EDUCATION MATHEMATICS ELEMENTARY

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders is the result of a long-term collaboration. These supplemental math modules for grades 1-6 bridge the unique knowledge of Yup'ik elders with school-based mathematics. This series challenges students to communicate and think mathematically as they solve problems. Problems are inquiry-oriented and the problems are constructed so that the possibilities are constrained and the students can understand mathematical relationships, properties of geometrical shapes, develop place value understanding, and state conjectures and provide proofs. Curriculum taps into students' creative, practical, and analytical thinking. Our classroom-based research strongly suggests that students engaged in this curriculum can develop deeper mathematical understandings than students who engage with the more procedure-oriented paper and pencil curriculum. MCC's research has shown that these modules have been effective in enhancing students' mathematical learning.

Also in this series:

Going to Egg Island: Adventures in Grouping and Place Values (2nd Grade)
Students learn to group objects in a variety of ways. In particular, they learn the Yup'ik system of counting and grouping (base 20 and sub-base 5). Students compose and decompose numbers. This hands-on and evidence-based approach to teaching numeration has produced good results. The complete package includes an accompanying story book called *Egg Island*, posters, two CD-ROMs, and a coloring book.

Picking Berries: Connections Between Data Collection, Graphing, and Measuring (2nd grade)
Students engage in a series of hands-on activities that help them explore measuring, data, and graphic representation. The complete package includes a CD-ROM, posters, two story books: *Big John and Little Henry*, about using traditional Yup'ik body measurements to build kayaks; and *Berry Picking*, about a Yup'ik family's berry picking trip, which incorporates a traditional story about mosquitoes.

Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area (6th Grade)
Students will explore what happens when the perimeter is held constant and area changes, and what happens when area is held constant and perimeter changes. Through model building, students explore properties of various quadrilaterals, including measurements of perimeter and area. The complete package includes posters and a CD-ROM.

Salmon Fishing: Investigations into Probability (6th & 7th Grades) The module engages students in exploring a variety of topics within probability, using activities that are based on salmon fishing in southwest Alaska. This module uses subsistence and commercial fishing as a contextual background. Students investigate the concepts of experimental and theoretical probability, the law of large numbers, sample space, and equally and unequally likely events. The package includes the module, two posters, a CD-ROM,
and an excel spreadsheet.

Forthcoming in this series:

Patterns and Parkas (tentative title) - 2nd Grade
The traditional repeating geometric border pattern sewn on Yup'ik fur parkas provide the basis for aseries of activities on patterns and shapes.

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The Supplemental Math Modules were developed at University of Alaska Fairbanks.

Part of the series *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders*

Designing Patterns: Exploring Shapes and Area

GRADE LEVEL
3-5
K 1 2 3 4 5 6

Designing Patterns

Exploring Shapes and Area

Daniel Lynn Watt
Jerry Lipka
Joan Parker Webster
Evelyn Yanez
Dora Andrew-Ihrke
Aishath Shehenaz Adam

MCC

Part of the series *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders*

Designing Patterns:
Exploring Shapes and Area

Part of the Series

Math in a Cultural Context:
Lessons Learned from Yup'ik Eskimo Elders

Grade 3, 4, and 5

Daniel Lynn Watt
Jerry Lipka
Joan Parker Webster
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Developed at University of Alaska Fairbanks, Fairbanks, Alaska

Designing Patterns: Exploring Shapes and Area

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Acknowledgements

From Jerry Lipka, Series Editor

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Introduction

Math in a Cultural Context:

Lessons Learned from Yup'ik Eskimo Elders

Introduction to the Series

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders is a supplemental math curriculum based on the traditional wisdom and practices of the Yup'ik Eskimo people of southwest Alaska. The kindergarten to sixth-grade math modules that you are about to teach are the result of more than a decade of collaboration between math educators, teachers, Yup'ik Eskimo elders, and educators to connect cultural knowledge to school mathematics. To understand the rich environment from which this curriculum came, imagine traveling on a snowmachine over the frozen tundra and finding your way based on the position of the stars in the night sky. Or, in summer paddling a sleek kayak across open waters shrouded in fog, yet knowing which way to travel toward land by the pattern of the waves. Or imagine building a kayak or making clothing and accurately sizing them by visualizing or using body measures. This is a small sample of the activities that modern Yup'ik people engage in. The mathematics embedded in these activities formed the basis for this series of supplemental math modules. Each module is independent and lasts from three to eight weeks.

From 2000 through spring 2005, with the exception of one urban trial, students who used these modules consistently outperformed at statistically significant levels over students who only used their regular math textbooks. This was true for urban as well as rural students, both Caucasian and Alaska Native. We believe that this supplemental curriculum will motivate your students and strengthen their mathematical understanding because of the engaging content, hands-on approach to problem solving, and the emphasis on mathematical communication. Further, these modules build on students' everyday experience and intuitive understandings, particularly in geometry, which is underrepresented in school.

A design principle used in the development of these modules is that the activities allow students to explore mathematical concepts semi-autonomously. Though use of hands-on materials, students can "physically" prove conjectures, solve problems, and find patterns, properties, short cuts, or generalize. The activities incorporate multiple modalities and can challenge students with diverse intellectual needs. Hence, the curriculum is designed for heterogeneous groups with the realization that different students will tap into different cognitive strengths. According to Sternberg and his colleagues (1997, 1998), by engaging students creatively, analytically, and practically, students will have a more robust understanding of the concept. This allows for shifting roles and expertise among students and not only privileging those students with analytic knowledge.

The modules explore the everyday application of mathematical skills such as grouping, approximating, measuring, proportional thinking, informal geometry, and counting in base twenty and then the modules present these in terms of formal mathematics. Students move from the concrete and applied to more formal and abstract math. The activities are designed to meet the following goals:

- Students learn to solve mathematical problems that support an in-depth understanding of mathematical concepts.
- Students derive mathematical formulas and rules from concrete and practical applications.
- Students become flexible thinkers because they learn that there is more than one method of solving a mathematical problem.
- Students learn to communicate and think mathematically while they demonstrate their understanding to peers.
- Students learn content across the curriculum, since the lessons comprise Yup'ik Eskimo culture, literacy, geography, and science.

Beyond meeting some of the content (mathematics) and process standards of the National Council of Teachers of Mathematics (2000), the curriculum design and its activities respond to the needs of diverse learners. Many activities are designed for group work. One of the strategies for using group work is to provide leadership opportunities to students who may not typically be placed in that role. Also, the modules tap into a wide array of intellectual abilities—practical, creative, and analytic. We assessed modules that were tested in rural Alaska, urban Alaska, and suburban California and found that students who were only peripherally involved in math became more active participants.

Students learn to reason mathematically by constructing models and analyzing practical tasks for their embedded mathematics. This enables them to generate and discover mathematical rules and formulas. In this way, we offer students a variety of ways to engage the math material through practical activity, spatial/visual learning, analytic thinking, and creative thinking. They are constantly encouraged to communicate mathematically by presenting their understandings while other students are encouraged to provide alternate solutions, strategies, and counter arguments. This process also strengthens their deductive reasoning.

Pedagogical Approach Used in the Modules

The concept of third space is embedded within each module. Third space relates to a dynamic and creative place between school-based knowledge and everyday knowledge and knowledge related to other non-mainstream cultural groups. Third space also includes local knowledge such as ways of measuring and counting that are distinct from school-based notions, and it is about bringing these elements together in a creative, respectful, and artful manner. Within this creative and evolving space, pedagogical forms can develop creatively from both Western schooling and local ways. In particular, this module pays close attention to expert-apprentice modeling because of its prevalent use among Yup'ik elders and other Alaska Native groups.

Design

The curriculum design includes strategies that engage students:

- cognitively, so that students use a variety of thinking strategies (analytic, creative, and practical);
- socially, so that students with different social, cognitive, and mathematical skills use those strengths to lead and help solve mathematical problems;
- pedagogically, so that students explore mathematical concepts and communicate and learn to reason mathematically by demonstrating their understanding of the concepts; and
- practically, as students apply or investigate mathematics to solve problems from their daily lives.

The organization of the modules follows ve distinct approaches to teaching and learning that converge into one system.

Expert-Apprentice Modeling

The rst approach, expert-apprentice modeling, comes from Yup'ik elders and teachers and is supported by research in anthropology and education. Many lessons begin with the teacher (the expert) demonstrating a concept to the students (the apprentices). Following the theoretical position of the Russian psychologist Vygotsky (cited in Moll, 1990) and expert Yup'ik teachers (Lipka and Yanez, 1998) and elders, students begin to appropriate the knowledge of the teacher (who functions in the role of expert), as the teacher and the more adept apprentices help other students learn. This establishes a collaborative classroom setting in which student-to-student and student-to-teacher dialogues are part of the classroom fabric.

More recently, we have observed experienced teachers use joint productive activity—the teacher works in parallel with students modeling an activity, a concept, or a skill. When effectively implemented joint productive activity appears to increase student ownership of the task and increases their responsibility and motivation. The typical authority structure surrounding classrooms changes as students take on more of the responsibility for learning. Social relations in the classroom become more level. In the case of this module the connections between out-of-school learning and in-school learning are strengthened through pedagogical approaches such as expert-apprentice modeling and joint productive activity when those are approaches of the community.

Reform-oriented Approach

The second pedagogical approach emphasizes student collaboration in solving "deeper" problems (Ma, 1999). This approach is supported by research in math classrooms and particularly by recent international studies (Stevenson et al., 1990; Stigler and Hiebert, 1998) strongly suggesting that math problems should be more in-depth and challenging and that students should understand the underlying principles, not merely use procedures competently. The modules present complex problems (two-step open-ended problems) that require students to think more deeply about mathematics.

Multiple Intelligences

Further, the modules tap into students' multiple intelligences. While some students may learn best from hands-on, real-world related problems, others may learn best when abstracting and deducing. This module provides opportunities to guide both modalities. Robert Sternberg's work (1997, 1998) in uenced the development of these modules. He has consistently found that students who are taught so that they use their analytic, creative, and practical intelligences will outperform students who are taught using one modality, most often analytic. Thus, we have shaped our activities to engage students in this manner.

Mathematical Argumentation and Deriving Rules

The purpose of math communication, argumentation, and conceptual understanding is to foster students' natural ability. These modules support a math classroom environment in which students explore the underlying mathematical rules as they solve problems. Through structured classroom communication, students will learn to work collaboratively in a problem-solving environment in which they learn both to appreciate alternative solutions and strategies and to evaluate these strategies and solutions. They will present their mathematical solutions to their peers. Through discrepancies in strategies and solutions, students will communicate with and help each other to understand their reasoning and mathematical decisions. Mathematical discussions are encouraged to strengthen students' mathematical and logical thinking as they share their ndings. This requires classroom norms that support student communication, learning from errors, and viewing errors as an opportunity to learn rather than to criticize. The materials in the modules (see Materials section) constrain the possibilities, guide students in a particular direction, and increase their chances of understanding mathematical concepts. Students are given the opportunity to support their conceptual understanding by practicing it in the context of a particular problem.

