hanging). Prepare the Temporary and Permanent Fish Racks and Fish to Body Measures Chart transparencies. Copy Model-Sized Pieces (2-page worksheet), two sets per student.

Duration

Two to three class periods.

Vocabulary

Approximate measures—measurements which are estimated or close, but not exact.

Estimate—to guess or calculate approximately; the value of a guess or approximation.

Model—(v.) to make a physical or mathematical model.

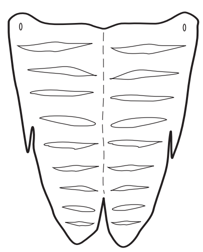
Practical measures—measurements that have practical meaning or develop within or out of a context with purpose.

Scale—ratio between the dimensions of a representation and those of an object.

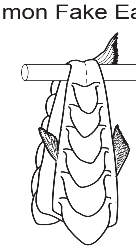
Iqelqin—the measure of both hands with outstretched fingers, between the tips of one's little fingers.

Patneq—the measure of one's hand width, from the outside of one's pinky finger to the outside of one's thumb.

King Salmon Fake Blanket Cut



Red Salmon Fake Earring Cut



Chum Salmon Fake Kite Cut

Fig. 9.1: Different ways to cut salmon for drying as done in the Kuskokwim region.

Instructions

1. Show the transparency of Temporary and Permanent Fish Racks, pointing out the differences between those shown. Point out the Hanging Salmon to Dry poster again. Explain to your students that today they'll be investigating how many fish can hang on a pole. Since there's not enough space in the classroom to build actual fish racks, we will use paper models of the poles and calculate how many poles are needed depending on the amount of fish.

2. Point out from the Hanging Salmon to Dry and Fish Cuts posters how each kind and size of fish requires a different type of cut to help expedite the drying process, also shown in Figure 9.1

3. Show the transparency, Fish to Body Measures Chart, and describe the various body measures. Have the students briefly practice the body measures, if necessary.

4. Have the students work in groups of three or four. Pass out the set of Model-Sized Pieces worksheets, one set to each student. Point out that the set includes one pole and multiple copies of each body measure and each fish needed. Explain that the model, based on Mary George's body measures, is scaled down. The pole actually represents roughly nine feet or 108 inches (~3 m), not the twelve feet (4 m) used earlier in the module. This is because the ends of any fish rack pole cannot be used. Nine feet (3 m) represents the usable space on a twelve-foot fish rack pole (Fig. 9.2).

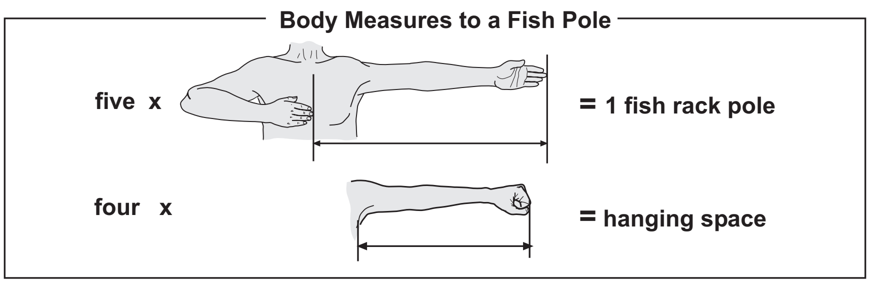


Fig. 9.2: Difference between fish rack pole lengths and usable space.

5. Hand out scissors to each student to cut out the model pieces. Have them discover how many of each salmon species (king, chum, or red) will fit on a fish rack pole using the model pieces. Encourage them to consider loading the pole with only one species at a time, as well as mixing them. Encourage them to measure the fish length with the proportional body measures and to determine how many body measures fit on the pole. Have students record their results and procedures for each scenario in their math notebooks.

6. Have students create a table in their math notebooks that combines all the data compiled in Step 5. Have one student in each group play the role of the teacher to aid in forming the table. The table should show the various relationships in more than one way—such as with picture equations (drawing the modeled results), algebraic equations (in the form of a simple equation or a proportion), and proportion notation. Note: A variety of tables may result. One table could look like Figure 9.5. The first pole could also be written as P:K::4:1, meaning the pole is four times the length of a king salmon.

Cultural Note

Spawned-out red salmon *(say-alleq)* also provide food for the Yup'ik people in Bristol Bay usually in September and early October. They are typically cut into strips and once dried *(tamuanaq)* are enjoyed throughout the long winter months.

Teacher Note

A nice touch to this activity involves making model fish cuts to hang on a model fish rack. Students can design and build model fish racks using skewers and gum drops. The fish cuts can be sized based on the paper cut-outs. Here are examples of small model fish cuts and a life-sized model fish cut.

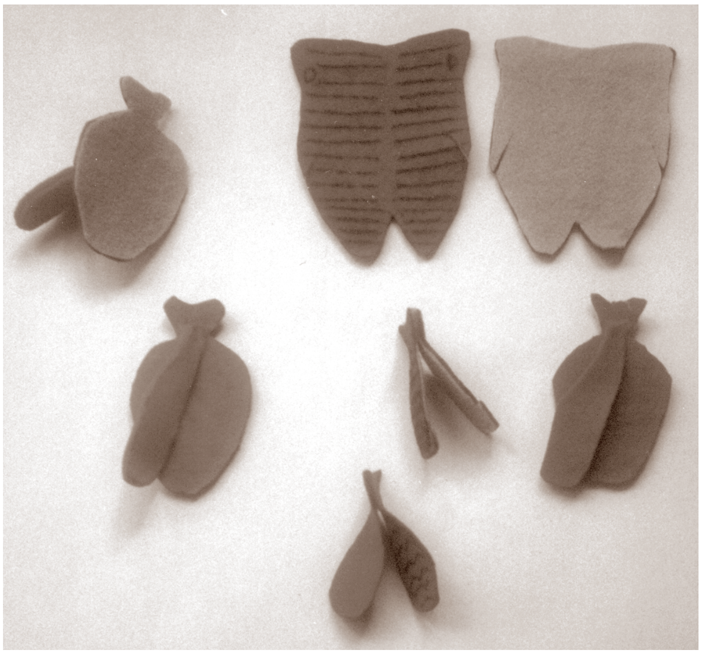


Fig. 9.3: Example of model fish cuts made from felt.

Fig. 9.4: A life-sized model of a chum salmon fake kite cut made from felt.

Table: Picture Equations and Algebraic Thinking

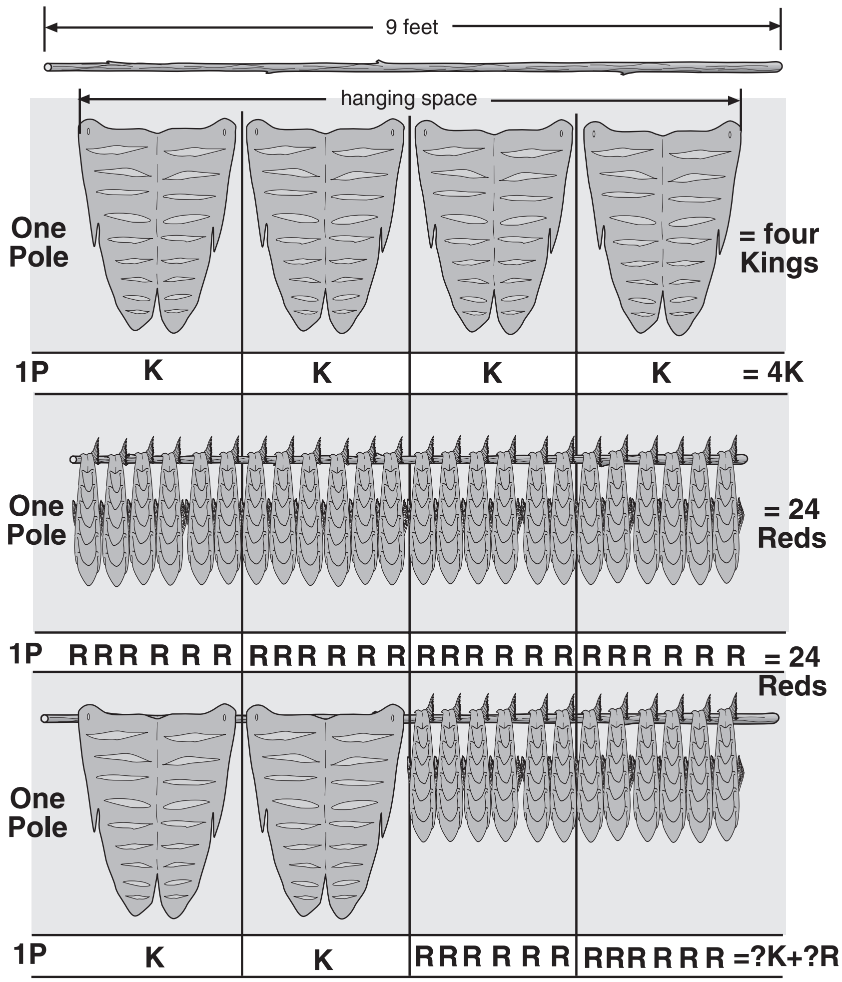
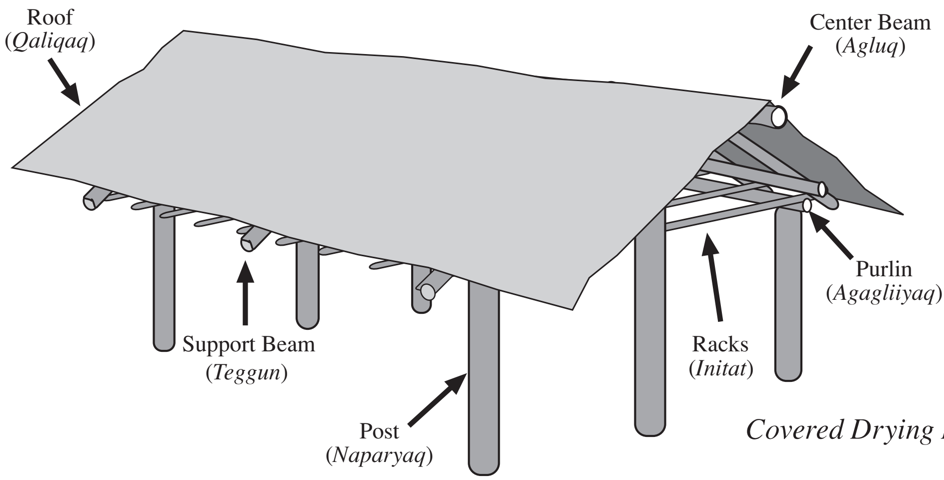


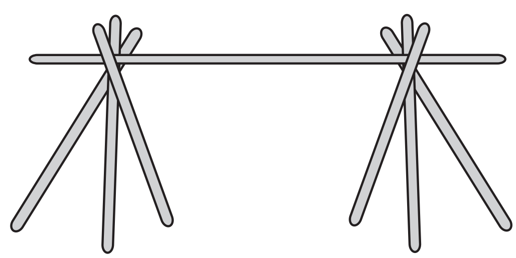
Fig. 9.5: Example of pictures and equations for different poles.

7. Have groups share their tables, how they constructed them, and explain any differences in counts. Discuss the usefulness of the algebraic equations, proportions, the variety of different species on one pole, and the arrangements allowing the most and the least fish. Discuss the ideas of practical and approximate. For example, the shoulder measure is easily applied when lifting a blanket-cut king salmon, as the fish neatly fits in the space between most adults' shoulders. However, when carrying a blanket-cut king salmon, the arm measurement is more useful for approximating the length.

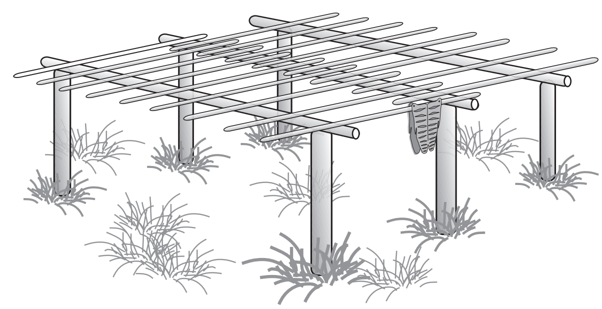
Temporary and Permanent Fish Racks



Covered Drying Rack (Talicivik)



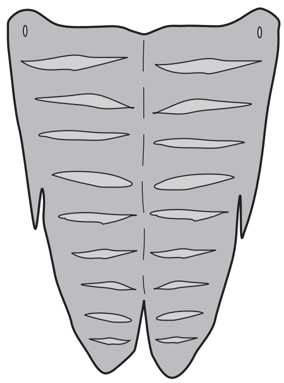
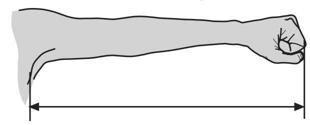
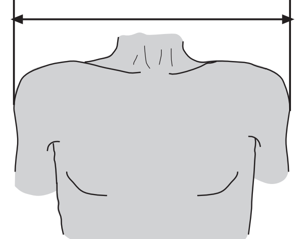
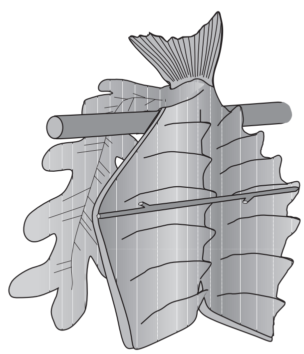
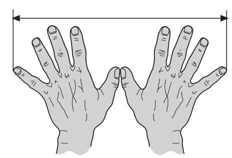
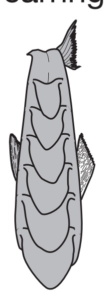
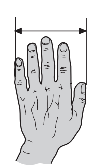
Temporary Fish Rack



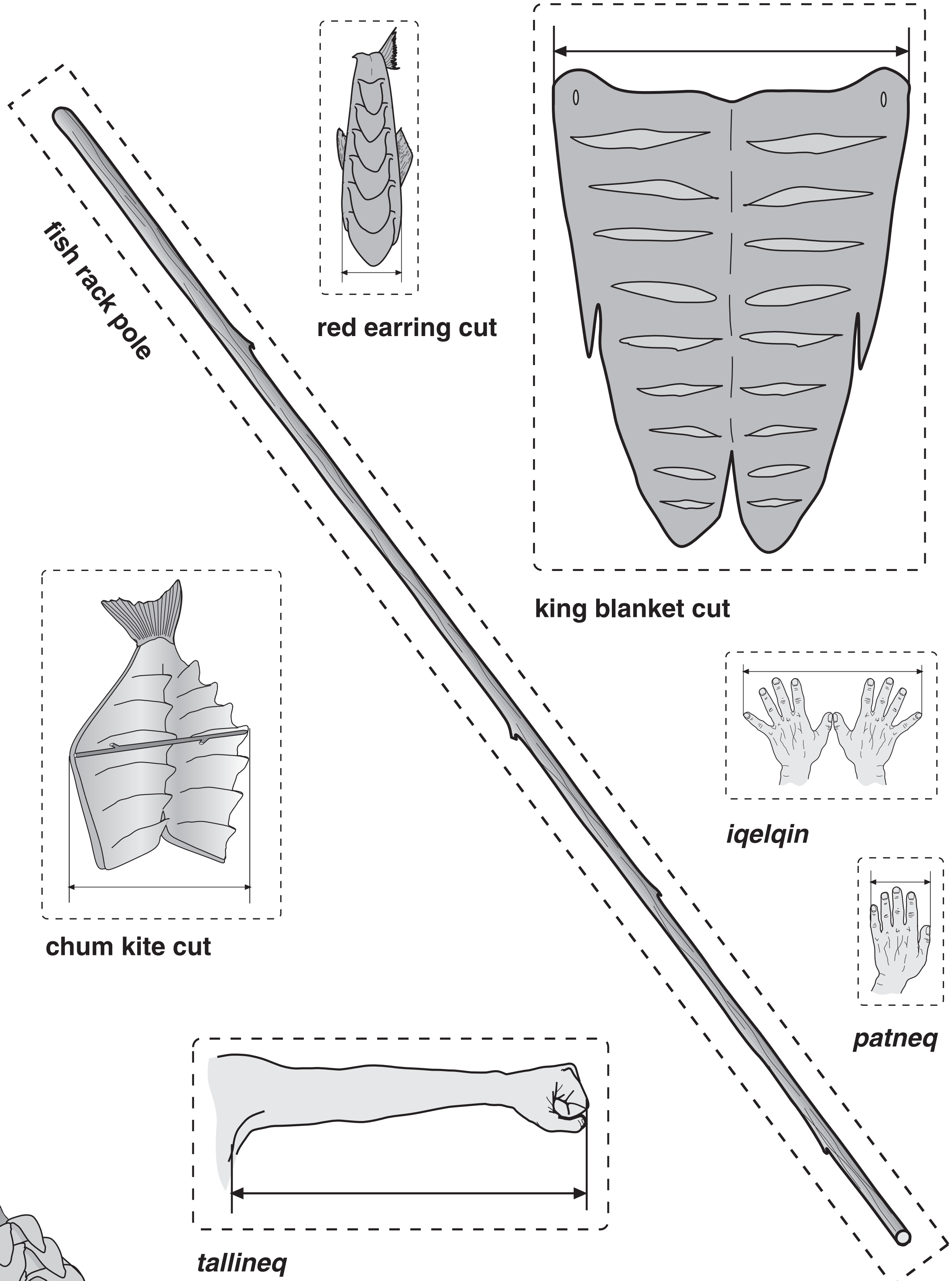
Permanent Open Fish Rack (Qer'aq)

Permanent Open Fish Rack (Qer'aq) *from Bethel, 1950*

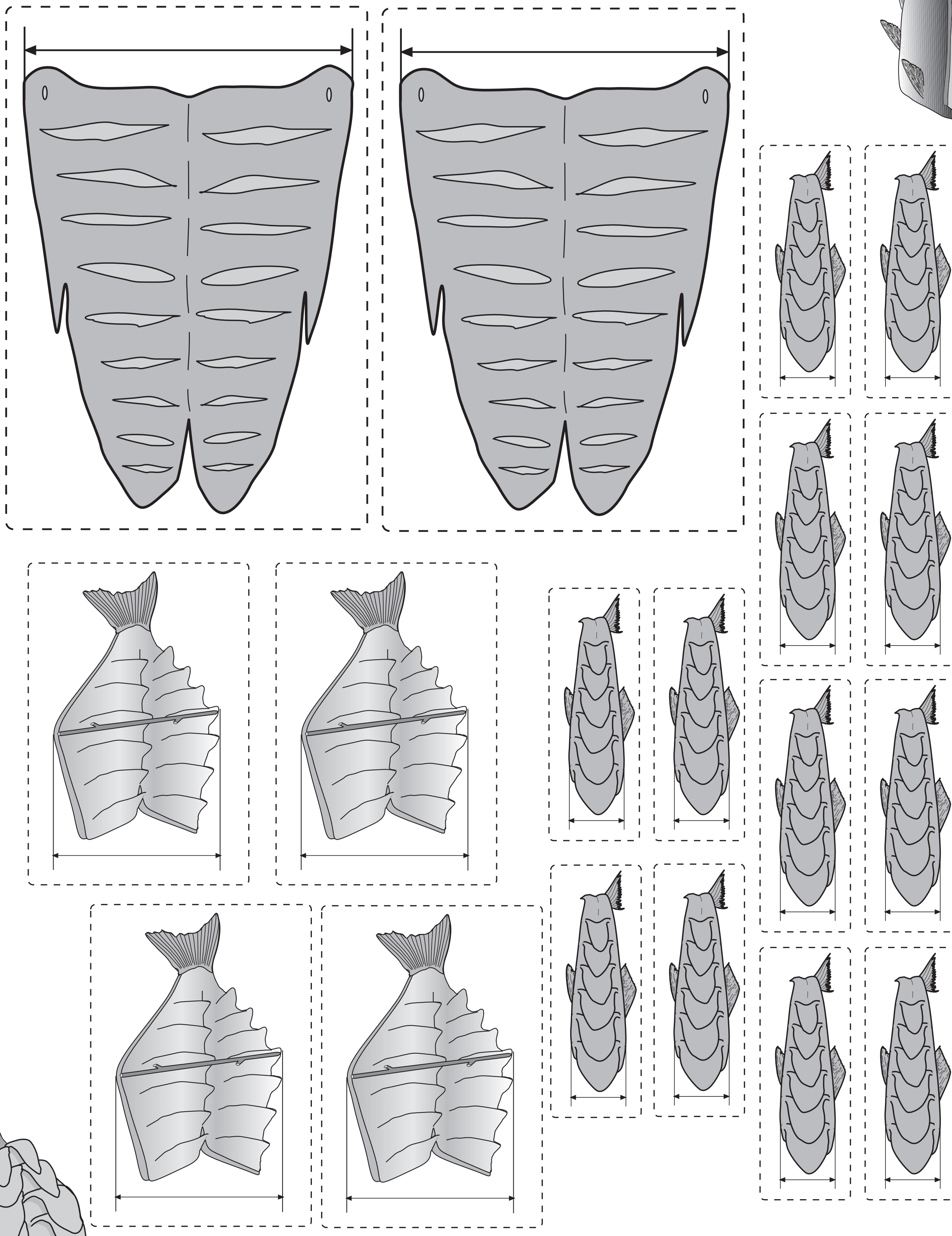
Fish to Body Measures Chart

Salmon	Cut	Body Measure with Equivalent Width of the Fish Cut
king	fake blanket cut 	*tallineq* (arm length)  *tusneq/tusenrek* (shoulder width) 
chum	fake kite cut 	*iqelqin* (two stretched out hands) 
red	fake earring cut 	*patneq* (hand) 

Model-sized Pieces (page 1)



Model-sized Pieces (page 2)



Activity 10
Feeding Your Family for the Winter

Your students are now prepared to play the fishing game. First, they will go fishing using a card game format. Then they will determine the most efficient way to hang the fish so as to minimize the number of poles used. They will have a target number of fish to aim for, given by real-world examples based on family size and village location. As the culmination of the game, students will put into practice the various methods of problem-solving they have learned throughout the module. Students will use their tables from Activity 9 with symbolic representation and manipulation to solve problems.

The game is designed to honor the Yup'ik values of efficiency and lack of waste. Since the salmon are dying as soon as they are caught in a net, they must be used; wasting fish shows a lack of respect for the cycle of life and the relationship between humans and animals. In fact, it is a common belief among the Yup'ik people that the fish may not return in succeeding years if they are mistreated. There is a limited time to fish and pressure to process the fish quickly and efficiently. Chance plays a large role in how many of each species are caught as do problems that arise during the season. The need to catch enough fish to subsist on is tempered by the desire not to waste any. Lastly, each group must act together as a family in order to survive.

You can either conclude the module with the Fishing Game in this activity or proceed on to Section 4 depending on the developmental level of your students. In Section 4, students investigate patterns in order to determine the algebraic equations used to describe the general pattern, such as $3n + 2$, where n is the number of cubes much like in Section 2 where N was the length of a fist measure, *naparneq*. The module culminates in the quintessential math discovery of how cutting an object increases its surface area but allows volume to remain constant, as is the case with the practice of drying salmon. We encourage you to progress through Section 4 as long as you have some students ready to move into this type of algebraic thinking. Using collaborative group work, students should learn from other students and may be able to engage with the mathematics at various levels, providing an opportunity for learning algebraic thinking that may not have been afforded earlier.

Goals

- To determine how many poles are needed to hang all the fish caught
- To hang all the fish using algebraic thinking to find the best combination of fish on each pole with the least amount of pole space wasted
- To compare the number of fish caught to that needed to survive—given on the family cards

Materials

- Transparency, Alaska's Southwest Region, from Activity 1
- Transparency, George's Yearly Salmon Catch, from Activity 1
- Worksheet, Game Instructions (one per group)
- Worksheet, Game Board (one per group)
- Worksheet, Fish Distribution Table (one for entire class)
- Worksheet, Family Cards, pages 1–3 (one set for entire class; this is enough for nine groups)
- Worksheet, Fish Cards (four copies of king, nine copies of red, eleven copies of chum for entire class)
- Worksheet, Scenario Cards (six copies for entire class)
- Standard 6-sided dice (two per group)
- Blank paper for score keeping and fish hanging
- Math notebooks

Preparation

Copy the transparencies Alaska's Southwest Region and George's Yearly Salmon Catch. Familiarize yourself with the game by reading Game Instructions. Make one copy per group of the Game Instructions and Game Board worksheets. Make four copies of Fish Cards: King; nine copies of Fish Cards: Red; eleven copies of Fish Cards: Chum; and six copies of Scenario Cards. Cut apart and stack in separate piles. Make one copy of the set of three Family Cards. Cut out the strips and place face down in a pile. Place a copy of the Fish Distribution Table next to the family cards. Have game instructions, game boards, family cards, fish and scenario cards, and dice located on the table, ready for students to pick up.

Duration

One class period.

Instructions

1. Explain to students that today they will play a game. In this game, they will be a Yup'ik family fishing and then drying the caught fish. Through

this game, they can practice the modeling and problem-solving skills learned and implemented throughout the module.

2. Show the Alaska's Southwest Region transparency and explain that depending on where a family lives, it may experience radically different fish runs than another family located as close as a mile away. Point to each of the three fishing areas (Bristol Bay, Kuskokwim River, and Lower Yukon River) given in the family cards. Explain that each group in the class will pretend to be a subsistence fishing family located in a specific region as given on the family card. This card will also tell them how many of each type of salmon they will need to catch to survive the winter comfortably. Remind students of the subsistence idea from Activity 1 by showing the transparency of George's Yearly Salmon Catch.

3. **Demonstrate.** Model the game for your class. For the purposes of the demonstration, the class will be one family.

   a. Have a volunteer pick a family card and read aloud the location and target number of fish the family (class) needs to catch.
   b. Have another student read from the Fish Distribution Table to find out how many of each species a run in that location typically yields.
   c. Then have a third volunteer remove from the stacks of fish cards the correct number of species of salmon, as directed by the distribution table. Note that six scenario cards should also be gathered.
   d. The teacher then shuffles the fish cards and the scenario cards together and places them on the game board so that the playing can begin.
   e. Have another volunteer pick a card from the deck. If he has picked a scenario card, have him follow the directions on it. If he has picked a fishing card, have him read the species depicted on it, then roll the dice. The number rolled is the number of that species caught.
   f. On the board, the teacher should keep a running total of how many of each type of salmon the class catches.
   g. Allow ten minutes to play the game. However, if the class feels that they are approaching the target numbers (which they do not want to exceed—part of the game is not wasting salmon), they can quit early.

4. Once the students get the feel for the game, have them break into groups of three to four. Have one group at a time:

   a. Collect two standard 6-sided dice.
   b. Collect the Game Board.
   c. Collect the Game Instructions.

d. Pick one Family Card and read it to identify the location of their fish camp and the target number of fish they need to catch.
e. Using the location on the Family Card, read from the Fish Distribution Table to find out how many fish of each species are typically caught in that location.
f. Collect the correct number of Fish Cards per species as directed by the Fish Distribution Table.
g. Collect six Scenario Cards.

Cultural Note

In the event that a family catches more than it needs, it will share what it is has with those who could not fish or possibly other families that did not do as well. However, the family must still process the fish quickly. If it cannot hang the excess fish to dry, alternative methods of processing fish are used.

5. Have each group shuffle the fish and scenario cards together and place them on the game board. Have each group choose a scorekeeper. Give them ten minutes for the fishing process. They may stop earlier if they feel they've come close enough to their target numbers on their family cards. If a group goes through the entire deck and needs to continue fishing, they can shuffle and reuse the whole set of cards again. Walk around to make certain groups are following instructions and keeping count of the fish properly. Encourage students to talk among themselves to work out any problems and only to ask you if the entire group is confused.

6. Once fishing is complete, explain to students that they need to tally up the numbers of salmon according to each species. Then using the model pieces from Activity 9, pretend to hang each fish on the nine foot fishing pole by writing out algebraic expressions.

   The end result of the game is a series of algebraic expressions, proportions, or equations—one for each pole. For example, four kings hanging on one pole could be expressed:

   4K = 1P   Pole #1:4K   P:K::4:1

   The family that comes closest to its target numbers without going over wins.

7. Encourage students to use the model pieces, their notes, and paper and pencil to organize their catch into a useful format.

8. Once groups are done fishing, have each group compare their actual catch to their target catch. Write the following questions on the board for students to consider. Have them write the answers in their math notebooks.

   a. Will you be able to survive the winter?
   b. How many poles did your group use?
   c. How did you rearrange the position of the fish to effectively use the pole space?

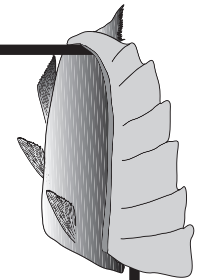
d. How did you justify having different numbers of each type of salmon compared to that needed to survive?

9. Have each group share their results and explain their thinking. Discuss the different representations—if any forms were easier to use, easier to understand, or more efficient than others. Follow this back to the differences seen in different Yup'ik fish camps.

10. **Optional.** If you are concluding the module here, be sure to encourage students to work through the problem-solving approaches with Assessment Activity C and discuss their results. The assessment provides practice in algebraic thinking and manipulation similar to what students did in this activity. Then take time to have your students synthesize all the pieces of information they have gathered throughout the module. They can do this either through journaling in their math notebooks, small group discussion, or in a whole class format. Students should be able to

    - describe the differences between a constant and a variable;
    - graph information using a bar graph and a two-variable scatter plot;
    - provide a convincing argument of the connection between cutting, drying, and preserving food;
    - write equations and expressions using variables with addition, subtraction, and multiplication as a shortcut to adding;
    - manipulate variables, expressions, and equations;
    - explain proportions and provide examples of proportional measures; and
    - use proportional notation to describe relationships.

Game Instructions



1. The first player chooses a card from the fishing deck. If a fish card is picked, the player rolls the pair of six-sided dice to determine how many fish were caught. If a scenario card is selected, the player follows the instructions on it.