Familiar and Unfamiliar Contexts Challenge Students' Thinking

By working in unfamiliar settings and facing new and challenging problems, students learn to think creatively. They gain con dence in their ability to solve both everyday problems and abstract mathematical questions, and their entire realm of knowledge and experience expands. Further, by making the familiar unfamiliar and by working on novel problems, students are encouraged to connect what they learn from one setting (everyday problems) with mathematics in another setting. For example, most sixth-grade students know about rectangles and how to calculate the area of a rectangle, but if you ask students to go outside and find the four corners of an eight-foot-by twelve-foot-rectangle without using rulers or similar instruments, they are faced with a challenging problem. As they work through this everyday application (which is needed to build any rectangular structure) and as they "prove" to their classmates that they do, in fact, have a rectangular base, they expand their knowledge of rectangles. In effect they must shift their thinking from considering rectangles as physical entities or as prototypical examples to understanding the salient properties of a rectangle. Similarly, everyday language, conceptions, and intuition may, in fact, be in the way of mathematical understanding and the precise meaning of mathematical terms. By treating familiar knowledge in unfamiliar ways, students explore and confront their own mathematical understandings and begin to understand the world of mathematics. These major principles guide the overall pedagogical approach to the modules.

Literacy Counts

Literacy Counts is an integrated approach that provides points of entry to math through literacy, particularly vocabulary development. Because the series Math in a Cultural Context (MCC) includes traditional, contextual, and embedded math stories, it is important to provide support and guidance on how to use these stories to promote students' mathematical knowledge. A *Teacher's Guide to Literacy Counts* is forthcoming.

The Organization of the Modules

The curriculum includes modules for kindergarten through seventh grade. Modules are divided into sections: activities, explorations, and exercises, with some variation between each module. Supplementary information is included in Cultural Notes, Teacher Notes, and Math Notes. Each module follows a particular cultural story line, and the mathematics connect directly to it. Some modules are designed around a children's story, and an illustrated text is included for the teacher to read to the class.

The module is a teacher's manual. It begins with a general overview of the activities ahead, an explanation of the math and pedagogy of the module, teaching suggestions, and a historical and cultural overview of the curriculum in general and of the specific module. Each activity includes a brief introductory statement, an estimated duration, goals, materials, any pre-class preparatory instructions for the teacher, and the procedures for the class to carry out the activity. Assessments are placed at various stages, both intermittently and at the end of activities.

Illustrations help to enliven the text. Yup'ik stories and games are interspersed and enrich the mathematics. Transparency masters, worksheet masters, and suggestions for additional materials are attached at the end of each activity. An overhead projector is necessary. Blackline masters that can be made into overhead transparencies are an important visual enhancement of the activities, stories, and games. Such visual aids also help to further classroom discussion and understanding.

Resources and Materials Required to Teach the Modules

Materials

The materials and tools limit the range of mathematical possibilities, guiding students' explorations so that they focus upon the intended purpose of the lesson. For example, in one module, latex sheets are used to explore concepts of topology. Students can manipulate the latex to the degree necessary to discover the mathematics of the various activities and apply the rules of topology.

For materials and learning tools that are more dif cult to nd or that are directly related to unique aspects of this curriculum, we provide detailed instructions for the teacher and students on how to make those tools. For example, in *Going to Egg Island: Adventures in Grouping and Place Values,* students use a base twenty abacus. Although the project has produced and makes available a few varieties of wooden abaci, detailed instructions are provided for the teacher and students on how to make a simple, inexpensive, and usable abacus with beads and pipe cleaners.

Each module and each activity lists all of the materials and learning tools necessary to carry it out. Some of the tools are expressly mathematical, such as interlocking centimeter cubes, abaci, and compasses. Others are particular to the given context of the problem, such as latex and black and white geometric pattern pieces. Many of the materials are items a teacher will probably have on hand, such as paper, markers, scissors, and rulers. Students learn to apply and manipulate the materials. The value of caring for the materials is underscored by the precepts of subsistence, which is based on processing raw materials and foods with maximum use and minimum waste. Periodically, we use food as part of an activity. In these instances, we encourage minimal waste.

DVDs

To convey the knowledge of the elders underlying the entire curriculum more vividly, we have produced a few DVDs to accompany some of the modules. For example, the *Going to Egg Island: Adventures in Grouping and Place Values* module includes DVDs of Yup'ik elders demonstrating some traditional Yup'ik games. We also have footage and recordings of the ancient chants that accompanied these games. The DVDs are also available on CD-ROM on request and are readily accessible for classroom use.

Yup'ik Language Glossary and Math Terms Glossary

To help teachers and students get a better feel for the Yup'ik language, its sounds, and the Yup'ik words used to describe mathematical concepts in this curriculum, we have developed a Yup'ik glossary on CD-ROM. Each word is recorded in digital form and can be played back in Yup'ik. The context of the word is provided, giving teachers and students a better sense of the Yup'ik concept, not just its Western "equivalent." Pictures and illustrations often accompany the word for additional clari cation.

Yup'ik Values

There are many important Yup'ik values associated with each module. The elders counsel against waste. They value listening, learning, working hard, being cooperative, and passing knowledge on to others. These values are expressed in the content of the Yup'ik stories that accompany the modules, in the cultural notes, and in various activities. Similarly, Yup'ik people as well as other traditional people continue to produce, build, and make crafts from raw materials. Students who engage in these modules also learn how to make simple mathematical tools fashioned around such themes as Yup'ik border patterns and building model kayaks, sh racks, and smokehouses. Students learn to appreciate and value other cultures.

Cultural Notes

Most of the mathematics used in the curriculum comes from our direct association and long-term collaboration with Yup'ik Eskimo elders and teachers. We have included many cultural notes to describe and explain more fully the purposes, origins, and variations associated with a particular traditional activity. Each module is based on a cultural activity and follows a Yup'ik cultural story line along which the activities and lessons unfold.

Math Notes

We want to ensure that teachers who may want to teach these modules but feel unsure of some of the mathematical concepts will feel supported by the Math Notes. These provide background material to help teachers better understand the mathematical concepts presented in the activities and exercises of each module. For example, in the *Perimeter and Area* module, the Math Notes give a detailed description of a rectangle and describe the geometric proofs one would apply to ascertain whether or not a shape is a rectangle. One module explores rectangular prisms and the geometry of three-dimensional objects; the Math Notes include information on the geometry of rectangular prisms, including proofs, to facilitate the instructional process. In every module, connections are made between the "formal math," its practical application, and the classroom strategies for teaching the math.

Teacher Notes

The main function of the Teacher Notes is to focus on the key pedagogical aspects of the lesson. For example, they provide suggestions on how to facilitate students' mathematical understanding through classroom organization strategies, classroom communication, and ways of structuring lessons. Teacher Notes also make suggestions for ways of connecting out-of-school knowledge with schooling.

Assessment

Assessment and instruction are interrelated throughout the modules. Assessments are embedded within instructional activities, and teachers are encouraged to carefully observe, listen, and challenge their students' thinking. We call this active assessment, which allows teachers to assess how well students have learned to solve the mathematical and cultural problems introduced in a module.

Careful attention has been given to developing assessment techniques and tools that evaluate both the conceptual and procedural knowledge of students. We agree with Ma (1999) that having one type of knowledge without the other, or not understanding the link between the two, will produce only partial understanding. The goal here is to produce relational understanding in mathematics. Instruction and assessment have been developed and aligned to ensure that both types of knowledge are acquired; this has been accomplished using both traditional and alternative techniques.

The speci c details and techniques for assessment (when applicable) are included within activities. The three main tools for collecting and using assessment data follow.

Notebooks

In recent years, NCTM has promoted standards that incorporate math journals as part of math instruction. Journaling has most often occurred as a tool for re ecting on what was learned. In contrast, math notebooks, which are incorporated in strand one of Literacy Counts, are used by students to record what they are thinking and learning about math concepts before, during, and after the activities in the modules. Through the use of math notebooks, students build their content knowledge while at the same time developing their literacy skills through reading, writing, drawing, and graphic representations. Math notebooks also play an important role in helping students develop math vocabulary.

Observation

Observing and listening to students lets teachers learn about the strategies that they use to analyze and solve various problems. Listening to informal conversations between students as they work cooperatively on problems provides further insight into their strategies. Through observation, teachers also learn about their students' attitudes toward mathematics and their skills in cooperating with others. Observation is an excellent way to link assessment with instruction.

Adaptive Instruction

The goal of the summary assessment in this curriculum is to adapt instruction to the skills and knowledge needed by a group of students. From reviewing journal notes to simply observing, teachers learn which mathematical processes their students are able to effectively use and which ones they need to practice more. Adaptive assessment and instruction complete the link between assessment and instruction.

An Introduction to the Land and Its People, Geography, and Climate

Flying over the largely uninhabited expanse of southwest Alaska on a dark winter morning, one looks down at a white landscape interspersed with trees, winding rivers, rolling hills, and mountains. One sees a handful of lights sprinkled here, a handful there. Half of Alaska's 600,000-plus population lives in Anchorage. The other half is dispersed among smaller cities such as Fairbanks and Juneau and among the over 200 rural villages that are scattered across the state. Landing on the village airstrip, which is usually gravel and, in the winter, covered with smooth, hard-packed snow, one is taken to the village by either car or snowmachine. Hardly any villages or regional centers are connected to a road system. The major means of transportation between these communities is by small plane, boat, and snowmachine, depending on the season.

It is common for the school to be centrally located. Village roads are usually unpaved, and people drive cars, four-wheelers, and snowmachines. Houses are typically made from modern materials and have electricity and running water. Over the past 20 years, Alaska villages have undergone major changes, both technologically and culturally. Most now have television, a full phone system, modern water and sewage treatment facilities, an airport, and a small store. Some also have a restaurant, and a few even have a small hotel and taxicab service. Access to medical care and public safety are still sporadic, with the former usually provided by a local health care worker and a community health clinic, or by health care workers from larger cities or regional centers who visit on a regular basis. Serious medical emergencies require air evacuation to either Anchorage or Fairbanks.

The Schools

Years of work have gone into making education as accessible as possible to rural communities. Almost every village has an elementary school, and most have a high school. Some also have a higher education satellite facility, computer access to higher education courses, or options that enable students to earn college credits while in their respective home communities. Vocational education is taught in some of the high schools, and there are also special vocational education facilities in some villages. While English has become the dominant language throughout Alaska, many Yup'ik children in the villages of this region still learn Yup'ik at home.

Yup'ik Village Life Today

Most villagers continue to participate in the seasonal rounds of hunting, shing, and gathering. Although many modern conveniences are located within the village, when one steps outside of its narrow bounds, one is immediately aware of one's vulnerability in this immense and unforgiving land, where one misstep can lead to disaster. Depending upon their location (coastal community, riverine, or interior), villagers hunt and gather the surrounding resources. These include sea mammals, sh, caribou, and many types of berries. The seasonal subsistence calendar illustrates which activities take place during the year (see Figure 1). Knowledgeable elders know how to cross rivers and nd their way through ice elds, navigating the seemingly featureless tundra by using directional indicators such as frozen grass and the constellations in the night sky. All of this can mean the difference between life and death. In the summer, when this largely treeless, moss- and grass-covered plain thaws into a large swamp dotted with small lakes, the consequences of ignorance, carelessness, and inexperience can be just as devastating. Underwater hazards in the river, such as submerged logs, can capsize a boat, dumping the occupants into the cold, swift current. Overland travel is much more dif cult during the warm months due to the marshy ground and many waterways, and one can easily become disoriented and get lost. The sea is also integral to life in this region and requires its own set of skills and specialized knowledge to be safely navigated.