2. The scorekeeper marks the number and type of fish caught or lost (from a scenario card) and keeps a running total.

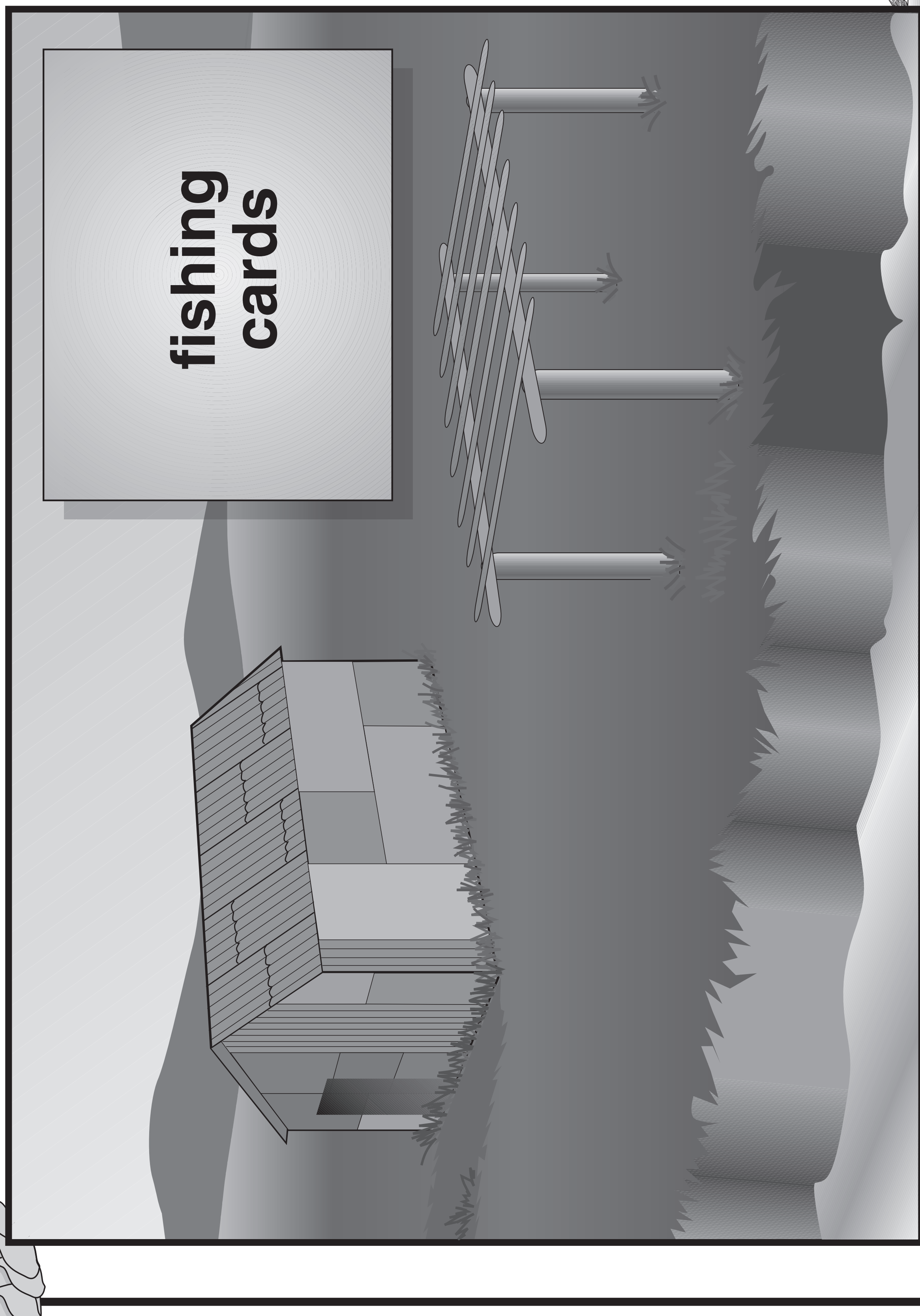
3. Once the player has finished his or her turn, the player to the left goes next.

4. The fishing session will last ten minutes. A group may choose to stop earlier if they feel they have reached their target numbers and all group members agree. If a group chooses to stop early, they can begin to calculate the number of fish to hang on each pole. No group can keep fishing after the allotted time.

5. Once the fishing is completed, each group must determine the most efficient way to hang its catch. Draw the relationships on paper specifying exactly how many of each species of salmon will be hung from each pole as algebraic expressions, ratios, or equations.

6. The group that comes closest to reaching its target numbers of species without going over wins.

Game Board



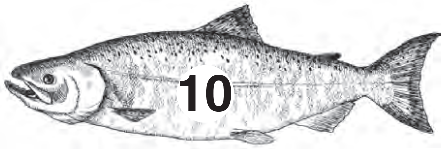
Fish Distribution Table

From the family card, find your region and pick up the number of cards given in the table.

Region	# king salmon	# red salmon	# chum salmon	# scenario cards
Kuskokwim	5	10	9	6
Bristol Bay	2	17	5	6
Lower Yukon	5	0	19	6

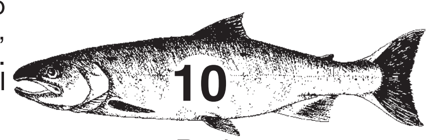
Note: These distributions have been adapted from the fish catch data given by the Alaska Department of Fish and Game, 2002.

Family Cards (page 1)

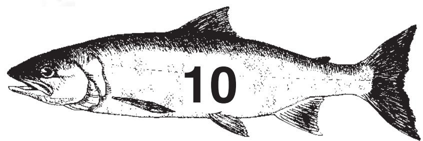


King

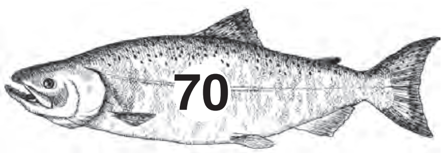
An old couple wants to catch 10 king, 10 chum, and 10 red salmon. They live in the Bristol Bay region.



Red

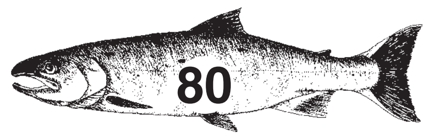


Chum

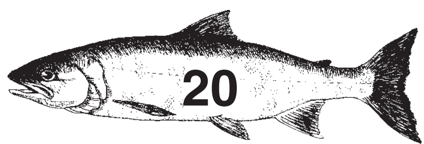


King

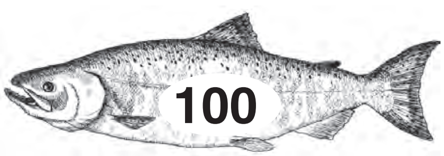
A husband and wife in their 40's have five children. They prefer to catch 70 king, 80 red, and 20 chum. They live in the Bristol Bay region.



Red

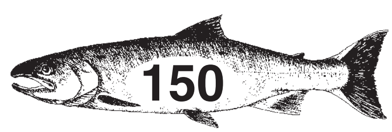


Chum

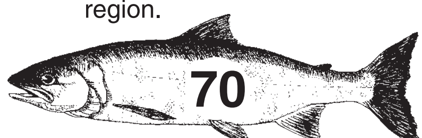


King

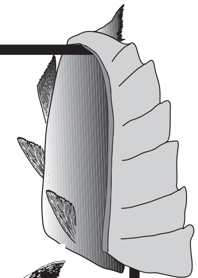
A grandmother is in charge of organizing and catching fish for her three daughters: one daughter has two children and a husband; another has four children all under ten; and another daughter lives alone. They typically need 100 king, 150 red, and 70 chum. They live in the Bristol Bay region.



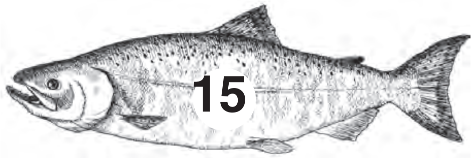
Red



Chum

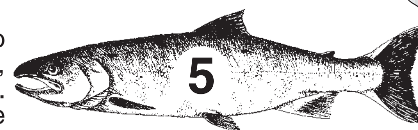


Family Cards (page 2)

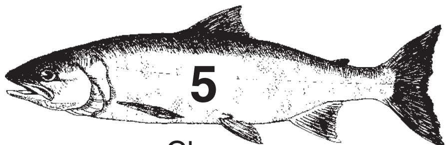


King

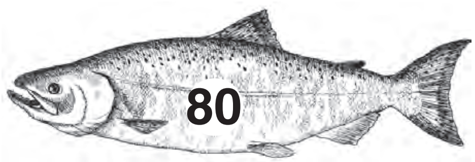
An old couple wants to catch 15 king, 5 chum, and 5 red salmon. They live along the Kuskokwim River.



Red

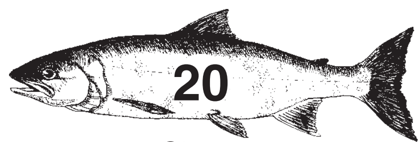


Chum

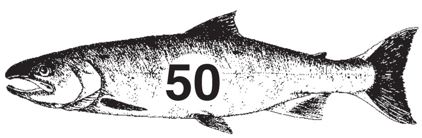


King

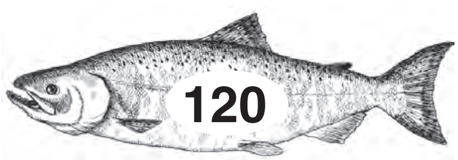
A husband and wife in their 40's have five children. They prefer to catch 80 king, 50 red, and 20 chum. They live along the Kuskokwim River.



Chum

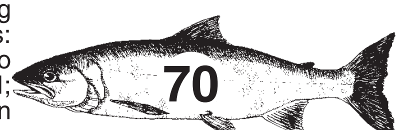


Red

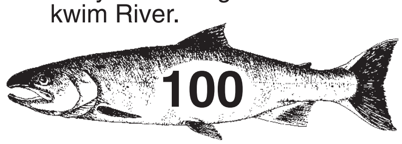


King

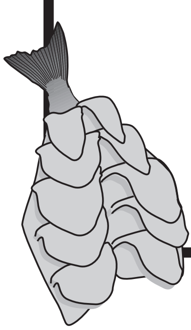
A grandmother is in charge of organizing and catching fish for her three daughters: one daughter has two children and a husband; another has four children all under ten; and another daughter lives alone. They typically need 120 king, 100 red, and 70 chum. They live along the Kuskokwim River.



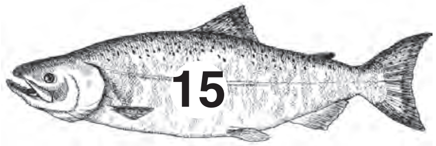
Chum



Red



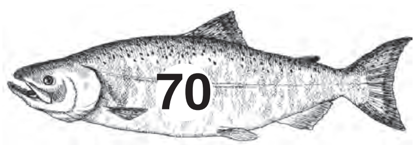
Family Cards (page 3)



King

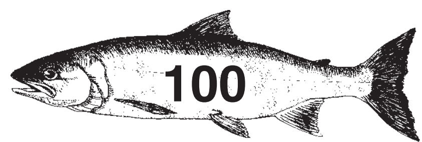
An old couple wants to catch 15 king and 10 chum salmon. They live along the Yukon River.

Chum

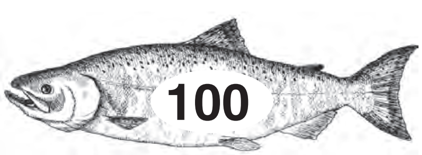


King

A husband and wife in their 40's have five children. They prefer to catch 70 king and 100 chum. They live along the Yukon River.

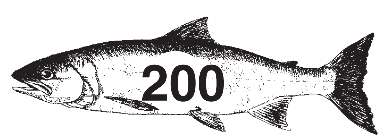


Chum



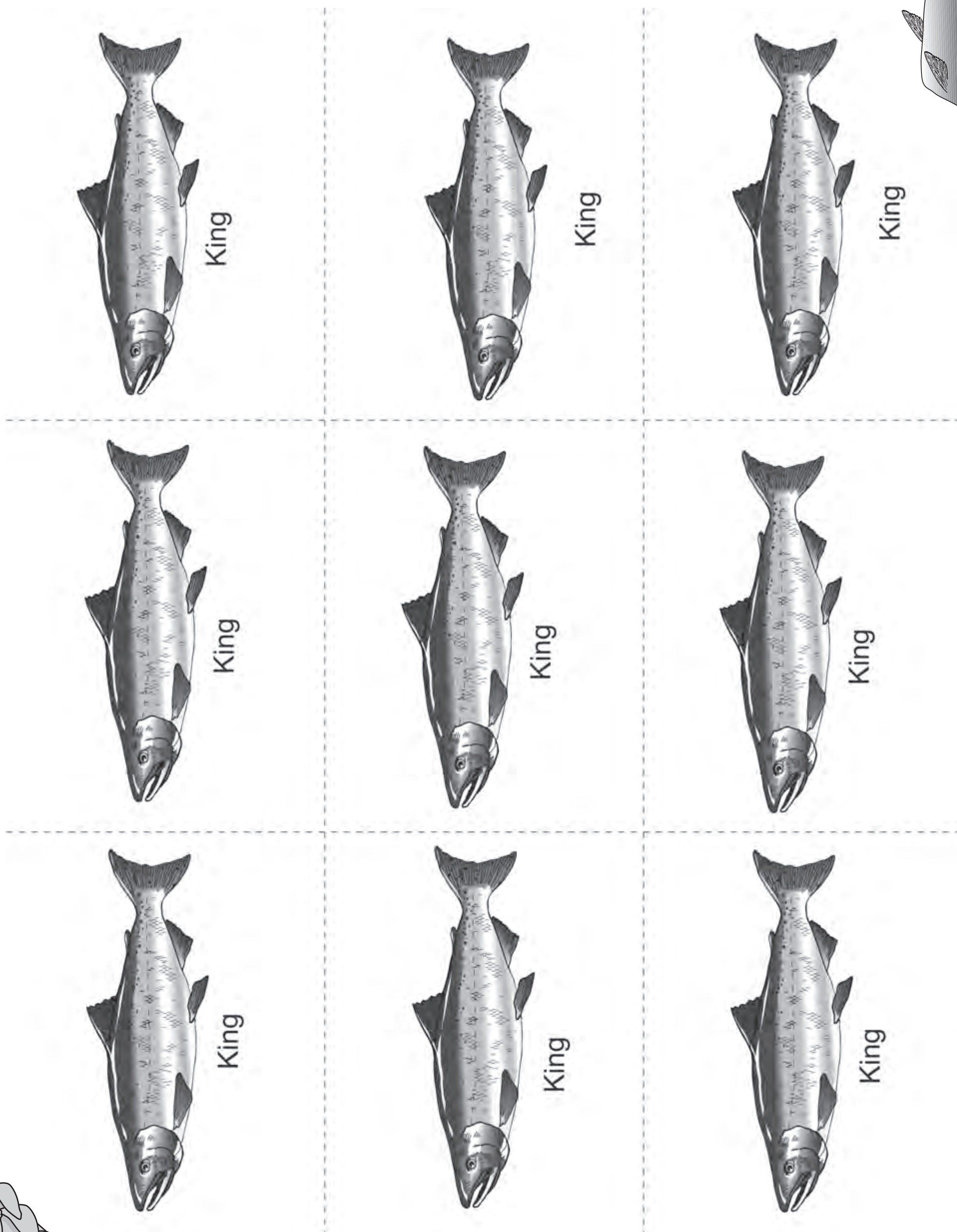
King

A grandmother is in charge of organizing and catching fish for her three daughters: one daughter has two children and a husband; another has four children all under ten; and another daughter lives alone. They typically need 100 king and 200 chum. They live along the Yukon River.



Chum

Fish Cards: King



Fish Cards: Red


Red


Red


Red


Red


Red


Red


Red


Red


Red

Fish Cards: Chum


Chum


Chum


Chum


Chum


Chum


Chum

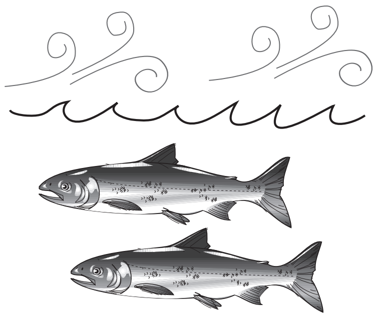

Chum


Chum

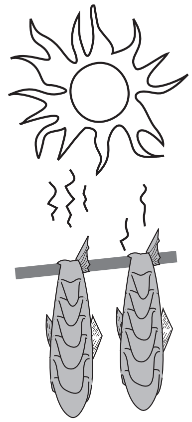

Chum

Scenario Cards

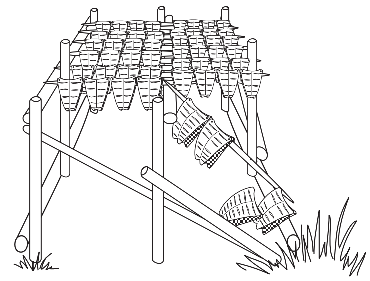
Beautiful day. Winds from the South East. Add two chum salmon to your catch.