The Importance of the Land: Hunting and Gathering

Basic subsistence skills include knowing how to read the sky to determine the weather and make appropriate travel plans, being able to read the land to nd one's way, knowing how to build an emergency shelter and, in the greater scheme, how to hunt and gather food and properly process and store it. In addition, the byproducts of subsistence activities, such as carved walrus tusks, pelts, and skins are made into clothing or decorative items and a variety of other utilitarian arts and crafts products and provide an important source of cash for many rural residents.

Hunting and gathering are still of great importance in modern Yup'ik society. A young man's rst seal hunt is celebrated; family members who normally live and work in one of the larger cities will often y home to help when the salmon are running, and whole families still gather to go berry picking. The importance of hunting and gathering in daily life is further re ected in the legislative priorities expressed by rural residents in Alaska. These focus on such things as subsistence hunting regulations, shing quotas, resource development, and environmental issues that affect the well-being of game animals and subsistence vegetation.

Conclusion

We developed this curriculum in a Yup'ik context. The traditional subsistence and other skills of the Yup'ik people incorporate spatial, geometrical, and proportional reasoning and other mathematical reasoning. We have attempted to offer you and your students a new way to approach and apply mathematics while also learning about

Yup'ik culture. Our goal has been to present math as practical information that is inherent in everything we do. We hope your students will adopt and incorporate some of this knowledge and add it to the learning base.

We hope you and your students will benefit from the mathematics, culture, geography, and literature embedded in the *Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders* series. The elders who guided this work emphasized that the next generation of children should be flexible thinkers and leaders. In a small way, we hope that this curriculum guides you and your students along this path.

Tua-i-ngunrituq [This is not the end].

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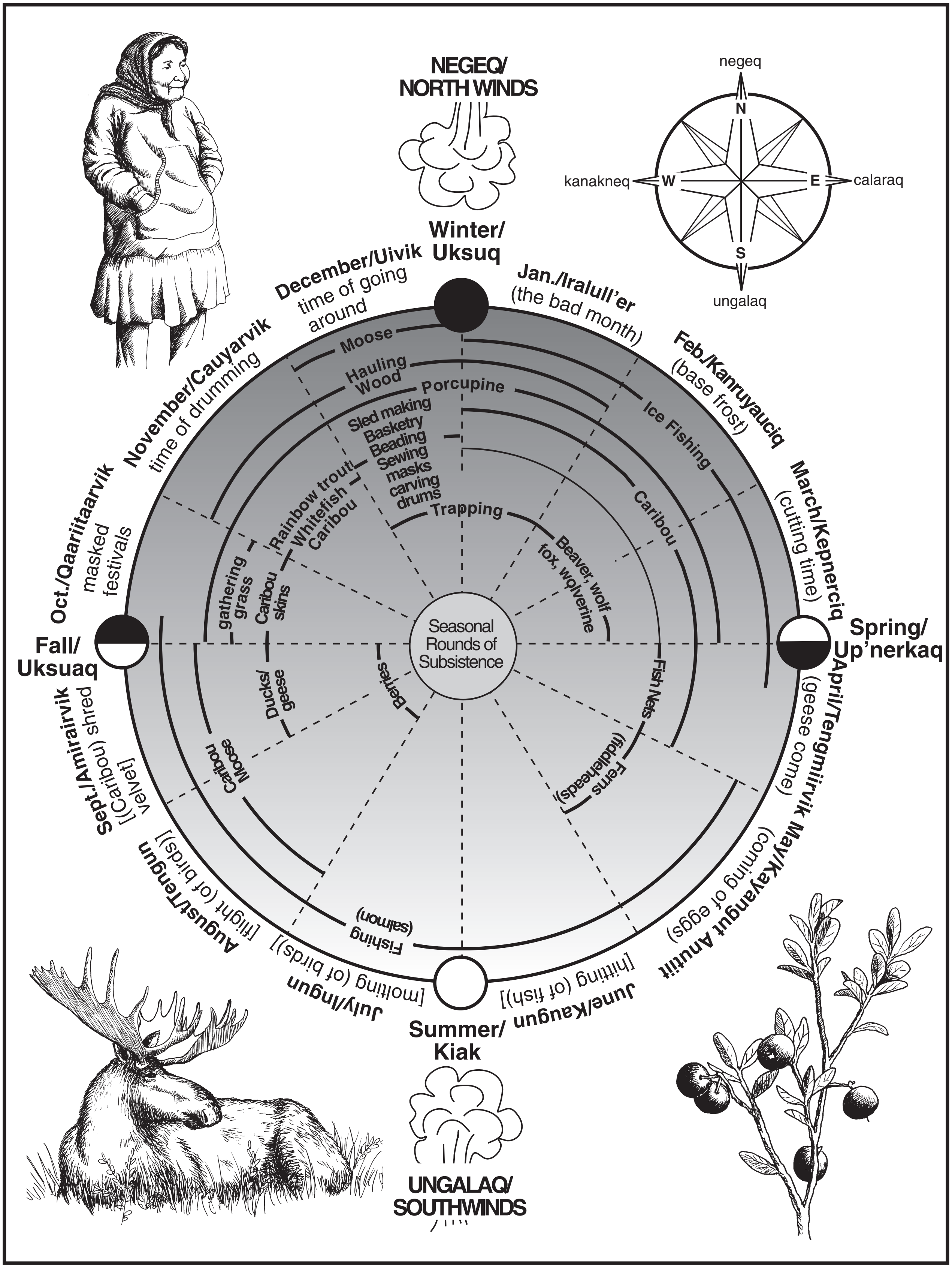


Fig. 1: Yearly subsistence calendar

Introduction to the Module

In this module, students design patterns to be used in a headdress or similar linear strip. As they create their pattern by cutting and folding a rectangle into smaller pieces they learn about geometrical and area relationships. Students learn how to make pattern pieces as they follow the methods of expert Yup'ik seamstresses. The module provides numerous opportunities for the teacher to extend and adapt this curriculum, from further explorations into fractions to Yup'ik cultural knowledge.

Upterrlainaryaraq: Always Getting Ready

Always getting ready (*upterrlainaryaraq)* is not just a Yup'ik word, but is a way of life. The harsh climate mandates being ready for any activity—fishing, berry picking, sewing, a potlatch, a walk to school—and they speak of always getting ready: *upterrlainarluta.* Yup'ik men and women have always had to take care of themselves and construct the things they need for living: tools, shelters, toys, and clothing, everything that is necessary in the village. For years, Yup'ik Eskimo women have been cutting and sewing clothing for members of their families. They make dresses (*qasperet*), overcoats (*atkuut*), pants (*ulruut*), and warm boots (*piluguut*), without reliance on precut or predesigned patterns or measuring devices. They carefully observe the future wearer, plan the design, visualize the proportions, cut the material, and sew it together into a garment that fits perfectly. They call this *tumartat,* "the putting together of pieces to form a whole." This module is about using pieces to understand shapes, area relationships, and to construct patterns.

Walkie Charles, a professor at the University of Alaska Fairbanks' Alaska Native Language Center whose home village is Emmonak, says:

> *Tumartat* is the gathering of pieces of anything and putting them together to make a whole. We look at what we sew—pieces of different types of furs, for instance—and we put them together to create one whole. We look at our patterns; different shapes are made and sewn together to create a whole. Everything around us is a pattern that creates our whole being. People have learned that they can *tumarciq* [put pieces together to make patterns] using shapes from their own surroundings and creating patterns that represent different aspects of their environment such as the animals, the mountains, and the stars. The representations of these shapes are commonly used to adorn clothing and tools. Designs are created for different cultural celebrations: mountains on headdresses and animals on women's dance fans. All of these are *tumartat*, the putting together of pieces to create a whole.

Traditionally these patterns were sewn by Yup'ik women on such items as parkas, headdresses, and dance belts. Mary Active, a Yup'ik elder from Togiak describes how she started sewing when she was very young.

> I started to make dolls (*sugat*) using a little piece of cloth. I was made to make them myself, and they began to improve. And I began to make parkas for them and the parkas began to improve. I got faster in making them. I was probably ten when I started to make little boots for real babies and when I was twelve my mother had me make a pair of adult boots. She had me tan caribou legs and make a pair for Willie Coupchiak. . . . The skin I tanned was probably tough. . . . My mother told me that when she died she wasn't going to come back from the graveyard and show me how to sew and that I had to try

to make them myself. So I tried very hard to sew by myself even though I would get very frustrated. Annie and I would tell stories with our story knives, and when we got tired of that we'd go inside and make dolls and parkas for them.

Without the use of rulers or similar tools, Yup'ik women devise decorative borders for parkas and designs for headdresses, and fur boots, consisting of a repeated motif of geometric shapes or more intricate figures. Similarly, your students will be following these processes throughout this module.

In this module, students will learn Winifred Beans' method of cutting a rhombus from a folded rectangle. Winifred Beans is a Yup'ik elder from St. Mary's. Before we discuss the mathematics of her method of cutting a rhombus from a rectangle, it is also important to note ways of teaching and learning that are embedded in this process in the community context. Winifred Beans, like other Yup'ik elders that we have worked with over the past two decades, taught us through her own engagement in activity. Academically, we call this expert-apprentice modeling. The elder has us observe so that we will learn. When we, the students, are ready we join in. We call this joint productive activity. We have observed a number of Yup'ik and non-Yup'ik teachers using this way of teaching and learning effectively and productively in school settings. Some times the teacher, or in this case the elder, may support that learning with verbal instruction or thinking aloud about the processes undertaken, problems encountered, and possible solutions. We call this cognitive apprenticeship. The Yup'ik culture as well as many others supports learning through apprenticeship and this is supported throughout this module.

Winifred Beans makes one cut on a folded rectangle to create a rhombus and four right triangles. Students will also explore the lines of symmetry of the rectangle and the cut-out rhombus and the four congruent triangles. They will explore and describe the properties of a rectangle and a rhombus and use those properties to distinguish between two shapes. In addition, students will explore the shapes within the folded rectangle by decomposing and recomposing the shapes to get an understanding of the relationships between the rectangle and the cut-out pieces, and among the cut-out pieces. In this way students will be exploring part-to-whole relationships and part-to-part relationships. They will investigate area first using the shapes they have created as nonstandard area units and then using square inches as standard area units. Students will construct a rhombus pattern puzzle and complete a project by creating a headband incorporating a symmetrical pattern of their choice.

To learn about the origins of the word rhombus, go to http://www.pballew.net/rhomb.html.[1]

Mathematics Concepts

This module helps students develop plane geometry concepts based on Yup'ik practices, drawing on the mathematical skills inherent in the culture. The Yup'ik people have developed visual conceptualization and estimation to a very high degree. They use multiple problem-solving strategies, critical thinking skills, and complex reasoning. For example, when designing pattern pieces for linear designs, many elders start from a basic shape such as a square, rectangle, or even a rhombus and then make all other pieces related to the starting piece. This is the

1. **Rhombus** (pl. **rhombi**). 1567, from L.L. rhombus, from Gk. rhombos "rhombus, spinning top," from rhembesthai "to spin, whirl," from PIE *wrembh-, from *werbh- "to turn, twist, bend" (cf. O.E. weorpan "to throw away"), from base *wer- (see versus). The Yup'ik word for rhombus is *takuralria ciliggluni*, literally "a box that has been stretched and tilted."

"genius" from a design point of view and from a pedagogical point of view—helping students learn geometry using this simple but versatile method. This approach allows students to explore:

- geometrical relationships;
- properties of shapes included within this module—rectangles, rhombi, right triangles and squares;
- symmetry, congruence, midpoint bisectors, and diagonals of different shapes;
- conjectures and proofs;
- area of geometrical shapes and area relationships among related shapes;
- part-to-part and part-to-whole relationships (fractions);
- multiplicative thinking; and
- proportional thinking.

In this module, the starting piece is a rectangle. All other pieces made from the starting piece open up explorations into geometrical properties and relationships. Students follow the example of Yup'ik elders to fold and cut a rectangle to form a rhombus. By partitioning the original rectangle students create pieces whose areas are proportional to each other and are a fraction of the whole. These investigations into part-to-whole relationships can form a foundation for a conceptual understanding of fractions and area. Students move toward multiplicative thinking through area explorations; structuring space by using symmetry, folding, and cutting; and numeric relationships. This spatial approach to math follows the lead of others who suggest that teaching through spatial and visual modalities before numeric relationships (Harris, 1991) may make more sense to students who have considerable experience with spatial tasks, than beginning with numeric relationships.

James and James (1968), in their classic *Mathematics Dictionary,* define mathematics as "the logical study of shape, arrangement, quantity, and related concepts" (p. 226). They speak further of the purely mathematical concepts of space and number. The Yup'ik people use shape, arrangement, and space extensively and creatively in many ways, including the design and construction of border patterns and other types of decoration. The geometry in this module encourages children to recognize, describe, and record the properties, symmetries, and relationships of squares, rectangles, and rhombi and of shapes derived from those basic shapes. They also work on composition and decomposition of shapes to create basic repeating units; they describe and construct symmetrical repeating patterns. As they cut shapes apart and investigate the part-to-whole relationships, they gain insights into conservation and measurement of area. Conjecturing, problem solving, offering proofs, formal and informal measurement, partitioning, and basic fraction concepts are introduced as students create their pattern designs. These activities prepare students for later work in symmetry and the ways in which symmetry affects mathematical modeling.

The skills and knowledge developed by these activities are directly related to the precepts of the NCTM Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000). Children are not "passive receivers of rules and procedures" but rather "active participants in creating knowledge." Most of the activities incorporate the following standards for grades three to five: problem-solving, communication, reasoning, estimation, number sense, whole number computation, geometry and spatial sense, measurement, and patterns and relationships. Connections are made between mathematical topics as well as to other areas of the curriculum (art, language arts, science, social studies) and to the children's own environment. The NCTM Curriculum and Evaluation Standards (NCTM, 2000) state further: "Children who develop a strong sense of spatial relationships and who master the concepts and language of geometry are better prepared to learn number and measurement ideas."

Properties and Symmetries of Quadrilaterals

The module starts with a very familiar shape, a rectangle. Most of us know that a rectangle is a quadrilateral (four-sided polygon) with four 90-degree angles (also called right angles). As a consequence of these basic properties, a rectangle's opposite sides are equal and parallel and it possesses two lines of symmetry (horizontal and vertical), as shown in Figure 2.

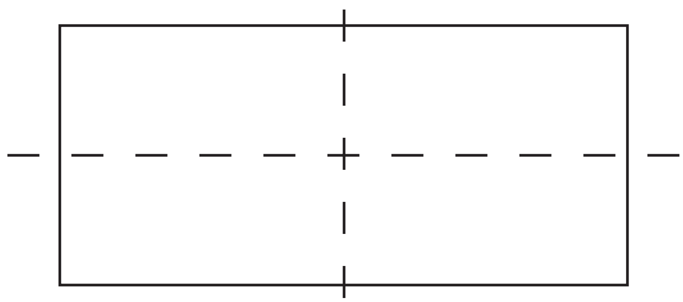


Fig. 2: Lines of symmetry in a rectangle

A line of symmetry divides a gure into two equal parts, in such a way that when the gure is folded along that line, the folded sides match each other perfectly. Yup'ik people use these properties—four right angles, opposite sides equal, two lines of symmetry—in order to construct rectangles from larger pieces of material.

If you fold down one edge of a rectangular or square sheet of paper so that the folded edge is parallel to the fold, and cut as shown in the diagram, the result will be a rectangular strip. If you fold part of the strip in the same way, you can make a smaller rectangle of any size you wish. You can check and improve the rectangle by folding it along its horizontal and vertical lines of symmetry and trimming off any portions that do not match up. Eventually your rectangle will be perfectly symmetrical with its opposite sides equal and all its angles 90 degrees.

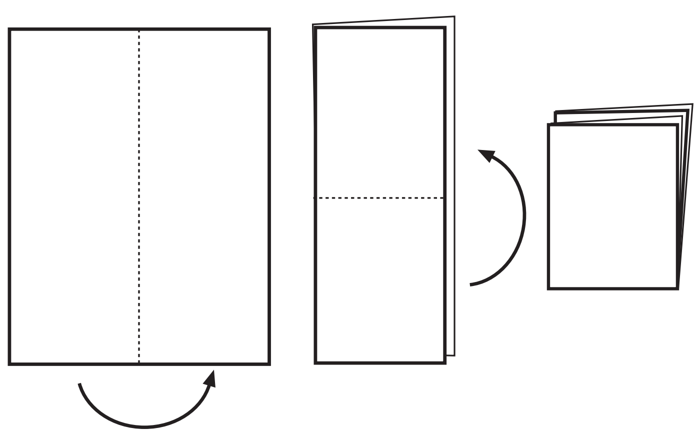


Fig. 3: Cutting and folding a rectangle from a sheet of paper

Cutting a Rectangle into a Rhombus and Four Right Triangles

Winifred Beans of St. Marys village is a Yup'ik elder who is known as a designer of fancy pattern borders. She makes her pattern pieces (*tumaqcat*) by rst constructing a rectangle and then dividing it into a rhombus and four congruent right triangles. She carefully folds the rectangle into quarters (a smaller rectangle), then just as carefully cuts the quarter rectangle from one corner to another, making sure to cut across the center of the original rectangle so that it remains intact. When the result is unfolded, Mrs. Beans has made a perfect rhombus and four congruent right triangles.

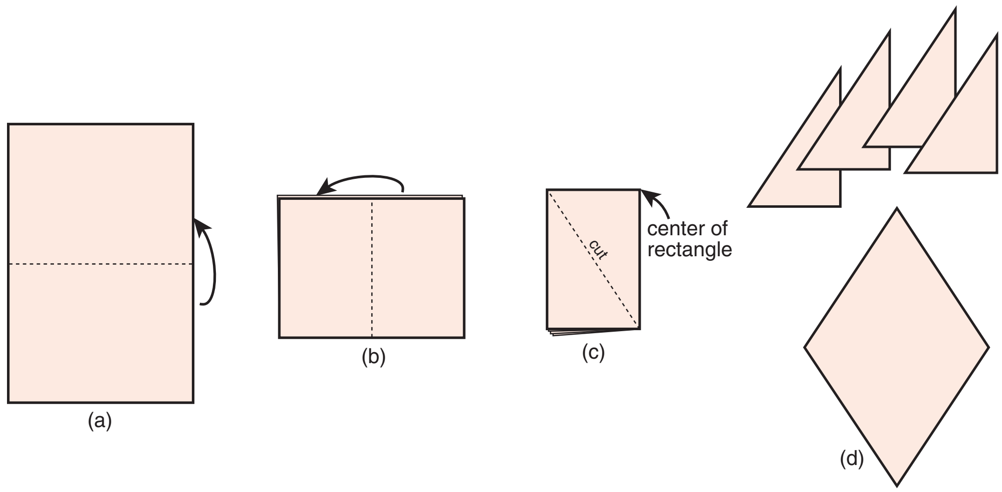


Fig. 4: Winifred's method of cutting a rhombus

Cultural Math Note

Winifred Beans' method of cutting a rhombus from a rectangle illustrates three important geometric principles used in her designs:

1. Take advantage of symmetry. By folding a rectangle into quarters, Winifred ensures that the resulting shape will have two lines of symmetry and that the four right triangles will be congruent.
2. Create shapes whose edges match. The edges of the right triangles and rhombi produced by this method will match up exactly when they are made in different colors and formed into patterns.
3. Divide shapes in ways that preserve geometric relationships. Yup'ik designs frequently contain shapes whose areas are related by simple ratios. The area of the rhombus is half that of the rectangle. The four right triangles can be rearranged to form a rhombus. Their areas are in a ratio of 1 to 4. The area of one right triangle is ⅛ of the original rectangle.

A rhombus is a quadrilateral with four equal sides and opposite angles equal. Like a rectangle, a rhombus has two lines of symmetry. Unlike a rectangle these go from vertex to vertex, rather than through the midpoints of the sides. As we compare the properties of a rectangle with those of a rhombus we see that they are quite complementary (see Figure 5, Properties of special quadrilaterals). You can think of a square as a special kind of rectangle with all sides equal—or as a special kind of rhombus with all angles equal. As a result, a square combines the properties of both shapes. All three of these quadrilaterals are also special cases of a parallelogram, a quadrilateral with opposite sides equal and parallel and opposite angles equal.

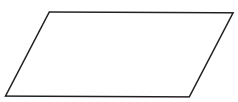
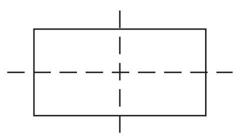
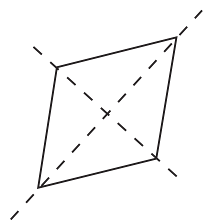
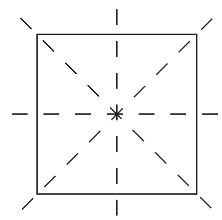
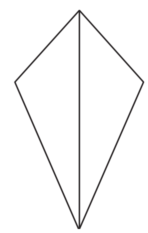
Properties of a parallelogram	Properties of a rectangle	Properties of a rhombus	Properties of a square	Properties of a kite
• Four sides • Opposite sides equal and parallel • Opposite angles equal • No lines of symmetry	• Four sides • Opposite sides equal • All angles equal to 90 degrees • Two lines of symmetry passing through sides	• Four sides • All sides equal • Opposite angles equal • Two lines of symmetry passing through vertices	• Four sides • All sides equal • All angles equal to 90 degrees • Four lines of symmetry passing through the sides and the vertices	• Four sides • Opposite sides are equal • Opposite angles are equal • One line of symmetry
				

Fig. 5: Properties of special quadrilaterals

Special cases: Notice that a rectangle has all the properties listed for a parallelogram, plus several additional properties. The same is true of a rhombus and a square. Therefore, rectangles, rhombi, and squares are all special kinds of parallelograms. A square has all the properties listed for a rectangle, plus several more, so a square is a special kind of rectangle. A square also has all the properties listed for a rhombus, plus several more, so a square is a special kind of rhombus. Figure 6 shows schematically how these special quadrilaterals are related.