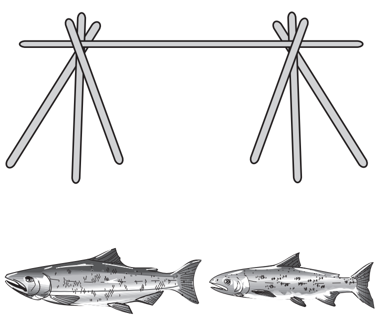
Too much sun on the fish. Lose two smallest salmon.



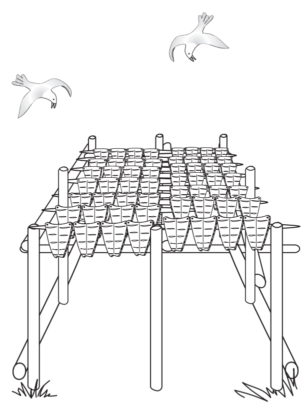
The fish rack collapsed. Lose two big salmon.



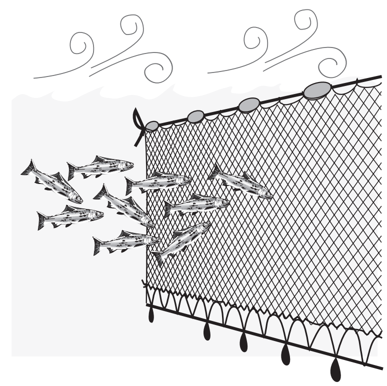
Worked hard and built a temporary fish rack to dry extra salmon. Add two chum and one king to your day's catch.



Seagulls got the fish on the racks. Lose one salmon.

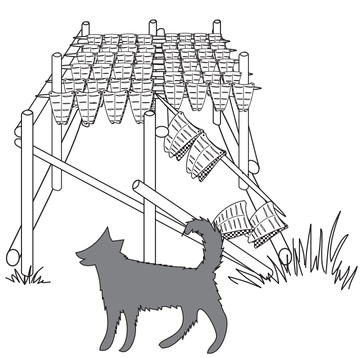


The wind was good and blew the salmon towards the nets. Double your day's catch.

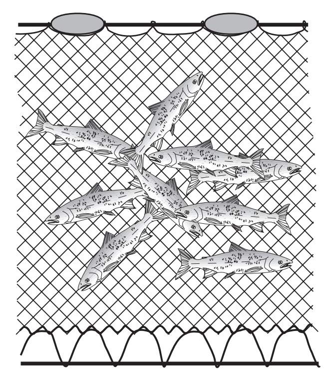


Thick fog. You couldn't fish and lost the day's catch.

One of the dogs knocked down a fish rack. Lose four salmon.



Nothing happened. Just a good day of fishing. Catch stays the same.



Assessment Activity C

As in the previous assessment activities, have students work in groups of three to four and use the problem-solving approach (outlined in the Introduction to the Module) to solve the following problems. Remind them again of how the group problem-solving strategy works—one student takes the leadership role to facilitate discussion about solving the problem, asking such questions as: What is the unknown? What type of problem is it? What information is missing? What do we need to find out? Another student facilitates the planning, asking such questions as: To help us with the problem, should we draw pictures? Should we use symbols? Should we make up a similar but easier problem to solve to help us with this problem? After students have worked individually on their answers, another student takes on the role of the evaluator and compares everyone's answers, leading the students to rework the problem if he/she thinks the answers are not correct. Students should switch roles for each question. It may be necessary to remind them to switch.

For your convenience the problems and solutions are given here. They are organized on a blackline master placed at the end of this activity (page 124) to copy and hand out to students.

Teacher Note

Students may use tables, graphs, or any manipulatives they find helpful as problem-solving tools. However, the goal of this assessment includes writing and working with proper notation. Encourage students to connect whatever tools they use to the notation.

Problem-Solving Challenge 1

William wants to measure rope for the nets he'll need for fishing. He usually creates three different nets—a smaller one for red salmon, a slightly bigger one for chum salmon, and a larger one for king salmon. To build his nets, William is going to use his body measures. He already knows the standard length for most of them; his *yagneq* is 6 feet, his *taluyaneq* is 3 feet, and his *ikusegneq* is 1 feet 6 inches long.

a. For his first net, he needs about 20 feet of rope. Provide three combinations of body measures that he could use to get close to this length. Express these combinations as equations using variables for the body measures. Which combination of measures comes closest to his desired length?

   Solution: For the red salmon net: 3Y = 18' *or* 2Y + 2T + 1I = 19.5' *or* 1Y + 3T + 2I = 18'. The second equation comes closest to the 20 feet length needed.

b. Next, William will need 30 feet of rope for a second net and 40 feet for his third. Provide three combinations of body measures he could use to get close to each of these lengths. Express these combinations as equations using variables for the body measures. Which total measure comes closest to his desired length?

Solution: For the chum net: 5Y = 30' *or* 10T = 30' *or* 20I = 30'. All are exactly the right length.

For the king net: 6Y = 36' *or* 6Y + 1T = 39' *or* 6Y + 1T + 1I = 40.5'. The last one is the closest even though it has gone over the required 40 feet.

c. For William to use any of his net ropes, he needs to place floats every 5 feet. Express the relationship between the length of the rope and the length between floats as a proportion.

Solution: r:f::20:5 *or* r:f::30:5 *or* r:f::40:5

d. How many floats will he need for his 20-foot rope? He uses floats on both ends of the rope. Draw a picture showing where each float is placed on this rope. How many floats does he need for the 30-foot and 40-foot rope? Suppose he had a long-liner rope that was 200 feet long. How many floats will he need?

Solution: For the small net: r:f::20:5

So, that means 4 sections of rope are needed. Since he uses floats on both ends of his rope, 1 more float is needed for a total of 5 floats (Fig. C.1).

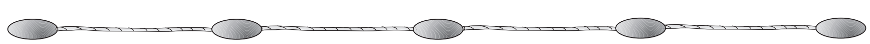


Fig. C.1: Drawing of rope and buoy.

For the chum net: r:f::30:5. So, that means 6 float sections and 7 floats are needed.

For the king net: r:f::40:5. So, that means 8 float sections and 9 floats are needed.

If he has a 200-foot rope, then r:f::200:5 and he will have 40 float sections and 41 floats are needed.

Teacher Note

There's a possibility that families or communities will prepare their nets differently. Keeping this in mind, you may want to adapt the assumptions (i.e., a float at both ends of the net or floats spaced five feet apart) to fit your students' experiences. For example, in the Bristol Bay region people place their floats only two feet apart instead of five. You may want to ask students to recalculate the problems again for this different assumption.

Problem-Solving Challenge 2

Two king salmon weigh the same as two chum and six red salmon. One chum weighs the same as three red salmon. How much does a king salmon weigh relative to a red salmon? How much does a king salmon weigh relative to a chum salmon?

Solution: Two king salmon weigh the same as two chum and six red salmon can be written as $2k = 2c + 6r$.

One chum salmon weighs the same as three red salmon is $c = 3r$.

Therefore:

$2k = 2(3r) + 6r$

$2k = 6r + 6r$

$2k = 12r$

$k = 6r$

Thus, one king salmon weighs the same as six red salmon.

In terms of chum salmon:

$2k = 2c + 3r + 3r$

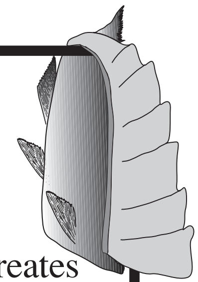
$2k = 2c + 1c + 1c$

$2k = 4c$

$k = 2c$

Therefore, one king salmon weighs the same as two chum salmon.

If it helps to use manipulatives, try using a banana to represent a king salmon, apple for a chum salmon, and cherries for red salmon as shown on page 10 in the Introdution to the Module.



Assessment Activity C

Problem-Solving Challenge 1

William wants to measure rope for the nets he'll need for fishing. He usually creates three different nets—a smaller one for red salmon, a slightly bigger one for chum salmon, and a larger one for king salmon. To build his nets, William is going to use his body measures. He already knows the standard length for most of them; his *yagneq* is 6 feet, his *taluyaneq* is 3 feet, and his *ikusegneq* is 1 feet 6 inches long.

a. For his first net, he needs about 20 feet of rope. Provide three combinations of body measures that he could use to get close to this length. Express these combinations as equations using variables for the body measures. Which combination of measures comes closest to his desired length?

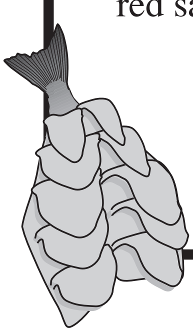
b. Next, William will need 30 feet of rope for a second net and 40 feet for his third. Provide three combinations of body measures he could use to get close to each of these lengths. Express these combinations as equations using variables for the body measures. Which total measure comes closest to his desired length?

c. For William to use any of his net ropes, he needs to place floats every 5 feet. Express the relationship between the length of the rope and the length between floats as a proportion.

d. How many floats will he need for his 20-foot rope? He uses floats on both ends of the rope. Draw a picture showing where each float is placed on this rope. How many floats does he need for the 30-foot and 40-foot rope? Suppose he had a long-liner rope that was 200 feet long. How many floats will he need?

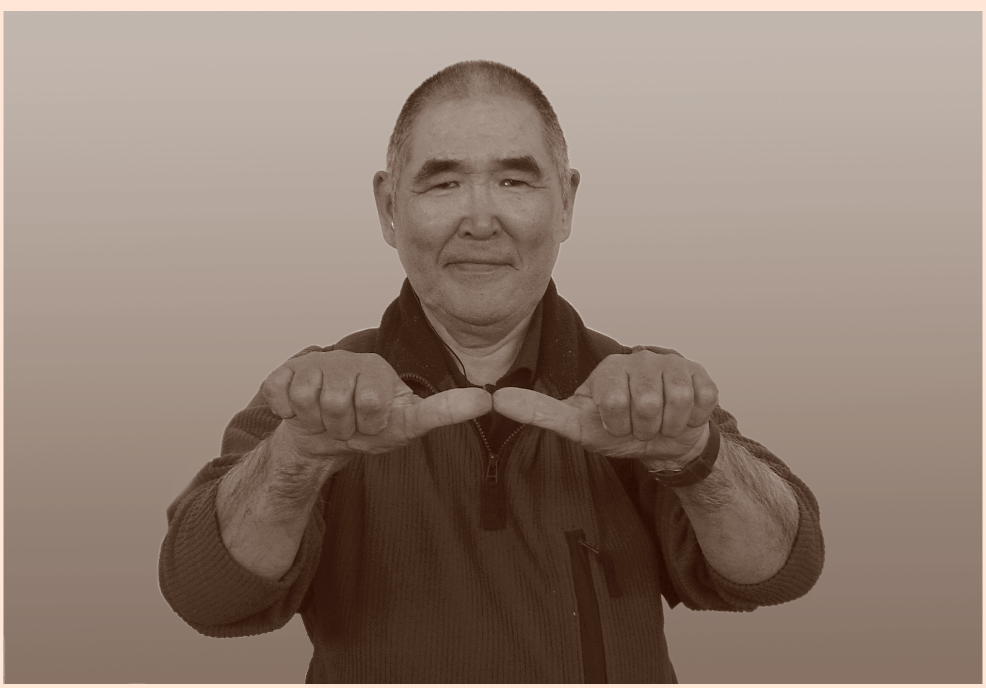
Problem-Solving Challenge 2

Two king salmon weigh the same as two chum and six red salmon. One chum weighs the same as three red salmon. How much does a king salmon weigh relative to a red salmon? How much does a king salmon weigh relative to a chum salmon?



Section 4: Connecting Patterns and Drying Salmon

This section provides a series of activities created to help students fully understand why the Yup'ik people cut the salmon before hanging it to dry. Students investigate the surface area and volume of a cube and a rectangular surface first, to be sure they understand these concepts. Then, students analyze patterns in the data found by considering the surface area and volume of an increasing number of centimeter cubes both separated and connected. Students are provided with multiple ways of viewing the problem so that they can create general patterns describing the change in surface area. The final exploration in this activity leads students to analyze the pattern between separated and connected cubes showing that surface area increases yet volume remains the same. Piecing this information together with the results of the science experiments, students are able to explain the specifics of how cutting salmon to dry is an effective method of food preservation.



Frederick George modeling the malruk naparnerek *measure.*

Activity 11
Measuring the Outside—Notion of Surface Area

For millennia, the Yup'ik people, as well as other people throughout the world, have faced the hazards of food spoilage from bacteria, flies, and maggots. To preserve their food supply for prolonged future use, many cultures have developed techniques to reduce food waste and spoilage, increasing its shelf-life. Some methods include drying or smoking foods, and storing them in cool places, such as in containers submerged in cold water.

For the Yup'ik people, preserving their winter supply of salmon begins as soon as the salmon are caught and removed from the nets. The person preparing the salmon first washes the freshly caught fish in a saltwater brine, then cuts it into various shapes and thicknesses, depending on the species of salmon, its size, and its intended use. One function of cutting the salmon into thinner slabs and strips is to increase the amount of meat exposed to the air (increasing surface area) which facilitates the drying process, which in turn helps to preserve the meat.

In this activity, your students will investigate the concept of surface area, specifically for rectangular solids. They will learn the formula for calculating the surface area of a rectangular solid with dimensions L, H, and W, where L represents the length, H the height, and W the width of the solid. As they study surface area, your students will better understand why the process of fish cutting is needed for food preservation in the Yup'ik culture.

Goals

- To define the general concept of surface area
- To discover the general formula for surface area of a rectangular solid
- To calculate the surface area of one object from three-dimensional and two-dimensional views

Materials

- Poster, Hanging Salmon to Dry
- Poster, Fish Cuts
- Worksheet, Surface Area Grid (one per student)
- Worksheet, Surface Area Worksheet (one per student–optional)
- Scissors (one pair per student)
- Colored markers (three per student)
- Tape—clear
- Math notebooks

Preparation

Cut out a box (to use as an example in Step 2) using the Surface Area Grid worksheet. Label it with a marker using length, L; width, W; and height, H, along each appropriate edge. Tape it together in box form to introduce the lesson. Copy the two worksheets, Surface Area Grid and Surface Area Worksheet, one for each student. Hang up the Hanging Salmon to Dry poster, if it's not still hanging.

Duration

One to three class periods.

Vocabulary

Dimension—the fewest number of independent coordinates required to specify uniquely the points in a space; the range of such a coordinate.

Edge—a one-dimensional intersection (line segment) of two faces in a three-dimensional solid.

Face—a two-dimensional polygonal surface (flat side) of a three-dimensional solid.

Rectangular solid—a three-dimensional shape having six rectangular sides, such as a shoebox.

Side—another term for the face within a three-dimensional solid.

Surface—the outside of a three-dimensional solid; each surface can be called a face or side.

Surface area—the sum of the areas of all the faces, or surfaces, of a solid figure, expressed in square units.

Instructions

1. Introduce today's lesson by briefly explaining to your students the different cuts made for hanging and drying fish. Refer to the Fish Cuts poster to do this. Note that the Appendix contains a step-by-step description of cutting king salmon. Explain to your students that to better understand the mathematics behind drying, they will learn about a concept called surface area. Ask them what they think surface area means (the total area of all the surfaces of a solid added together).