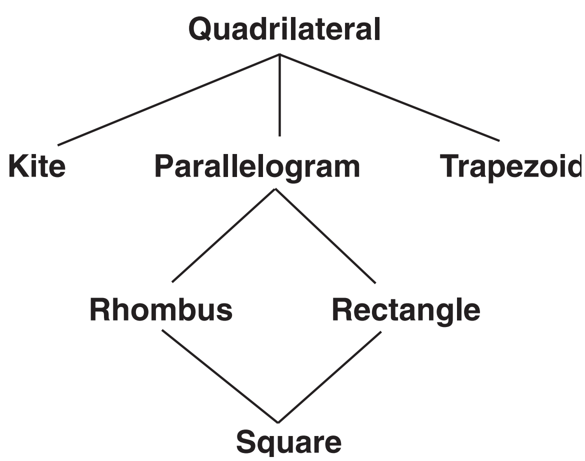


Fig. 6: Family relations of quadrilaterals

Properties and Symmetries of Triangles

When Winifred Beans cuts a rhombus from a rectangle as described above, the “leftover pieces” form four congruent right triangles. A right triangle is a triangle that contains a right angle. Two shapes are congruent if all their corresponding sides and angles are equal. You can test for congruence by picking up one of the triangles and seeing whether it fits exactly on top of another one.

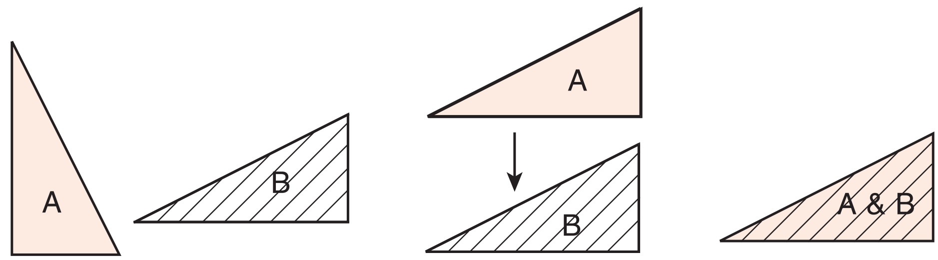


Fig. 7: Triangle A can be rotated and moved to fit on top of triangle B, proving they are congruent.

If you were to cut the rhombus along both its diagonals, you would have four additional triangles, also congruent to each other and to the first four triangles. You could prove mathematically that all eight triangles are congruent—but you can also demonstrate this informally. Think about how the folded rectangle was cut along its diagonal, dividing it into two congruent triangles. You could also pick up all eight triangles and demonstrate that they all fit exactly on top of each other.

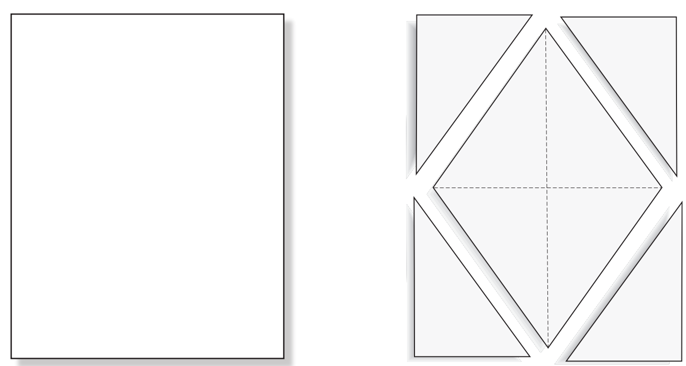


Fig. 8: Cut-out rhombus and triangles from the rectangle

This is also a way of determining that each triangle covers ⅛ of the area of the original rectangle. The area of the rhombus, made up of four such triangles, equals $4/8$ or ½ the area of the original rectangle.

Right Triangles

The triangles that are formed when you cut a rhombus from a rectangle are right triangles, that is, triangles that include one 90-degree angle. A typical right triangle, such as the ones shown above, has no lines of symmetry. The triangles that are formed when you cut a rhombus out of a square are *right isosceles* triangles, sometimes called half-square triangles. These have two equal sides with a 90-degree angle between them. The two other angles are equal to 45 degrees. A right isosceles triangle has one line of symmetry, with the very nice property that if you fold and cut along the triangle’s line of symmetry, you create two smaller congruent right isosceles triangles. You may continue this process, folding and cutting, to make smaller and smaller right isosceles triangles.

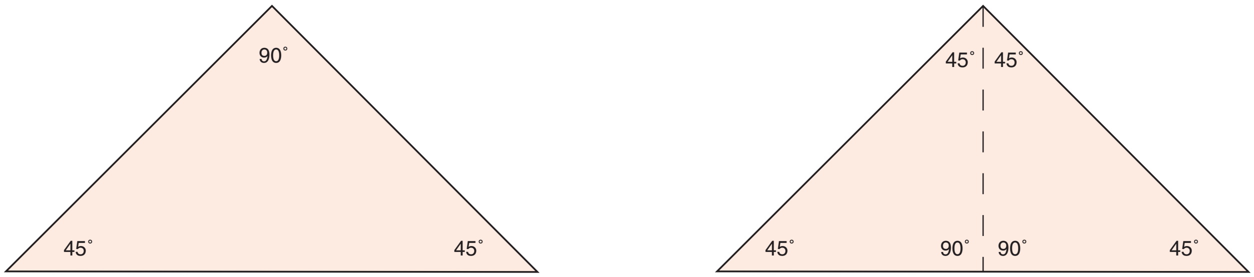


Fig. 9: A right isosceles triangle divided along its line of symmetry into two smaller right isosceles triangles

Squares and rectangles can also be divided into smaller congruent squares and rectangles. The simplest way is to fold and cut the original shape along its horizontal and vertical lines of symmetry to make four smaller shapes, each one quarter of the area of the original square or rectangle.

Some Yup'ik parka designers divide squares or rectangles into three congruent rectangles by folding the original shape into thirds and cutting it. The folding is done by trial and error: you have to be careful to make sure all three sections are equal before making the nal fold. Each of the rectangles can also be folded in thirds, to make three smaller congruent shapes, each ⅑ the area of the shape you started from.

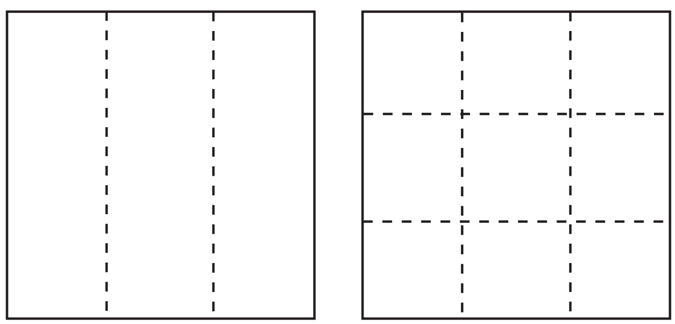


Fig. 10: A square divided into three congruent rectangles and nine congruent squares

There is one more shape derived from rectangles that is used in some traditional Yup'ik patterns. This is a parallelogram made by dividing a rectangle into two congruent triangles, and then connecting the edges to form a quadrilateral with opposite sides parallel. This parallelogram covers the same area as the original rectangle.

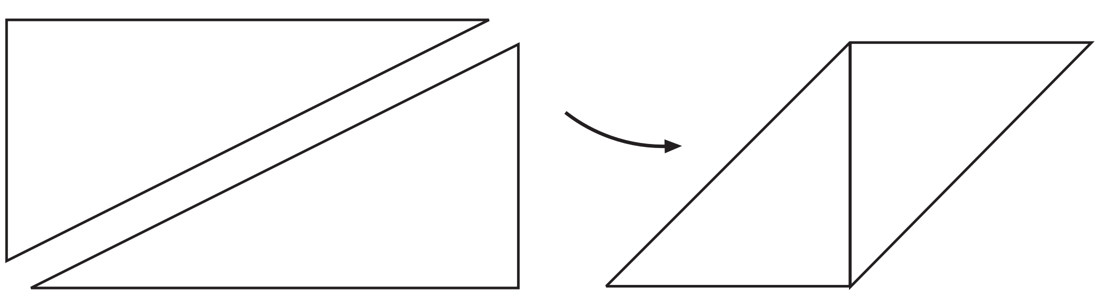


Fig. 11: Two congruent triangles cut out from a rectangle, forming a parallelogram with the same area as the rectangle

This basic set of shapes (*tumaqcat*), all derived from rectangles and rhombi, are used to create all the traditional designs that appear throughout this module. Figure 12 shows a sample of some of the possible *tumaqcat* shapes in contrasting colors that can be derived from a rectangle and rhombus.

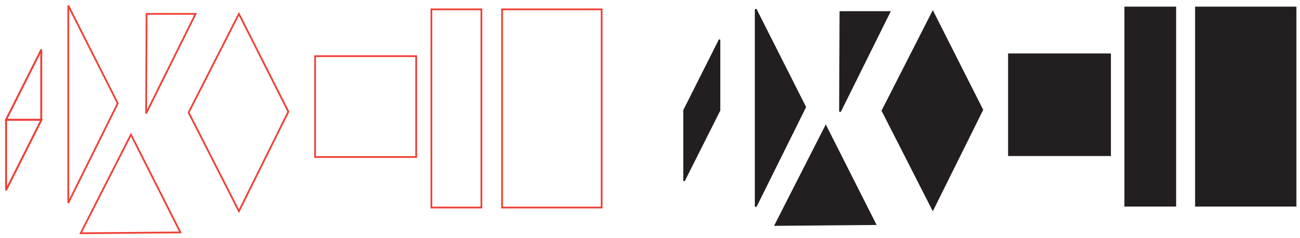


*Fig. 12: Basic set of shapes (*tumaqcat*)*

Investigating Area and Part-Whole Relationships

Area is de ned as the amount of surface covered by a shape, measured in square units appropriate for the size of the region being measured. Square inches are used to measure pieces of paper, square yards for a rug, and square miles for a city, county, or state. In this module students explore area in three stages, taking them from intuitive understandings to formulas for the areas of shapes they are studying.

Conservation of Area

The concept of conservation of area underlies all development of area concepts. In its simplest form it means that if you take any shape and subdivide it, the total area of all the pieces is the same as the original shape. Although this seems to be simple common sense, the perception that some shapes look bigger may require students to verify this for themselves by recombining shapes into their original form, as shown in Figure 13.

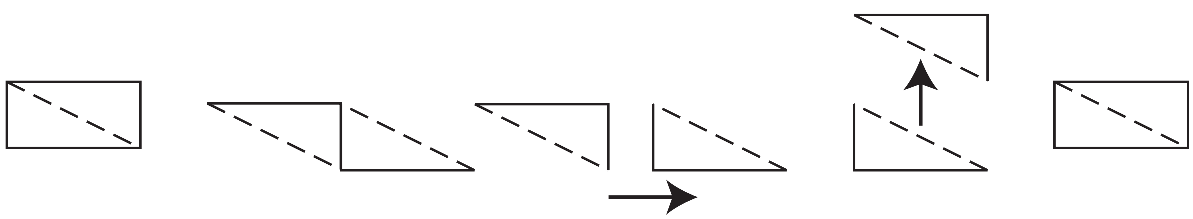


Fig. 13: Recombining shapes into their original form to prove conservation of area

Comparing Areas by Cutting and Folding

The simplest area measurements in this unit involve comparing the areas of related shapes. We have already seen how we can use eight congruent triangles to show that the area of a rhombus is four times as large as each triangle, and one half the area of the rectangle it was cut from. We can either cut the rhombus into four congruent triangles along its diagonals, or fold it along its diagonals to see that the folded shape is congruent to the four triangles made by the rst cut (see Figure 8).

Both rectangles in Figure 14 have the same area. This can be determined by folding them in half in different ways and comparing the folded shapes with each other. For example, fold rectangle A in half so the long edges match, and fold rectangle B so the short edges match. The two folded rectangles will be congruent, which shows that rectangles A and B cover the same area. Or, you may cut one of the rectangles in half and place the pieces so they match the other rectangle. At rst, students will actually need to cut and fold the shapes to discover these area relationships. Later, they may be able to nd out the same information using measurements and area formulas.

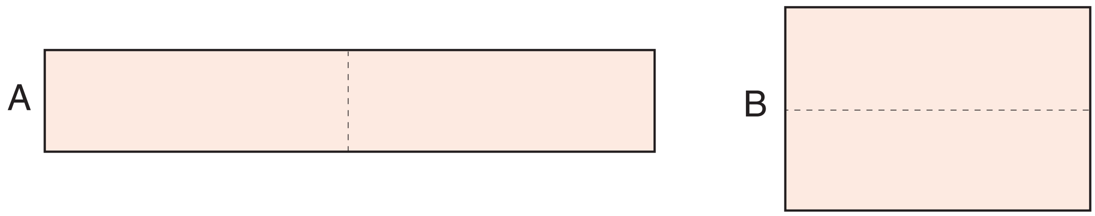
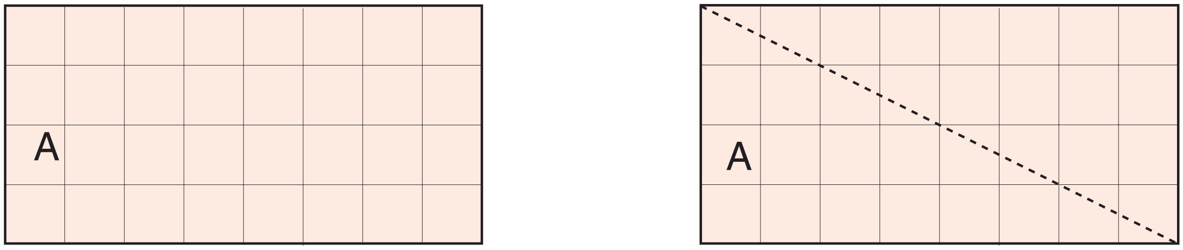


Fig. 14: Two rectangles with the same area

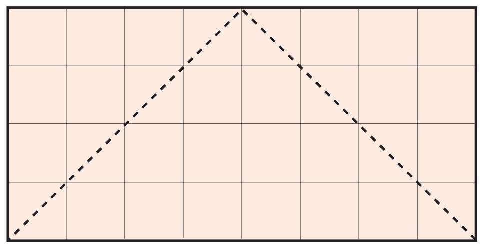
Students next learn to measure area using nonstandard units that arise from the constructions they are making. For example, when a rectangle is cut into a rhombus and four congruent right triangles using Winifred Beans' method, each of the right triangles covers exactly ⅛ of the area of the original rectangle. The area of the rectangle is equal to eight right triangle units. The same units can also be used to nd the area of the rhombus: four right triangle units. (See Figure 8.)

The third way students measure area involves using grid paper and counting squares. In Figure 15, the rectangle has an area of 32 square units. When the rectangle is divided along a diagonal it is easy to prove that the triangles formed are congruent; therefore the area of each triangle is equal to half the area of the rectangle, 16 square units. This leads to an easy way to nd the area of any right triangle: draw a rectangle around it, then count the squares in the rectangle and divide by 2. This reasoning can be extended to nding the area of an isosceles triangle, because it can be divided into two right triangles as shown below. Once again the area is half the area of the surrounding rectangle.

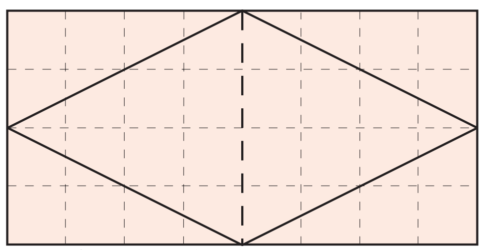
We can easily extend this idea to a rhombus because a rhombus can be split into two isosceles triangles. Once again, the area of a rhombus is one half the area of the surrounding rectangle.



(a) Divide a rectangle with an area of 32 square units into two right triangles, each with an area of 16 square units.



(b) Divide an isosceles triangle into two right triangles. The area of the isosceles triangles is the sum of two right triangles. Each of those is ½ the area of the surrounding rectangle (16 square units). The total area of the isosceles triangle is 8 + 8 = 16 sq. units.



(c) The area of the rhombus can be found by dividing it into two isosceles triangles. Each triangle has an area of 8 sq. units, so the rhombus has an area of 16 sq. units, half of the surrounding rectangle.

Fig. 15: Finding the area of triangles and rhombi by drawing rectangles around them

Finally, students have the opportunity to learn formulas, or shortcuts, to measure the area of any rectangle, right triangle, isosceles triangle, or rhombus. Students can easily discover a shortcut for the area of a rectangle on grid paper: count the number of squares in a row and multiply it by the number of rows. In traditional terminology this is usually written as:

Area of a rectangle = base x height

This leads to the shortcut for the area of a right triangle or an isosceles triangle:

Area of triangle = ½ x base x height

It turns out that this shortcut works for any triangle, which can be shown by drawing rectangles and dividing the triangle into pieces that can be surrounded by rectangles.

One last extension—the shortcut for the area of a rhombus:

Area of rhombus = ½ the product of its diagonals

Parts and Wholes

The study of area comparisons also connects naturally to the study of parts and wholes (in other words, fractions). We've already seen that the area of a rhombus is half the area of the rectangle it was cut from and that each of the "left over" triangles has an area equal to ⅛ that of the rectangle. When students are folding and cutting up shapes, it's relatively simple to ask them to use fractional notation to describe the areas of the cut-up pieces.

Combining Shapes to Make Patterns

Yup'ik pattern makers use the rectangles, squares, rhombi, and right triangles in different sizes to create a variety of interesting symmetrical patterns. They use shapes of two contrasting colors to produce visually pleasing effects. The shapes derived from rectangles or squares can be t together in several different ways. This allows people to make many different patterns using the same basic shapes. A few examples are shown below in Figure 16.

Symmetrical Linear Patterns and Basic Repeating Elements

Yup'ik designers use linear patterns for parka borders (parka bottoms and sleeves), headbands, and boots. These patterns all follow a few simple rules (these rules were explained by Theresa Mike, a Yup'ik elder from St. Marys):

- Start with a basic combination of shapes in contrasting colors. This becomes the basic repeating element; these shape combinations are usually arranged so that shapes line up edge to edge.
- The basic repeating elements are repeated over and over in a linear sequence to complete the pattern.
- Each pattern has a "balance point" at the center, with an equal number of design elements on either side of the balance point. In a parka bottom border, the balance point is at the center where the sides of the border meet.
- Usually (but not always) the balance point is a line of symmetry for the entire shape.

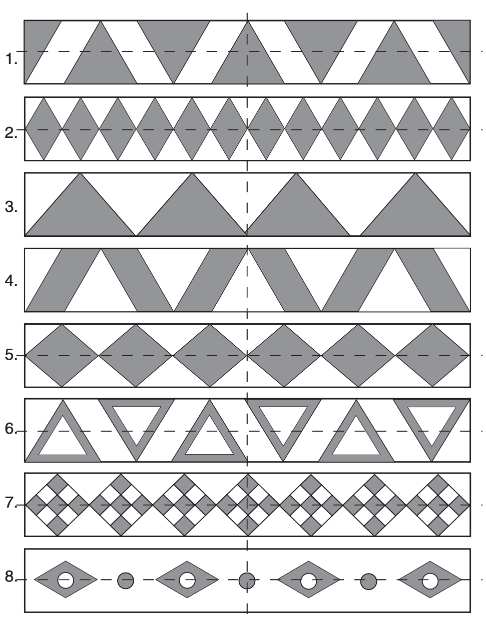


Fig. 16: Sample headband patterns designed according to Yup'ik principles. The balance point (symmetry line) is shown as a dashed line.

As students learn to identify the repeating elements and construct linear patterns, they are developing an important mathematical skill. Repeating patterns in one, two and three dimensions, as well as repeating rotational patterns, are found throughout the natural world, the constructed world, and the artistic world in which students live and grow up. Students' experiences with linear patterns in this module will help them understand patterns they see everywhere in the world around them.

Mathematically, a repeating linear pattern could continue forever. However, a practical pattern has a beginning edge and an ending edge. These edges may be constructed using only part of the basic repeating element in order to end with the edge vertical. Headband patterns usually end in such a way that the pattern would be continuous if one edge could wrap around to meet the other.

Cultural Note

Yup'ik people don't make patterns like this (Figure 17) because they do not have meaning. Patterns are named after things that they resemble. For example, the pretend mountains (Figure 16, the third panel) resemble mountains, and the braid resembles braids, etc. Evelyn Yanez stated, "we didn't use the trapezoid shape." Also, this pattern lacks balance between white and dark. The small dark triangles are out of balance with the white larger trapezoids. Aesthetically, this is not pleasing to a Yupiaq eye. (The words Yupiaq and Yup'ik are both used to mean the "people.")

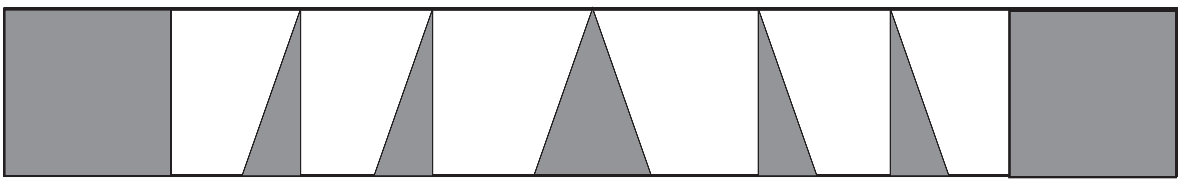


Fig. 17: A pattern using trapezoids, not Yup'ik style

Estimation, Measurement, and Partitioning

Most Yup'ik pattern strips are designed to go around a parka's bottom edge or its sleeve. The parka designers plan carefully so that as the pattern completes the circle, the two ends match, giving the sense of a pattern that continues forever. Headband patterns, however, are designed to leave room for an elastic or tie to secure the headdress at the back.