2. Have students work in groups of four. Hand out the scissors, markers, and Surface Area Grid worksheets to each student. Have each student cut out the shape, fold on the dotted lines to make a box, and lightly tape the box together (they will be unfolding their boxes later in the activity as shown in Figure 11.1). Show the box you've prepared before class.

Teacher Note

Depending on your students' familiarity with surface area, you may want to have them draw a shape on the grid first to estimate the surface area of the shape as well as the box. Some teachers had students draw the outline of a fish on the larger face to help students realize this is a model for a fish.

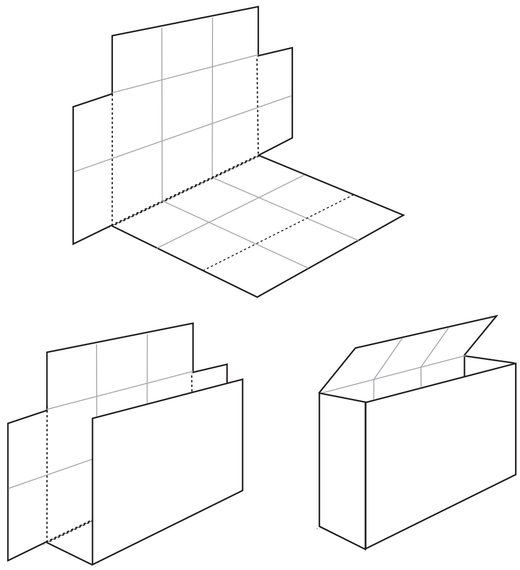


Fig. 11.1: Folding the surface area grid into a box.

3. After students have each completed their boxes, tell them that in mathematics boxes are also called rectangular solids. Explain that each surface is called a surface, face, or side. Point to each face on your model. Have students copy you as you run your finger along each edge and explain that this box has three distinct dimensions: length, width, and height. Explain that we would say this box has the following dimensions: 3 units by 2 units by 1 unit. Ask your students to name other objects or structures that can be described in this way. For example, a book could be described as having the dimensions 12 x 8 x 3 inches.

4. Have the students determine which faces on their boxes are opposite, then untape their boxes, and color the opposite sides with the same color. The box should end up with three colors (see Figure 11.2). Then, using three more colors, have them draw three lines along the appropriate edges to represent each of the three dimensions and label them (L, W, H) (see Figure 11.2). When they have completed this, have them find the surface area. (SA = 22 square units, notice there are 22 of the 1 unit x 1 unit squares.)

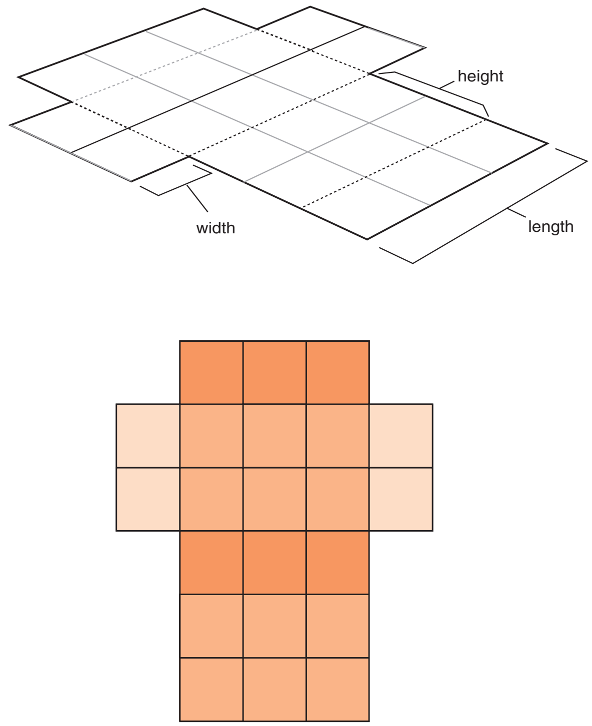
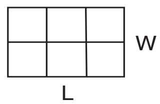
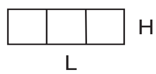
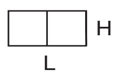


Fig. 11.2: Coloring the sides and labeling the dimensions of the box.

5. Next, have students share with the class what they discovered. They should have found: three sets of opposite faces, three different values representing the dimensions of length, width, and height, and at least one method of calculating the surface area (i.e. counting all the squares or applying the formula). They should discover that the surface area is 22 square units and has the same dimension as area (square units).

6. Encourage your students to organize their thoughts around the following ideas to discover the formula for surface area of a rectangular solid. If you feel it's appropriate, hand out the Surface Area Worksheet. Ask your students: How can we represent this three-dimensional box on a two-dimensional piece of paper? To draw the box, how many faces would we have to draw? (Only three faces need to be drawn.) Have the students draw the three faces inside the boxes given on the worksheet. Have them label the faces with the dimensions L, H, W. Have them finish the worksheet, ending with the formula for surface area (Fig. 11.3).

Surface Area Worksheet

drawing of face with dimensions (L, W, H)	number of similar faces	area of face	area of face in symbols (L, W, H)
	2	3 • 2 = 6	L • W
	2	3 • 1 = 3	L • H
	2	2 • 1 = 2	W • H

Value of Surface Area (using numbers)

SA= 6 + 6 + 3 + 3 + 2 + 2 = 22 units^2

Formula for Surface Area (using symbols)

SA= L•W + L•W + L•H + L•H + W•H + W•H
= 2(LW + LH + WH)

Fig. 11.3: Completed worksheet for surface area of 3 x 2 x 1 box.

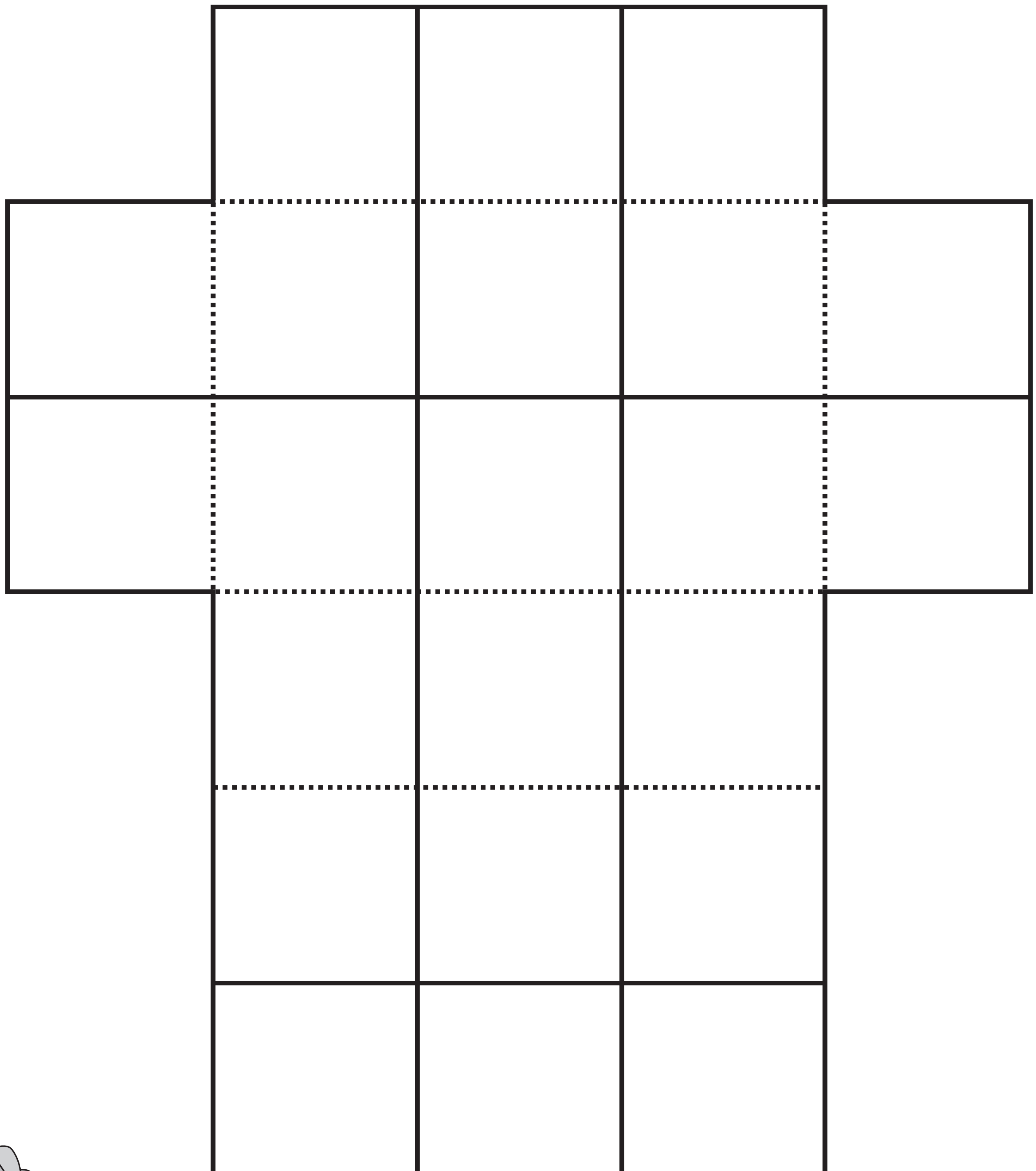
7. Have students share their results and methods with the class. Have them paste or tape the flattened box in their math notebooks and write about what they learned.

8. **Optional.** You may want your students to explore surface area using other shapes as well. Allow them to construct three-dimensional shapes from graph paper to continue this investigation.

Teacher Note

You may end up with a variety of answers among your students. For example, one student might label an edge H, while another might label the same edge L. As long as each student is consistent, that is, as long as he/she doesn't label an edge, L, in one drawing and the same edge, H, in another drawing, then the results hold true. This example is helpful in that we have three different values for the dimensions, so you can make sure the student is correct by checking that L is always the same number (2, for example).

Surface Area Grid



Surface Area Worksheet

drawing of face with dimensions (L, W, H)	number of similar faces	area of face	area of face in symbols (L, W, H)

Value of Surface Area (using numbers)

SA=

Formula for Surface Area (using symbols)

SA=

Activity 12
Outside and Inside—Calculating Surface Area and Volume

Students started investigating surface area in the previous activity. Although surface area helps to explain the drying process, the concept of volume is just as important. The meat of the fish that is consumed can be described mathematically as the volume of the fish. If the cutting process allowed waste of fish or decrease of volume, the Yup'ik people may very well have found alternative ways of processing their fish.

In this activity, students will continue to investigate the concept of surface area while also learning about volume. By considering the surface area and volume together for any sized rectangular solid, students investigate how these measurements relate to each other through the dimensions of the solid. This activity provides the building block needed for students to further examine the relationship between surface area and volume in Activity 13. Optional challenge exercises are provided at the end of the activity and may be used during class or given to students for homework.

Goals

- To model a rectangular solid using centimeter cubes to investigate surface area and volume
- To calculate surface area and volume of different solids
- To introduce the formula for calculating the volume of a rectangular solid

Materials

- Worksheet, Extra Challenges (one per student–optional)
- Surface Area Worksheets from Activity 11 completed
- Teachers only: the paper box from Activity 11
- Butcher paper (optional)
- Centimeter cubes (twelve per student)
- Calculators (optional)
- Math notebooks

Duration

Two to three class periods.

Vocabulary

Model—(n.) a small object, usually built to scale, that represents in detail another, often larger, object.

Rectangular solid—a three-dimensional shape having six rectangular sides, such as a shoebox.

Three-dimensional—having length, width, and height.

Volume—the measure of the amount of space inside of a solid figure (cube, ball, cylinder, or pyramid), expressed in cubic units.

Instructions

1. Review the ideas of the previous lesson by demonstrating with the folded box. Ask students to explain the difference between flattening out the paper and folding it to form the box (flattened is a two-dimensional object; folded is a three-dimensional shape). Ask them: How many centimeter cubes can fit into this box? How do we go about figuring that out? (Fill it up using centimeter cubes, and then count how many fit.) Explain that this measurement is volume and is a concept that can only be used with three-dimensional objects.

2. Distribute twelve centimeter cubes to each student, along with their completed worksheets from the previous lesson. Have them work in groups of four for discussion purposes, although students should work individually with their own cubes.

3. Have the students make models of a rectangular solid of any size using their cubes. (You may need to review the term, 'rectangular solid.') Students within each group may choose to make the same sized box to ease discussion. Have them complete a table (like that in the previous activity) in their math notebooks, including drawings of each face with labels, size of the solid, surface area calculated, and volume. Let them show and explain their results to the class. Be certain that they include the units of square centimeters (sq cm or cm^2) for surface area and cubed centimeters (cm^3) for volume.

4. Discuss the students' answers in class. If students are having trouble recognizing the formula for volume, create a table on the blackboard or butcher paper, using several of the students' boxes as examples. The boxes should be different sizes (1 cm x 1 cm x 12 cm; 1 cm x 2 cm x 6 cm; 1 cm x 3 cm x 4 cm; 2 cm x 2 cm x 3 cm, for example), and the table should include the dimensions of each box. Students may soon recognize that multiplying all three dimensions (L x W x H) produces the rectangular solid's volume.

5. Ask the students to provide some real-world examples of both surface area and volume. Note their comments on the board or on butcher paper.

If students are having trouble, provide some examples. Surface area examples might include determining the exterior surface area of a house for insulation, painting, or installing siding. Increasing the surface area of a radiator or wood stove to increase heat transfer is another example. Some examples of volume might include finding the volume of a water tank for filling and calculating the volume of a proposed slab of concrete to mix the correct amount of gravel and cement needed to build it.

6. Allow the last ten minutes of class for this wrap-up activity and discussion. Ask the students to determine the surface area and volume of a rectangular solid with dimensions 5 cm x 6 cm x 8 cm, using the formulas they learned. Have them share their results.

   Result: SA = 2 x (30 + 40 + 48) = 236 cm^2

   V = 5 x 6 x 8 = 240 cm^3

   Ask them to determine the surface area and volume of an indoor basketball court measuring 500 ft x 256 ft x 27 ft.

   Result: SA = 2 x (13500 + 128000 + 6912) = 296,824 ft^2

   V = 500 x 256 x 27 = 3,456,000 ft^3

Teacher Note

If needed, explain to students that formulas work well in situations that are not easy to replicate physically or with drawings. You may even want to ask them if they would want to draw either of the examples in Step 6 or model them with cubes (hoping they will say "No"). The wrap-up activity includes examples of large items to motivate your students to use formulas instead of building or drawing the objects. Explain that the formula is a major step toward algebraic thinking. This is the power of algebra—being able to begin with an easier problem, develop a formula, or generalization, and apply that formula to a more difficult problem.

Extra Challenges

You may want to provide these extra challenges to your students if there is time in class; they could also take them home. Encourage them to use the problem-solving approach and whatever methods are helpful (including manipulatives), to write down as much of their methodology as they can, and to look for patterns. For your convenience, the problems and solutions are given here. On page 139 they are organized on a blackline master to copy and hand out to students.

You may want to ask students to share solutions for one or all of these with the whole class. If so, discuss the various approaches, ideas, and obstacles students used while solving.

1. Given twenty-seven centimeter cubes, build a structure with the smallest surface area using all the cubes. What is the surface area? What is the volume?

   Solution:

   A 3 by 3 by 3 structure gives the smallest surface area of
   SA = 9(6) = 54 sq cm

   V = 27 cm^3

2. Each of the shapes are constructed from centimeter cubes. Calculate the surface area and volume of the three shapes. Assume the faces not visible are identical to those that are visible. Discuss answers and justify results.

Solution:

(a) SA = 48 cm^2 since each of the six sides has eight squares on it and 6 x 8 = 48.
V = 20 cm^3 because if no blocks were missing the volume would be 27 cm^3, but seven of those cubes are missing—one on each face and the very center cube.

(b) SA = 78 cm^2 since each of the six sides has thirteen squares on it and 6 x 13 = 78.

V = 33 cm^3 because a normal 3 by 3 by 3 cube has a volume of 27 cm^3 and six extra cubes have been attached.

(c) SA = 82 cm^2 because the top and bottom layers have SA = 21 cm^2 and the middle ring has SA = 35 cm^2. Note that since the side of the ring shows only four cubes, there is no extra ring in the back. Thus, an additonal five squares for the middle layer must be added on. So 21 + 21 + 35 + 5 = 82 cm^2.

V = 38 cm^3 because the regular cube has volume of 27 cm^3 and the partial ring has eleven extra cubes.

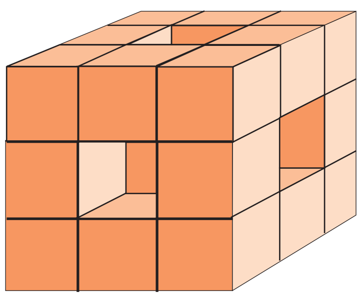
Extra Challenges

1. Given twenty-seven centimeter cubes, build a structure with the smallest surface area using all the cubes. What is the surface area? What is the volume?

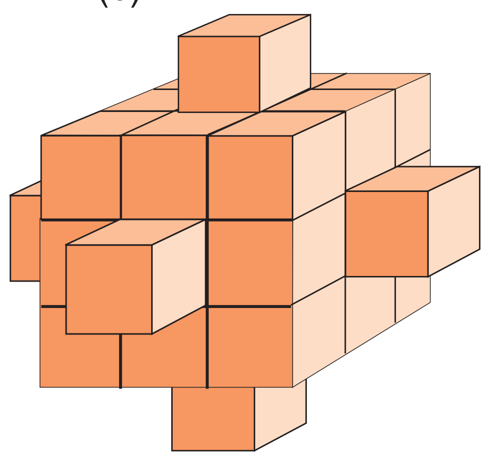
2. Each of the shapes below are constructed from centimeter cubes.

   Calculate the surface area and volume of the three shapes below. Assume the faces not visible are identical to those that are visible. Discuss answers and justify results.

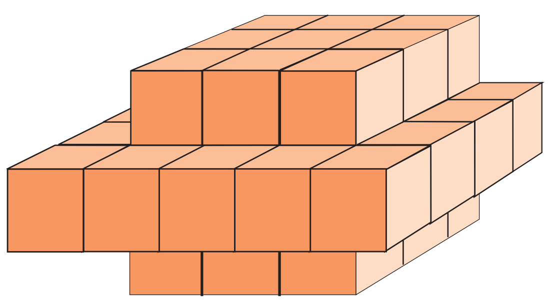
(a)



(b)



(c)



Activity 13
Increasing Surface Area of a Fixed Volume

In this activity, students will investigate how to increase the surface area of solids while allowing the volume to remain constant. This will provide the mathematical foundation needed to understand the process of cutting fish to increase surface area for better drying while not losing any meat. Students model this problem by analyzing the change in surface area and volume for several centimeter cubes separated and connected. This mathemathical modeling of drying salmon challenges students to describe the changes in surface area during the process of separating pictorially, with manipulatives, and through algebraic thinking to obtain the general pattern. As students connect these results with the results of the apple experiment, they learn that the more surface area exposed to the air, the quicker the salmon will dry and avoid spoilage.

At the end of this activity, we have supplied a section titled "Teacher Solutions—Patterns." This section explains several patterns that students could discover when completing the table in this activity. There are many patterns and understanding all of them is challenging. To prepare for this activity, we suggest you read this section first. Then, two extra challenges are given. The second of these includes the story of the mathematician, Carl Friedrich Gauss, who came up with a pattern for adding the numbers one to one hundred as a young schoolboy. You may choose to use this extra challenge as an introduction to this section, additional class work, or homework. If so, we suggest you copy the relevant pages for your students.

Goals

- To discover patterns for surface area and volume given a specific model
- To create a formula representing patterns for surface area and volume
- To create an algebraic formula (general) from the arithmetic approach

Materials

- Worksheet, Surface Area and Volume Patterns (one per student)
- Worksheet, Arithmetical Progressions (one per student, two pages–optional)
- Centimeter cubes (ten per student)
- Math notebooks

Preparation

Read "Teacher Solutions—Patterns" at the end of this activity (page 146). Copy Surface Area and Volume Patterns worksheet, one for each student. Copy Gauss Story worksheet if extra challenge is used.

Duration

Two or three class periods.

Vocabulary

Equivalent—having the same value, measure, or meaning.
Formula—a rule that is expressed using symbols.
Independent variable—the variable in a relationship that can change freely, usually on the *x*-axis.
Pattern—recognizable regularities in situations such as in nature, shapes, events, sets of numbers.
Recursive pattern—a number pattern in which one or more previous terms must be used to determine the next term.

Instructions

1. As an introduction to the activity, explain to the class that they will model the process of cutting salmon using centimeter cubes. Create a 1 x 1 x 4 rectangular solid using four 1-centimeter blocks. Review with the whole class surface area and volume using the 1 x 1 x 4 model. (Students should be able to say that volume measures how much fits inside; surface area measures how much covers the outside. They should also be able to explain how to calculate each.) Break apart the solid into two sections. Ask students how they think the surface area and volume have changed. Note their comments on the board and explain that today's lesson will investigate this question.

2. Divide the students into groups of four. Have them work individually, but encourage them to discuss their ideas and methods with fellow group members.

3. Pass out ten centimeter cubes and the Surface Area and Volume Patterns worksheet to each student.

4. Have the students divide their cubes into two sets, each set consisting of five cubes. Have students determine the surface area and volume of one set, keeping the cubes separate from one another. Then, have them build a long, rectangular solid with the other set (1 cm x 1 cm x 5 cm) and determine the surface area and volume of it (see Figure 13.1).

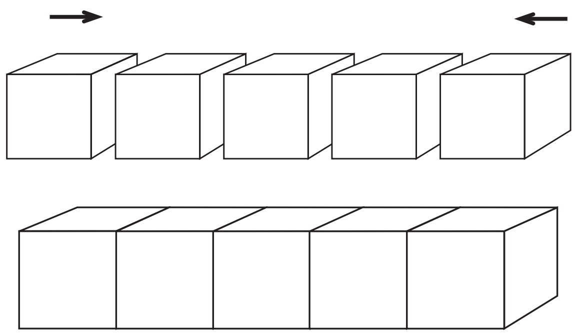


Fig. 13.1: Example of separate and connected cubes.

5. Have students complete the information on their worksheets row by row until the five cubes are connected. They should write any observations inside the given boxes.

6. Once students have completed the table, have them explain their results to the class, beginning with the values within each column and then moving on to the patterns. It is important for each group (student) to teach its method to the class to help develop the thinking of the rest of the class. Students may arrive at their answers by different methods, but still be correct. For example, as "Teacher Solutions—Patterns" notes, three different formulas express the surface area of the connected cubes. Have your students discuss how the various results they arrived at are equivalent. If students are having trouble coming up with formulas, you may want to go over the section, "Teacher Solutions—Patterns," with them.

7. To conclude, encourage students to think about how this relates to cutting salmon for the drying process. To aid the discussion, have them focus on the pattern for "Difference in Connected and Separate." This models the relationship between whole fish and cut-to-dry fish. The volume remains the same when the fish is cut into pieces, but the surface area increases, exposing more meat to the air, thus allowing for quicker drying time. Have students write what they learned about the mathematics of drying salmon in their math notebooks.

Teacher Note

To build mathematical thinking, one needs to look for patterns. Usually at least three terms in a series are needed to create a pattern, although often at least four may be necessary to fully explain the pattern.

Patterns can be found throughout this activity. A student can compare the volume of the separate blocks as they increase in number or the volume of the connected blocks as they increase in number. In addition, a student can compare the surface area of the separate blocks as they increase in number or the surface area of the connected blocks as they increase in number. Finally, a student can compare the surface area and volume of the separate blocks, or the surface area and volume of the connected blocks.

Finding all the patterns could be a difficult part of the lesson, so walk through the classroom and give hints to students if they get stuck or frustrated. Assess when you need to discuss the exercise and when you can allow students to work freely. You may need to interrupt their work several times to keep them on track.

Transitioning from the arithmetic patterns to the general patterns moves students into algebraic thinking essential for higher mathematics.

Teacher Note

Students may notice that the volume for both sets is the same, but the surface area decreases by 2 cm whenever two cubes are connected together. Have students complete the table (see Figure 13.2), starting with determining the surface area and volume of one 1-cm cube. Then they should determine the surface area and volume for two 1-cm cubes, both as separate units and connected together. From the patterns noticed, have them guess at the results for thirty cubes and then for the general pattern with an unknown number of cubes, *n*. Have them look for patterns in the table and write these observations inside the boxes given. For example, they may notice that the surface area of the connected cubes increases by four each time a block is added. By the end of the table, they may be able to express the pattern as a formula (SA of connected cubes = 4n + 2, for example, where n = number of cubes). The section, "Teacher Solutions—Patterns," on page 146 explains in detail how to arrive at this particular formula, as well as others.

If you are teaching in a multi-grade class, you may find that your fifth graders can describe what is going on as + 4 each time. This is appropriate for the younger students, but you will want to motivate sixth and seventh graders to build up to the general pattern of 4n + 2.

Some students will only be able to see the arithmetic process and describe it but not generalize. Use the resources of your students to help spread ways of thinking and viewing to make these transitions.

Surface Area and Volume Patterns

Fill in the values for each case. Use the space below to write out any observations you notice. The end result should be a formula for each pattern based on the number of cubes, n.

# of cubes	surface area separate (cm^2)	volume separate (cm^3)	surface area connected (cm^2)	volume connected (cm^3)
1	6	1	6	1
	Six sides all with area of 1 cm^2, so total is 6 cm^2	Multiply 1 x 1 x 1 to get 1 cm^3	Six sides all with area of 1 square cm, so total is 6 cm^2	same
2	12	2	10	2
	Two cubes each with 6 cm^2 surface area gives 12 cm^2	2 x 1 x 1 gives 2 cm^3	Connecting the two together loses 2 sides from the 12 separate	same as separate
3	18	3	14	3
	Just keep multiplying by six	Same as number of cubes	We lost 2 sides twice now	
4	24	4	18	4
			Seems to always add 4 each time	
5	30	5	22	5
30	180	30	122	30
	= 30 x 6	= 30	= 30 x 4 + 2	= 30
n	6n	n	4n + 2 or 6 + 4(n - 1) or 6n - 2(n - 1) or 2(5) + 4(n - 2)	

Fig. 13.2: Completed worksheet for surface area and volume patterns.

Extra Challenges

The following challenges are optional. You may choose to use them during class or hand them out as homework.

1. Have your students algebraically determine how the various patterns found in the previous exercise are equivalent. For example, have them show algebraically that $6 + 4(n - 1) = 4n + 2$. Other equivalencies exist as well.

2. Either read or distribute the Arithmetical Progressions worksheet containing the story of the famous German mathematician, Carl Friedrich Gauss, and his discovery of the formula for the sum of numbers 1 to n. This can be interesting historical information or used as an additional exploration by passing out circles and having students build the triangular numbers or draw them to better understand the sequence.

Math Note
Teacher Solutions—Patterns

Determining the general patterns relies on a certain way of viewing the problem. Encourage students to use the problem-solving strategies working together in groups to help each other to think algebraically. Solutions and multiple ways of viewing each pattern are provided in this Math Note.

Volume: Separate or Connected

Perhaps the easiest and most likely pattern for students to notice first is the one for volume. They should notice that the volume is the same for the same number of cubes, whether the cubes are connected or not. They should also notice that the value for volume is the same as the number of cubes.

Discuss how to write the formula for this. Explain that the formula is written in terms of the number of cubes, since they are the element that can be changed. In algebra, what can be changed is called the "independent variable." Tell your students to use a symbol that might help them remember what the variable represents, such as "n" for number. Result: V = n where n = number of cubes.

Surface Area of Separate Cubes

For the separate cubes, students should notice that the surface area is six times the number of cubes, since SA = 6 cm for one cube, SA = 12 cm for two cubes, and so on. Result: SA = 6n, where n=number of cubes. Students should be able to recognize this pattern and write this formula.

Students may conceive of this differently, such as adding six each time. On the board, you can show them how to determine the pattern by using a table as shown in Figure 13.3. Start by writing the headings of the columns and filling in the data for perhaps the first two rows. Then allow the students to continue independently.

number of cubes	SA separate	pattern	pattern for formula
1	6	= 6	= 6 x 1
2	12	= 6 + 6	= 6 x 2
3	18	= 6 + 6 + 6	= 6 x 3
4	24	= 6 + 6 + 6 + 6	= 6 x 4
5	30	= 6 + 6 + 6 + 6 + 6	= 6 x 5

Fig. 13.3: Example of method to determine pattern for surface area of separate cubes.

Surface Area of Connected Cubes

Under the "connected" category, the pattern might be recognizable, but it may be more difficult to write in a formula. Note that students may discover a different but equivalent formula for each pattern. Some students may consider only the numbers themselves, whereas others may look to the physical cubes to help determine a pattern. All approaches are useful and promote good algebraic thinking.

If students are able to discover the formula, have them present their results and thinking to the class. Otherwise, you can help them by writing a table on the board with the appropriate headings (as shown in Figures 13.4–13.9 in Alternatives 1–4) and filling in the data for the first two or three rows. Allow students to continue independently. They may complete the table correctly, but still may not be able to make the leap from the numbers to composing an actual formula.

Alternative 1

To obtain surface area, we see that we add four each time a cube is added to the solid (for one cube SA = 6 cm^2, for two cubes SA = 10 cm^2, or 6 + 4; for three cubes SA = 14 cm^2, or 10 + 4, or 6 + 4 + 4). Remember, to discover a pattern, you need to consider at least three items in the sequence and often more items are better. Let's look at four cubes as well: SA = 18 cm^2, = or 6 + 4 + 4 + 4. Now we notice that the pattern always starts with 6, and more 4s are added each time. Notice that for two cubes we add 4 only once, for three cubes we add 4 twice, for four cubes we add 4 three times. So, we always add 4 one time less than the number of cubes. Result: SA = 6 + 4(n-1), where n = number of cubes. Note that this formula can be simplified algebraically to SA = 6 + 4n – 4 = 4n + 2.

number of cubes	SA connected	pattern	pattern for formula
1	6	= 6 + 0	= 6 + (4 x 0)
2	10	= 6 + 4	= 6 + (4 x 1)
3	14	= 6 + 4 + 4	= 6 + (4 x 2)
4	18	= 6 + 4 + 4 + 4	= 6 + (4 x 3)
5	22	= 6 + 4 + 4 + 4 + 4	= 6 + (4 x 4)

Fig. 13.4: Example of method to determine pattern for surface area of connected cubes.

Alternative 2

Since there is a change of four each time, begin with the formula 4n. For the first cube, we are off by 2 (since 4 x 1 = 4 and the surface area is 6 cm^2) If we look at two cubes, 4(2) = 8, we are off by 2 again (since the surface area of two cubes connected is 10 cm^2). It looks like the formula is 4n + 2. Does it work for three cubes? 4(3) + 2 = 14. It does work.

number of cubes	SA connected	change of 4 ⟶ 4n	add 2 ⟶ 4n + 2
1	6	4	6
2	10	8	10
3	14	12	14
4	18	16	18
5	22	20	22

Fig. 13.5: Example of another method to determine pattern for surface area of connected cubes.

It may help to work with the cubes. Picture all five cubes connected; each center cube has four faces, but the end cubes each have one extra face at the ends as shown in Figure 13.6. The formula then is SA = 4n + 1 + 1 = 4n + 2.