Some of the practical questions a designer has to ask are: (1) How big should the starting squares or rectangles be so that when all the pieces are cut and sewn, the pattern will "close" perfectly as it circles the bottom of the parka or completes the headband? And (2) How many pieces of each type will I need to complete my pattern?

Look at the parka in Figure 18. You can tell this is not a trivial question to answer. You can also see that the same basic pattern element is used to make a smaller length pattern that circles the sleeves of the parka.

Fig. 18: Squirrel parka with pretend window border

The task of determining how many repeating elements can fit within a given pattern and how large they should be is called partitioning. Yup'ik designers partition their borders intuitively, without formally measuring. Because they have made and seen many parkas and headbands of different sizes, they know from experience that they will need to start with a basic shape of a particular size to fit a particular height and length of parka border.

First they decide on the height of the border strip; this is often measured using fingers. Once the height is determined, the size of the basic shape, usually a square or rectangle, is also determined.

A more conventional mathematical way to partition a headband into equal sized parts is to use a measuring tape and standard measures. First measure the length of the headband. Next decide on the height of the strip. This will determine the size of your pieces (*tumaqcat*). In the headband pattern shown in Figure 19, the height of the strip is the height of the rhombus, which is the same as that of the starting rectangle. This size also determines the size of the light colored isosceles triangles. Once you know the height, you can determine the length of the basic repeating unit. The number of repeating units equals the total length of the headband divided by the length of the repeating element. If this does not come out to be a whole number, you'll probably want to increase or decrease the size of your basic pattern element slightly, to make a slightly wider or narrower band, so that you will be able to complete the pattern.

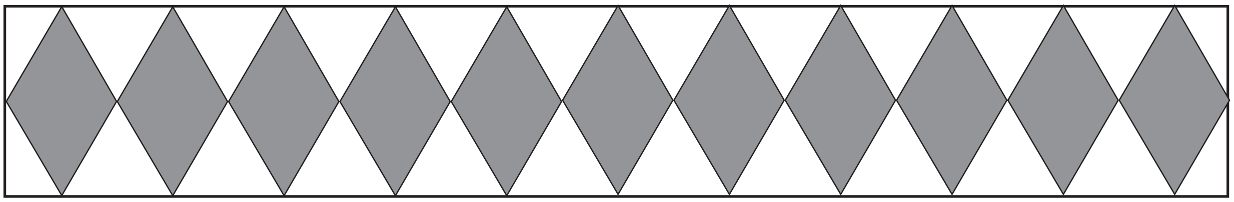


Fig. 19: This headband pattern contains 11 repeats of the basic element (a dark rhombus and two right isosceles) with one set of triangles divided in half to form the ends.

After you know the number of repeating elements, you can calculate the total number of pieces needed and thus the area of dark- and light-colored material.

Students confront the partitioning problem during the module when they make their own patterns. The module provides suggestions for helping students work this out by rst cutting a strip of the length needed, and then repeatedly folding each section in half to divide it into equal small sections. This is an intuitive method to partition a strip into equal segments that determine the size of the repeating elements. Theresa Mike, a Yup'ik elder from St. Marys, cautions that the folding will be more accurate if students fold each section separately, because lines become less accurate when the same piece of paper is doubled more than once.

Mathematical Reasoning: Conjecture and Proof

In addition to the practical problem solving involved in this module, students are sometimes asked to prove whether a certain statement is true or not. This is a big step in developing mathematical understanding and is a basic component of all mathematical thinking.

Mathematical investigations often take the following form:

- You observe a certain situation and look for patterns.
- When you think you've found a pattern, make a conjecture: a statement about a mathematical fact, relationship, or generalization that is based on careful observation or experimentation but that has not been proven.
- You try to prove your conjecture using previously known facts and logical systems of reasoning.

In this module, students make a type of conjecture when they rst construct their rhombi. Once they have cut out what they think is a rhombus, they can prove whether it is a valid rhombus by folding it in half along both symmetry lines to show that all sides match up. They are using known facts—that a rhombus has four equal sides, along with the symmetry properties of a rhombus—to prove that a shape they have constructed is or is not a rhombus.

Students are also asked to determine the area of their rhombus in relation to the original rectangle. One way they can do this is by cutting the rhombus along its symmetry lines and showing that the four triangles that result are identical to each other and to the four other triangles cut from the rectangle when they made the rhombus.

Since the entire area of the rectangle can be covered by eight triangles, and four triangles cover the rhombus, they have in effect proven that the area of the rhombus is half that of the rectangle.

A more formal proof of the same relationship can be given once students have learned formulas for the area of a rectangle (base x height) and a rhombus (½ the product of the diagonals). Using a drawing like Figure 20, students can show that the height and base length of the rectangle are the same lengths as the diagonals of the rhombus.

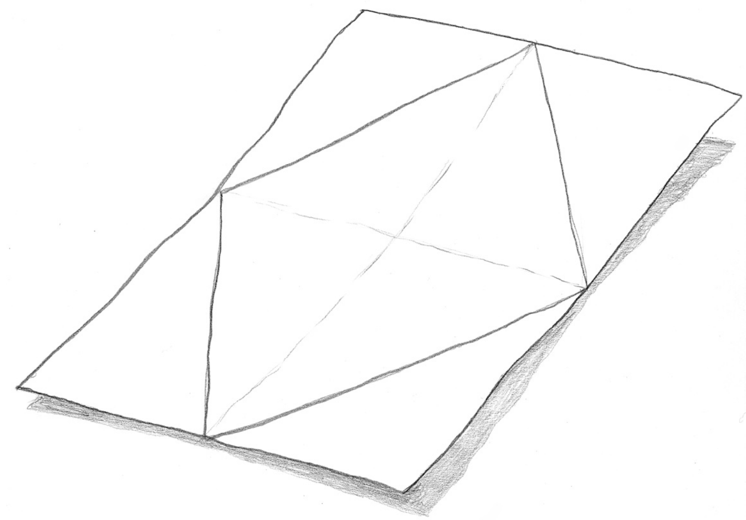


Fig. 20: A rectangle with an inscribed rhombus. The base and height of the rectangle are equal in length to the diagonals of the rhombus. The area of the rhombus is equal to ½ base x height.

Another type of conjecture involves comparing the areas of different pieces. This is important for determining the amount of material needed to construct a pattern.

For instance, in the "pretend windows" pattern (Figure 21), students may want to decide how many squares they will need in order to cut their pieces. A student may make a conjecture that the two white triangles could be cut from a single square, the same size as the "checkerboard" square. They could prove or verify this conjecture by cutting out two triangles of the same size. If they fit exactly on top of the square, that would represent a proof of the conjecture.

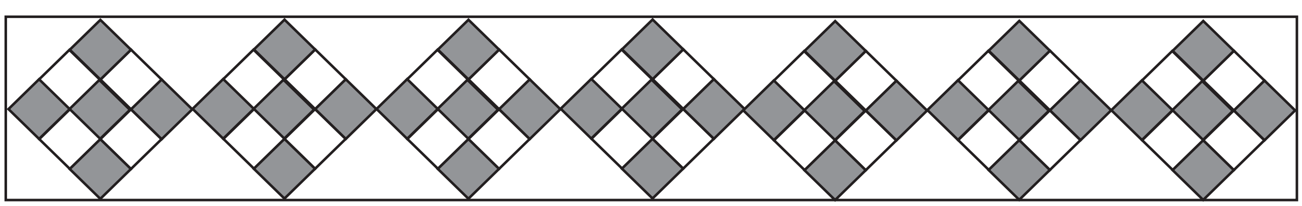


Fig. 21: Pretend windows pattern

These ways of proving geometric facts and relationships are not as rigorous as the formal proofs used by mathematicians—but for the purpose of this module and elementary age students they do satisfy the criteria for proof because they are based on known properties and logical reasoning.

Pedagogy of the Module

The pedagogy of the module begins with Ms. Beans demonstrating the method of how to make a rhombus from a rectangle. Observational learning is part of the context of expert-apprentice modeling. Students have ample opportunities to learn from the expert, from the teacher, and from other students. In this manner, the classroom becomes a socially supportive one. Knowledge and processes that support that knowledge become embedded in the social organization and structure of the classroom. A part of expert-apprentice modeling is joint productive activity. By this we mean the teacher and the students are simultaneously engaged in the same activity. Teacher products are made both to model for the students but are a product in their own right. Joint productive activity requires the teacher to work in parallel to students, allowing students further opportunity to observe from the teacher and from other students. Joint productive activity encourages cognitive apprenticeship. Teachers may talk aloud when they face problems and how they may solve that problem. Evelyn Yanez has noted this as particularly important for some learners, motivating them to engage in the activity and to solve problems. Additionally, expert-apprentice modeling, joint productive activity, and cognitive apprenticeship alter the social hierarchy of the classroom. The teacher, of course, maintains his/her leadership role; however, student autonomy increases as the classroom dynamic changes when students perceive that "we are all in this together."

Additionally, the module encourages the use of peer tutoring and cross-age tutoring. Research indicates that cross-age tutoring supports the tutee's learning. We encourage you to take advantage of having your students work with younger-aged students. One major reason that peer and cross-age tutoring is effective is that tutors and their students often speak a more similar language than do teachers and students (Cazden, 1986; Hedin, 1987). Peer tutoring usually resulted in signi cant cognitive gains for both the tutor and the tutee (Britz, Dixon, and McLaughlin, 1989).

Literacy Counts: Developing Language and Literacy in MCC

The story of Iluvaktuq, which is about a famous Yup'ik warrior, is commemorated in parkas worn by many Yup'ik people who are his descendants. We suggest that you use the storybook *Iluvaktuq and Paluqtalek* to accompany this module. Literacy Counts guides you in effectively using this story. Literacy Counts is a new and developing addition to the Math in a Cultural Context series. Why use Literacy Counts in a math module? We have multiple reasons for connecting literacy to the teaching of mathematics as a way to improve students' mathematical performance. First, math vocabulary is a necessary component in the process of learning mathematics. Math vocabulary is precise and differs from everyday usage. Not having the necessary mathematical vocabulary can be a barrier to students' progress. The inclusion of Literacy Counts in this module is one step toward increasing students' access to mathematics. Second, problem-solving oriented mathematics such as this series supports requires students to think conceptually and to solve math problems. Understanding the problem and learning problem-solving strategies is also a component of Literacy Counts and another connection to students' learning mathematics. Third, in this module we use the story of Iluvatuq and Paluqtalek as a way to make cultural connections to students and increase their motivation and interest in learning. Lastly, Literacy Counts taps into some of the pedagogical ways in which Yup'ik people learn and communicate with each other. This is explained in detail on page 19.