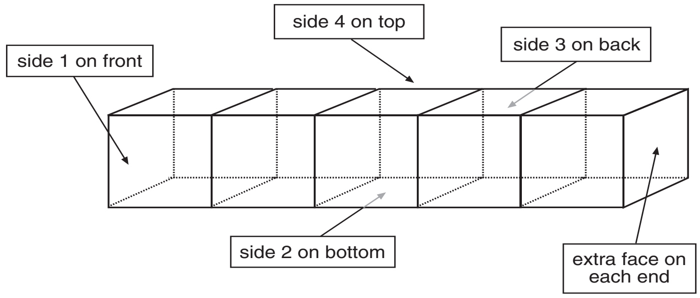


Fig. 13.6: Each center cube has four faces, but the end cubes have one extra face.

Alternative 3

It may be helpful for some students to watch how the surface area changes as the cubes are connected as shown in the table in Figure 13.7. Each cube separately has six faces, so begin with 6n. Every time two cubes are connected, two faces are lost—one for each cube (see Figure 13.8).

number of cubes	SA connected	pattern	pattern for formula
1	6	= 6 - 0	= (6 x 1) - (2 x 0)
2	10	= 12 - 2	= (6 x 2) - (2 x 1)
3	14	= 18 - 4	= (6 x 3) - (2 x 2)
4	18	= 24 - 6	= (6 x 4) - (2 x 3)
5	22	= 30 - 8	= (6 x 5) - (2 x 4)

Fig. 13.7: Alternative approach to finding a pattern for the surface area of connected cubes.

Therefore, we have the pattern 6n – 2L, where L is the number of links. When there are two cubes, there is only one link; three cubes have two links, and so on. The number of links is always one less than the number of cubes, so L=n-1. This is also seen in the table since the value multiplied by 2 is always 1 less than the number of cubes. The final formula would be SA = 6n – 2(n – 1). Note that this formula can be simplified algebraically to SA = 6n – 2n + 2 = 4n + 2.

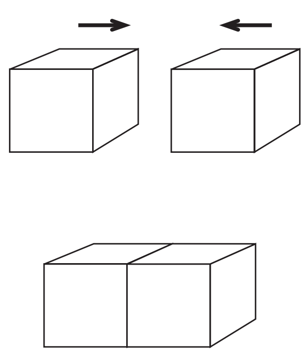


Fig. 13.8: Connecting two cubes causes two faces to be lost.

Alternative 4

Some students may prefer to think about the end cubes and the middle cubes separately. For a length of three cubes together, the two ends each have five faces. The middle cube has only four faces. Thus, SA = 5 + 5 + 4 = 14 cm^2. For four cubes, the two ends still have five faces. Since there are two middle cubes with only four faces, the SA = 5 + 5 + 4 + 4 = 18 cm^2. For five connected cubes, SA = 5 + 5 + 4 + 4 + 4 = 22 cm^2, since there are two end cubes and three middle cubes as shown in Figure 13.9. If we continue with this pattern, we see that it always begins with 5 + 5, and then adds 4 for each middle cube. Result: SA = 10 + 4n where n = the number of middle cubes. This could also be written as SA = 10 + 4(n – 2) where n = number of all cubes, since there are always two middle cubes less than total cubes. Note that this final formula can be simplified algebraically to SA = 10 + 4n – 8 = 4n + 2.

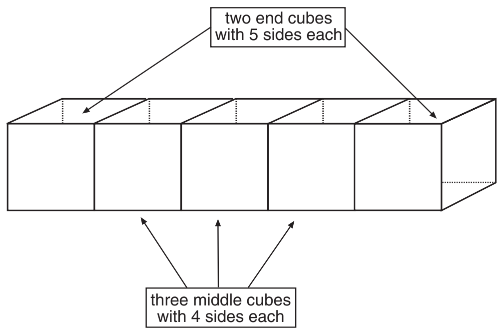


Fig. 13.9: Another view of the surface area of the five connected cubes.

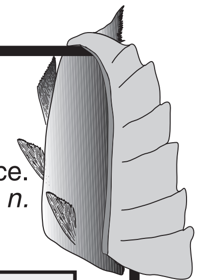
Change in Surface Area from Connected to Separate Cubes

Your students may notice the pattern of how much surface area is gained from keeping cubes separate rather than connecting them. If no one mentions this difference, you should direct their attention to it, as it demonstrates the reason for cutting salmon. Remember that they are looking for a pattern, and therefore should consider at least three terms in the sequence. Again, if your students have difficulty determining the pattern, creating a table will be helpful. On the board, write the headings as shown in the table in Figure 13.10 and allow students to proceed on their own. Result: Gain = 2(n – 1).

number of cubes	difference in SA	pattern for formula
1	0	= 0 x 2
2	2	= 1 x 2
3	4	= 2 x 2
4	6	= 3 x 2
5	8	= 4 x 2

Fig. 13.10: Table of differences in surface area between connected and separated cubes.

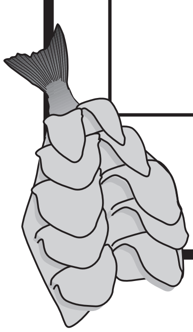
Remind your students that this is the concept employed when cutting salmon for drying; more surface area is exposed to the air, and thus the fish dries more quickly. However, no meat is lost, since, as noted earlier, the volume of connected and separate objects remains the same.



Surface Area and Volume Patterns

Fill in the values for each case. Use the space below to write out any observations you notice. The end result should be a formula for each pattern based on the number of cubes, *n.*

# of cubes	surface area separate (cm^2)	volume separate (cm^3)	surface area connected (cm^2)	volume connected (cm^3)
1				
2				
3				
4				
5				
30				
n				



Arithmetical Progressions, Page 1

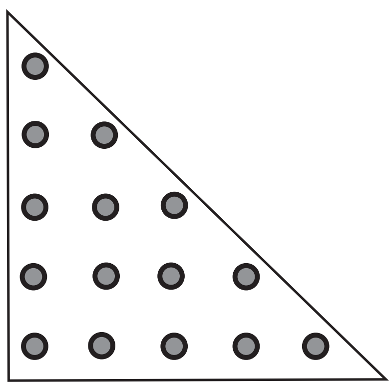
Carl Friedrich Gauss: April 30, 1777– February 23, 1855, Germany

Gauss was one of the greatest mathematicians of his time. It is said that at the age of eight he astounded his school teacher when he was asked to add up all the numbers from one to a hundred and gave the correct answer within a few seconds.

Gauss did not actually add up all the numbers in his head, but instead relied on a formula to quickly arrive at the solution. Before seeing the formula for the sum of the first *n* numbers, let's take a look at the first few terms in this sequence.

n	adding	sequence
1	1	1
2	1+2	3
3	1+2+3	6
4	1+2+3+4	10
5	1+2+3+4+5	15
6	1+2+3+4+5+6	21
7	1+2+3+4+5+6+7	28

The terms in the sequence: 1, 3, 6, 10, 15 . . . were studied by the ancient Greeks. The Greek mathematicians were interested in the ways in which numbers could be represented as shapes. These numbers are called triangular numbers because they can be arranged as a triangle as shown below.



This figure shows the first five triangular numbers. If you begin at the top and go row by row, each row adds one more dot than the previous row. So, the fourth row has four dots and the fifth row has five dots. The *n*th row will have *n* dots.

If we have a triangular number, such as the fifth triangular number, we could get the 6th triangular number by adding 6. This method is called the recursive method. To find the *n*th number you need to know the number before it, or the ($n - 1$) number. A general formula would allow us to find the nth number right away, as Gauss did, to find the answer when *n* equals 100 (the 100th triangular number).

One way to find a general formula is to look at the problem for a small number in a different way. Let's use $n = 10$ and consider the following setup. List out the sum of numbers from 1 to 10 in two rows, but put the second row in opposite order. Instead of adding across the numbers, add them up and down.

This gives us:

$$\begin{array}{rrrrrrrrrrrrrrrrrrrrr}
 & 1 & + & 2 & + & 3 & + & 4 & + & 5 & + & 6 & + & 7 & + & 8 & + & 9 & + & 10 \\
+ & 10 & + & 9 & + & 8 & + & 7 & + & 6 & + & 5 & + & 4 & + & 3 & + & 2 & + & 1 \\
\hline
 & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11 & + & 11
\end{array}$$

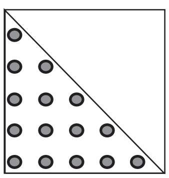
Now we have the sum from 1 to 10 listed twice with each column equaling 11. There are two important aspects to notice. First, 11 can be written as (10 + 1) which uses the number we started with ($n = 10$). Also, there are 10 of those 11s. So, instead of adding them together we can just use the result 10 x 11 = 110. In symbols, this would be written as $n \times (n + 1)$.

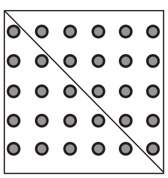
Arithmetical Progressions, Page 2

Remember, the sum of 1 to 10 was listed twice, so to get the actual sum of the first 10 digits we need to divide by 2, giving us 110/2 = 55. In symbols, the final result would be written as $n \times (n + 1) / 2$.

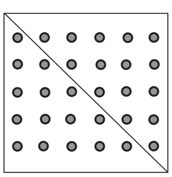
So $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 10 \times (10 + 1) / 2 = 55$.

This result can also be arrived at geometrically, using the idea of triangular numbers. The figure below represents a general triangular number.





We know from geometry that a triangle is half of a rectangle. If you attach another equal triangular number, it will form a rectangle with dimensions n by $(n + 1)$.



Using the formula for area of a rectangle, we can count $n \times (n + 1)$ dots inside the rectangle. Since the rectangle is composed of two of our triangles, we again divide by 2 to find the same formula: $n \times (n + 1) / 2$.

Appendix: Cutting King Salmon

by Nastasia Wahlberg

This appendix provides a step-by-step description of how the Yup'ik people from the Kuskokwim region process king salmon for drying.

Photo of a young king salmon drying in a fake kite cut.

King Salmon Cuts

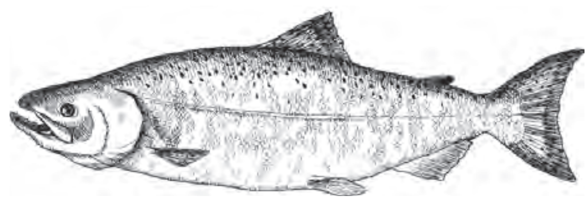


Fig. X.1: King salmon.

Kings *(Taryaqvak)*

Preparation

When a fairly large king salmon is caught, you can compare the size of the fish to your own body, beginning from the shoulders and going down to just above or below the knees. Some kings are even larger. The younger ones reach down to the hips or above the knees. You can compare their size to reds or chums. You can identify the maskings by the dark spots on their back.

After the kings are caught, people bring them up to the fish camp or to a place where it is convenient to do the cutting and curing (*segvik*—place to cut fish). The kings are put into large, homemade, rectangular wooden boxes, cut-out 50-gallon drums, and/or they can be placed into the plastic bins that commercial fishermen use nowadays. Once the fish are all brought up to the *segvik*, water is poured over the fish in the drums or bins, and they are covered to prevent spoilage from the sun's heat, and to hinder flies from laying their eggs on them.

To prepare the cutting area, one has to sharpen the women's cutting knives *(uluat*, plural for 3 or more *uluaq)* and put a place mat on the table to prevent the fish from sliding. The mat makes it easier to manipulate the fillets and other cuts. Old cut-out rugs, gunny sacks, and canvas cloth tarps have been used as mats. Three containers are prepared, usually buckets or old wash tubs. One is for discarding the guts. The second is to salt the fish. The third is for the heads .

A wash pan of some sort is usually placed next to the cutter to occasionally wash the accumulated slime off of the *uluaq* and the hands. A large rock or a knife sharpener is also laid on the table for occasional sharpening. Then, of course, having a radio to pass the time makes the work much more pleasant. Usually people have mosquito repellent on to prevent being bothered while cutting. They also use a waterproof apron if possible, or rain pants and rubber boots to prevent fish slime from getting all over their clothes.

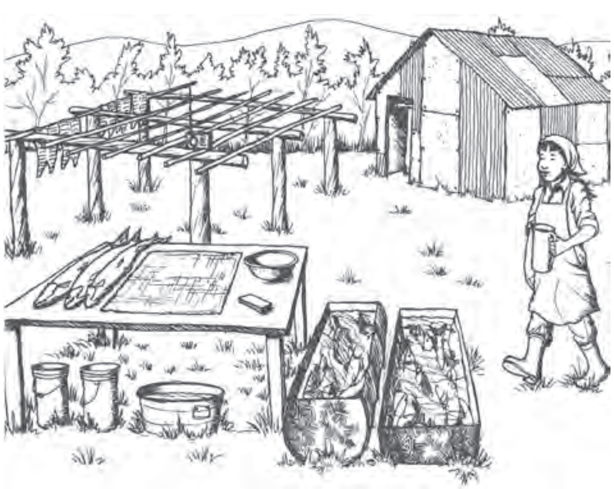


Fig. X.2: Fish camp scene.

The following instructions are provided as an explanation of how to handle all the various parts of a king salmon, walking through a four-phase process.

Phase I: Beheading and Cleaning the Salmon

A. First, place the salmon on the table and cut each fish's head off beginning at the connected gill in the front neck region (isthmus).

1. Tilt the head back and make a cut about 1/3-inch long between the adjacent neck bones. See Figure X.3.
2. Next, cut around the gills.
3. Make one long horizontal cut along the belly, from the isthmus all the way down to the anus.
4. Finally, continue the cut around the anus.

B. Throw the cut head into an old wash tub or other container, if possible. The heads are normally saved and frozen, cooked fresh, dried, cooked in the earth, or salt cured. Fish heads are also great for dog food. Using separate receptacles for the heads and the guts also prevents a single receptacle from filling up too quickly. When a family feels they have an excess amount of fish, the heads, along with the guts, are often thrown into a decomposition pit to use for fertilizer, so the next spring the garden will grow healthy vegetables.

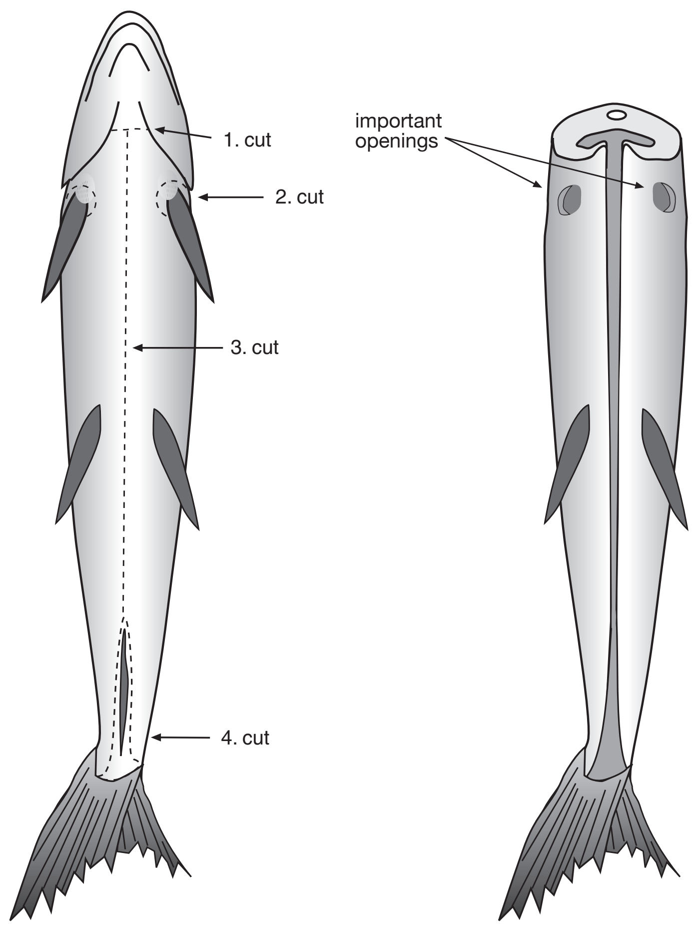


Fig. X.3: Beheading and cleaning a salmon.

C. Next, cut out the guts with the *uluaq* and discard them into the bucket you designated for the guts. If desired, separate out the salmon eggs for food or fish bait. You can dry or freeze them. Some people use them in soups or salads.

Some cutters, especially seasoned workers (*segta; seg-* is a stem meaning "to cut fish for drying"), like to make preparatory cuts during the first phase. Preparatory cuts are optional and can be done later when filleting. If you choose to make preparatory cuts, take off the two front fins. Later, use the two important openings (Fig.X.3) to handle the cut fish that forms what is called a blanket cut while hanging. Use the two openings to position the blanket on the rack while drying the skin. Those two openings are also useful to pair the kings by tying two blankets together through those openings once the meat is dry enough. The meat in the tail region is filleted as shown in Figure X.4.

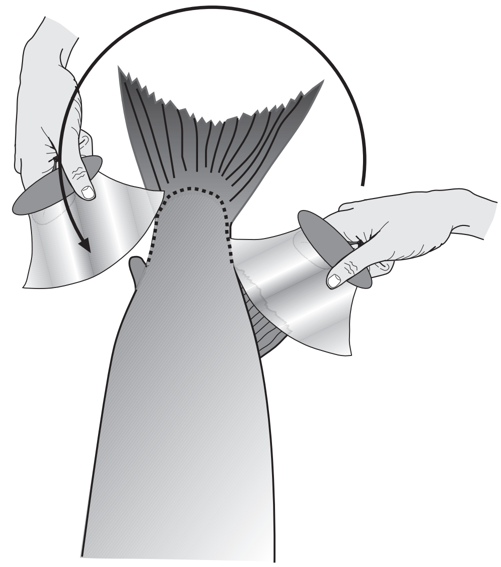


Fig. X.4: Cutting around the tail region, which is only done with mature king salmon.

D. When you begin filleting in the tail area, begin cutting on each side of the backbone. With the *uluaq,* make a smooth slice along each side of the spine beginning from the anus and continuing all the way around to the back of the tail. The intent is to cut all the meat away from the bone. Once both sides are cut, the spine and the bones should be free and the tail will look like a rounded "W."

Next, change the position of the fish so that the dorsal fin on the back is accessible to you. Cut around the dorsal fin in an ocular fashion as shown in Figure X.5.

Once you are done, place the fish back into the water-filled tank or bin. After you have completed beheading and cleaning all the fish, you are ready for Phase II.

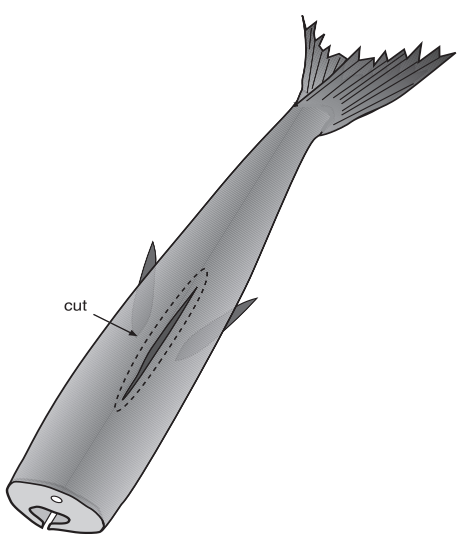


Fig. X.5: Cutting around the dorsal fin.

Phase II: Making the "Blanket" Cuts

A. Clean the cutting area of remains and wash the *uluaq*. Scrape and clean the slime and blood off the mats; otherwise the fish will slide around and make it difficult for filleting. Place as many fish as you would like on the table. You can place them there one at a time; however, when a person is cutting masses of fish, placing a few fish on the table at a time speeds up the process, as there is less labor and interruption.

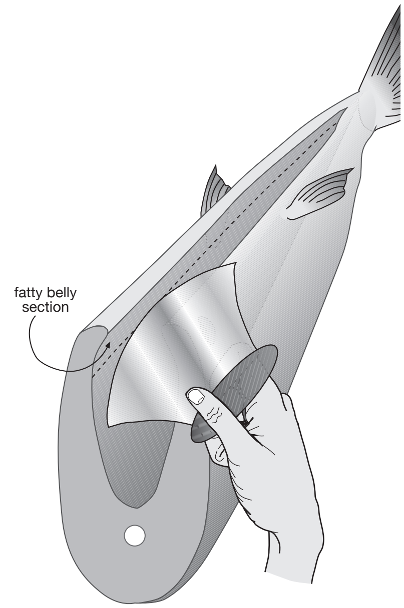


Fig. X.6: First cut.

B. Make the first cut into the meat at the neck bone area, close to where you made the two holes earlier. Cut into the thin film of membrane between the fatty belly section from the tip of the delicate ribs of the salmon to just beyond the middle fin on the opened belly. Next, make several small, approximately 1/8-inch horizontal cuts across the fatty belly portion of the slab. Make about 1/2- to 1/3-inch slanted cuts all the way up to the mid-fin on one side of the blanket. Then change the angle of the *uluaq* from that point and start slanting toward the neck area and cut across the same way. Try not to cut too hard through the skin so that when the blanket dries the meat will still cling to the skin. See Figure X.6.

This entry opens the flesh and allows the cutter to begin making a fillet as evenly as possible, separating the backbone from the skin side. Once all the cutting is done, the meat with the skin is left remaining.

The *uluaq* should be held evenly. In the beginning of this portion of the fillet, use your left hand to lift the opening of the meat to help regulate the cuts being made by the *uluaq* on the right hand. Move the *uluaq* back and forth, from neck to tail. Work gently at first, because the belly is thin compared to the rest of the fish. At the midway point, when you reach the thicker portion, use your left hand on the skin to sense and guide your cuts evenly. At this midway point, separate the bone from the flesh by making long smooth movements back and forth until the slab from the left side is completely off the backbone. See Figure X.7.

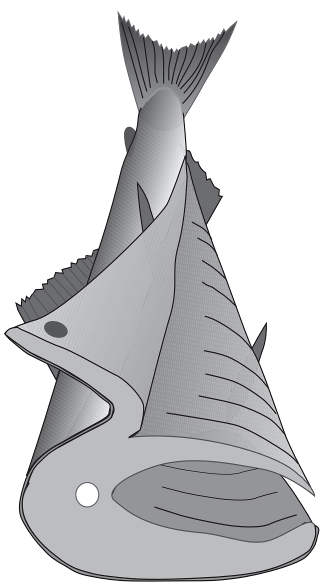


Fig. X.7: Separating the backbone from the skin side.

C. Then, make the same cuts on the other side of the meat portion. Most of the meat will need to stay even with approximately ¾ inches of meat on the skin. Slab thickness will vary because each cutter has her own preference. Make the cuts in a back and forth curving motion, especially toward the central back, where the meat becomes thicker. Continue cutting until half of the slab is filleted.

The slanted cuts are important because when the whole finished blanket cut is hanging on the fish rack, the cuts will separate, allowing for easier drying. Proceed onto the other side of the slab and make the same kind of cuts until both sides have the same precision cuts (Fig. X.8).

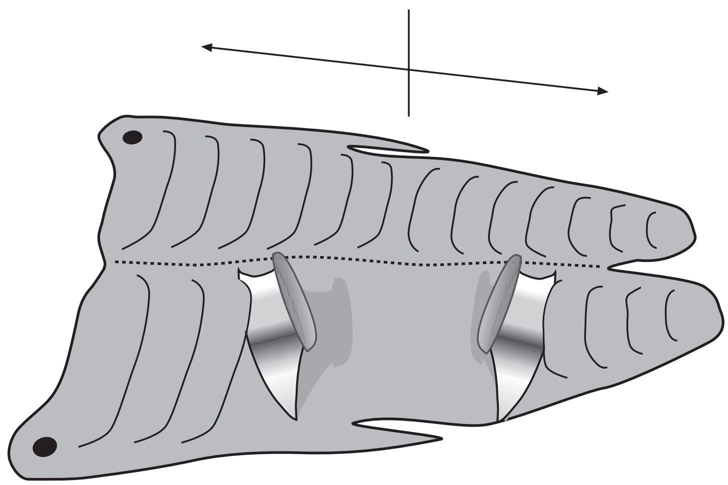


Fig. X.8: Slanted cuts.

D. Put the blanket into a salted water solution (brine) for a specified short period. Most cutters have their own recipes for brine. The salt helps to discourage blowflies from laying their eggs. The brine is absorbed into the salmon and creates a salted seasoning flavor. The brine also takes out any excess slime accumulation and cleans the king.

Then the king salmon blanket can be hung like it was a blanket draped over in a fold on a fish rack (Fig. X.9). Check the salmon daily and change the position of the blankets until they are dry.

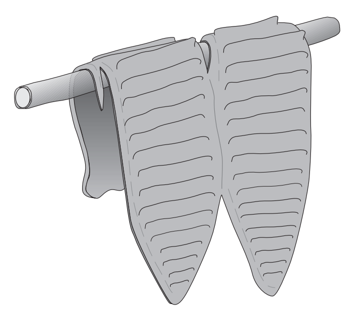


Fig. X.9: King salmon in a blanket cut hanging on a fish rack pole.

E. When you first hang the king blankets, they are wet; so, hang each one at least three fingers apart (*pingayuneq*). The space between each rack should be about the length of a forearm (*ikusegneq*) or somewhere within that range, so that air can flow between the fish and dry them.

F. Depending on the width of the space between the two posts and the length of the fish rack, about three to five blankets can be hung in one row. Soak each blanket as it is finished and hang it to dry.

Phase III: Backbone Cuts

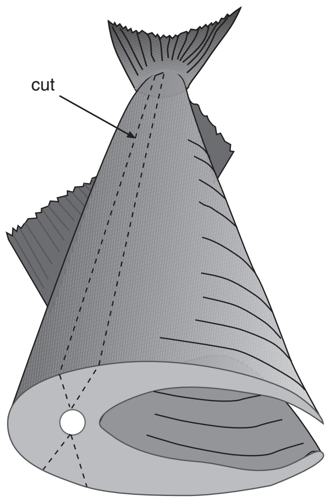


Fig. X.10: Cutting the backbone and meat out.

A. Since the king backbone has a lot of meat, remove a slab from along the spine on both sides. Make a V-shaped split from the neck spine up to the thinnest part of the meat (Fig. X.10). This piece of meat is called the *kiarneq*—an unsalted strip or fillet of fish flesh without skin that is cut from along the backbone and hung to dry. Lay the *kiarneq* aside to hang later.

Cut deep curved slanted slices that are angled from the tail and down toward the neck region. As you make each cut, two curved flaps will develop on the thin rib frame of the stomach and prominent backbone. The curved flaps will become noticeable when the backbones are pared and hung on the crossbeams of the fish rack as shown in Figure X.11.

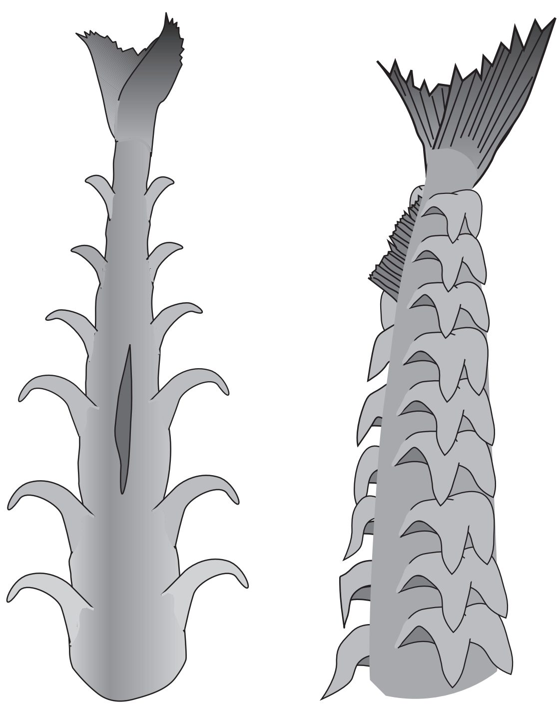


Fig. X.11: Example of completed backbone cuts from two views.

B. Hang all the *kiarneret* (plural) either on the fish rack or on the crossbeam (Fig. X.12). If it seems like the slabs will not hang well, the crossbeams will be a better location for them. These are the first fresh-dried fish of the season that are eaten. You can put them into a brine, if you desire.

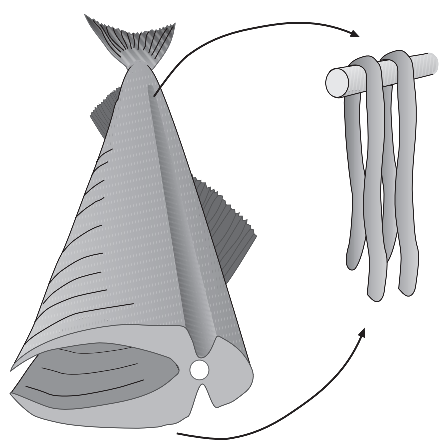


Fig. X.12: Hanging kiarneret *on the fish rack.*

Phase IV: Observing the Drying of the Fish

A. Once you finish cutting all the fish, be sure to check them daily to adjust and reposition them. The blankets will need to be shifted on the pole, sometimes with the tail end hanging lower, then into a reversed position the next day with the neck hanging lower. Sometimes it is good to hang them in a slanted position so that the skin can have a chance to dry in areas that need exposure. If this drying process isn't done properly, the wet areas of the skin will eventually break apart and the whole blanket will fall to the ground. If the blanket remains wet, the flesh will age. Shift the backbones and the *kiarneret* around until a hard crust is formed as the fish become drier. The thin curved *kiarneret* can be woven together with pliable wide string or grass and hung for further drying. Figure X.13 shows what this might look like.

B. A good week and a half with steady winds and sunshine is enough time for the kings to dry properly. The best way to check that each blanket is appropriately dry is to see if the meat is firm and crusty. When the king blankets have a hard crust, they are ready to be paired with string as explained in Step D below.

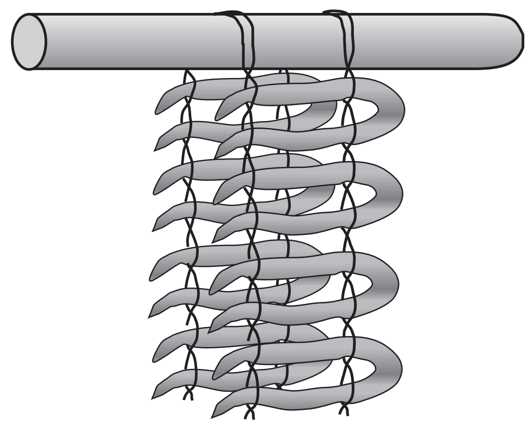


Fig. X.13: Example of weaving together kiarneret *with string.*

C. If it has been rainy, the person caring for the fish can bring them into the smokehouse and make a fire underneath the fish to dry and smoke them. If the fish stay out too long, they can become slimy and begin to spoil. Another method is to start a fire under the fish rack and close off the sides. The fish should be smoked daily until they become dry.

D. When the crusts are evident, pair the kings by tying two blankets together, and then push them back on the fish racks where they were originally hung, positioning them closer together. The fresher ones can then hang in the front where the airflow is better.