In keeping with National Council of Teachers of Mathematics (NCTM) standards that emphasize movement away from teaching by telling and rote memorization toward student-centered activities that focus on inquiry through problem posing, problem solving, and communicating mathematically, we have developed Literacy Counts, a comprehensive approach to developing language and literacy. There are two strands within the Literacy Counts approach: (strand 1) developing literacy (speaking, writing, reading, listening, and presenting) in math; and (strand 2) developing multiple literacies (linguistic, visual, kinesthetic, oral storytelling, etc.) through the use of traditional Yup'ik stories.

Studies have indicated that literacy and literacy instruction are integral and necessary to meaningful mathematics instruction (Carpenter & Lehrer, 1999; Gallimore & Tharp, 1990). This is supported by the *Principles and Standards for School Mathematics* (NCTM, 2000): "Students who have opportunities, encouragement, and support for speaking, writing, reading and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" (p. 60).

Strand 1: Literacy and Math: Developing the Math Notebook

What is a Math Notebook?

The purpose of math notebooks is to help students build conceptual content and math skills, while at the same time the notebook serves as a context for developing literacy—reading, writing, and vocabulary development, as well as listening and speaking. The math notebook can be used as part of any math program to record students' mathematical findings, understandings of mathematical processes and vocabulary, and questions and reflections on their experiences with mathematical concepts. Math notebooks can also be a reference that students can use during large or small group discussions about math. Likewise, math notebooks can provide a space for students to extend their individual learning by recording additional insights that may be developed through large- and small-group discussions.

How is a Math Notebook Different from a Math Journal?

In recent years, math journals have been incorporated into math instruction as a response to NCTM standards and testing requirements that focus on students' written explanations of their mathematical processes. However, math journals are often employed after a math activity, as a way for students to record their reflections about their learning. Typically, students respond to prompts that may be something like, "In math today, I learned …" In this context, math journals are not necessarily records of data gathered at the time of their mathematical explorations.

In contrast, math notebooks are tools your students can use:

- to record what they already know before the hands-on mathematical problem-solving activity,
- to record data as they are involved in the math activity,
- to pose questions throughout the process,
- for vocabulary development,
- to reflect on their thinking and learning after the inquiry, and
- as a resource for large or small group discussions.

What Kinds of Information Should I Ask Students to Record in Math Notebooks?

A teacher must first consider what information will be recorded by the students during each math activity. You may decide on a basic template and then add additional entry requirements, depending on the goals of your math lesson. For example, possible elements to include as part of the basic template might be:

- date,
- materials (needed or used),

- topic of lesson,
- goal for lesson,
- how to make shapes,
- geometrical relationships of shapes, and
- repeating linear patterns.

A basic template with this information can provide your students with a point of reference to locate information for discussion groups and to review previous lessons.

There are several different methods available for students to represent information presented in the math module activities. The processes (observing, comparing, writing proofs, etc.) and specific tasks (measuring, counting, dividing, etc.) suggest different ways information can be represented. Some of the methods your students can use with the MCC activities are:

- taking notes about the topic,
- making lists,
- creating charts, tables, and graphs,
- writing down observations as they are doing the activities,
- making technical drawings,
- writing definitions of vocabulary words, and
- writing equations, proofs, and conjectures.

Model and Practice

Before asking students to record any entries using these methods, model and practice each one in large-group as well as small-group settings.

It is important to provide instruction and modeling of each of these methods *before* you ask students to record entries in their math notebooks. Through modeling, students can become comfortable with each method and feel more confident to begin recording on their own.

In MCC modules, there are learning goals for math lessons. As you plan the lesson from the module, take time to note the different kinds of explorations students will be involved in during the lesson. For example, students may be asked to draw different shapes or write and illustrate a set of directions. These are data that can be recorded in the math notebook.

Strand 2: Developing Multiple Literacies Through Stories

In an effort to address literacy development and reading comprehension, which are important to all disciplines, MCC includes traditional Yup'ik stories and culturally relevant stories with embedded math in the modules. Anecdotal data gathered from related research projects about the implementation of MCC between 2000 and 2004 suggests that stories associated with traditional Yup'ik cultural and subsistence activities such as egging, included in *Going to Egg Island* (2003), and berry picking in *Picking Berries and Gathering Data* (2004), play an important role in students' engagement in the traditional storytelling genre as well as help students connect to the math activities and concepts presented in the modules.

Accompanying the Rhombi module is the traditional story of Iluvaktuq, a mighty warrior and hunter. Iluvaktuq was a real person who lived in the Togiak region during the time of the Great War, referred to as the War of the Eye, which began during a dart game involving two boys. One of the boys put out the eye of the other. The father of the injured boy then put out both eyes of the one throwing the dart. If he had taken just one eye, the matter would have been settled. However, the result was the death of the two fathers and a long-reaching conflict that developed throughout the region.

Iluvaktuq was one of a group of great warriors who lived during that time. There is historic documentation in the travel journals of Petr Korsakovskiy (1988) of a meeting with one of the warriors, Apanuugpak, who describes the war with the Aglurmiut (People of the Penninsula):

> You should know that the Russian Aglurmiuts go on campaigns against us every year. They kill us. This makes us grieve for our relatives and we try to get revenge by killing for the death of our kinsmen. Most of us must go to the hills to hunt, and even if we don't go far from our settlements we are in great fear.

The story, Iluvaktuq, connects to the module primarily because of the connection to the *tumaqcat* patterns often found on Yup'ik parkas. Patterns are highly important markers indicating family lineage. The story of Iluvaktuq describes the origin of the distinctive element that can still be found on parkas in the Togiak region today. That element is the *miryaruak*, white fur sewn onto the shoulder area of the parka, which represents what Iluvaktuq vomited as he was running away from enemy warriors.

Yup'ik traditional narratives are often divided into the two categories of *qulirat* and *qanemcit* (plural forms of *quliraq* and *qanemciq*). Explanations of the differences between the two categories vary considerably (Orr et al., 1997). One explanation is that *quliraq* (singular) is a story that "originates in the distant past, is passed down from generation to generation and has the stamp and authority of collective tradition and authorship, and *qanemciq* (singular) is a personal and historical narrative that can be attributed to an individual author, even though he or she has been forgotten" (p. 214). In this classi cation then, what is identi ed as myth or fairytale in the Western or European tradition is a *quliraq*, and personal and historical narratives are *qanemciq* (Orr et al., 1997). However, the alternative classi cation that Orr et al. subscribe to is:

> A *quliraq* is a traditional narrative that has a framed and formulaic introduction (A long time ago there was a village, which was situated on the banks of a river, which owed out into the ocean. On the far extremity of the village there was a grandmother and grandchild…). A *quliraq* is perceived as being ctional, and therefore the actuality of events and characters is not predicated. On the contrary, a *qanemciq* tells of events that are perceived as actually having occurred (p. 214).

Storyknifing

A storyknife was used by little girls as they retold each other stories they had heard from elder storytellers, their mothers or older sisters. As a girl relates the story, she illustrates the events in a cleared area of mud or wet sand, using the tip of the storyknife, a wood or ivory knife typically made for her by her father.

When telling a knifestory, symbols representing the geographical setting, buildings, animals, objects and people described in the story are used. While each girl may draw the symbols in a slightly different way, they are understood by all participating in the storyknifing event (Fitzhugh & Kaplan, 1982).

Also, storyknifing was used to design parkas and border patterns. Evelyn Yanez said:

> We played games while storyknifing, "this is how my parka will look." Mary Active [an elder from Togiak] said one time that she would draw what she was going to make. When she made them later on she remembered what she drew. We would draw things that we would make. I would even draw how my parkas would look like and I would watch my mom. This is how my parka would look and I would draw it with the storyknife.

Therefore, in the above, alternative classi cation, myths and legends are *qanemcit*, events that actually took place, and *quliraq* are framed stories that everyone understands generally as narrations of ctional events.

The preferred classi cation of stories in the Togiak region is the rst explanation; a *qanemiciq* is a personal/historical narrative about a person, and *quliraq* is similar to the Western classi cation myth or legend that has been handed over generations. Annie Blue, Yup'ik elder and storyteller of Togiak, says that some stories start out as *qanemciq* and then become *quliraq* over time. According to Annie Blue, Iluvaktuq is such a story. It began as a *qanemciq* told about this great warrior during his lifetime, and as it was handed down over generations, the story became a *quliraq*, telling of Iluvaktuq's super-ordinary and heroic accomplishments.

It is important to recognize that stories in the oral tradition represent a well-developed literary genre that contains unique stylistic and rhetorical features and structures. These features and structures are comparable to the rhetorical devices found in Western literary genre (e.g., foreshadowing and repetition) and at the same time re ect devices like voice in ections, pacing and gestures, which are only available to the oral storyteller. Thus, because traditional oral storytelling is a literary genre, it seems important for children, regardless of linguistic, ethnic or cultural backgrounds, to experience this genre as part of the curriculum of school (Parker Webster & Yanez, forthcoming).

Because the story has been created from transcriptions and translations of live storytelling events and represents a close adaptation to written text, we suggest you rst read the story aloud in its entirety. This will provide students with an experience that is similar to listening to a traditional oral story being told at a storytelling event. It will also give students a preview to the story, so that when they are asked to read it on their own or with a buddy, they will have had experience with the style and vocabulary of the story.

Story Circle Roles

Summarizer—summarizes the story or the part of the story that was read for the Story Circle that day.

The rst task of the Summarizer is to identify and record the following literary elements.

- Main Characters—Who?
- Setting—Where?
- Problem—What?
- Solution—How?
- Theme or Main Idea—Why?

During the Story Circle meeting, the Summarizer reads his/her summary paragraph to the group and asks for any details that might have been missed.

Word Gatherer—chooses words from the story recorded on the Vocabulary Bookmark during independent or buddy reading.

During the Story Circle meeting, the group picks one word to study. Using the Vocabulary Map, the group completes each box.

Questioner—asks questions about the story. Beginning questions may involve things directly in the text. For example:

- How many bowls did Iluvaktuq's wife use to prepare the sh?

Other questions may require inferencing:

- When did Iluvaktuq get the idea that he could escape?

Connector— nds connections to her/his own life experiences. The Connector can also describe connections to other texts (like a history book or another ction novel or a science textbook) or media such as lm, TV, magazines, Internet sites and music.

Responder—chooses a short passage or sentence of the story to illustrate either through drawing, dramatization, dance or other non-linguistic medium.

Reader—locates a favorite part of the story to read aloud to the other group members. The Reader must rst tell why she/he chose the passage and then read it aloud to the group. The Reader can also ask group members to participate in reader's theatre or choral reading of the chosen passage.

Iluvaktuq and Paluqtalek: A Story of Two Warriors also contains text boxes that are embedded throughout. These text boxes amplify terms within the text and can be sources for further student inquiry that can allow for interdisciplinary study. There is also a glossary of terms at the end of the story to help develop vocabulary.

Developing Vocabulary

An emphasis is placed on vocabulary development both in the math and literacy strand and in the stories and literacy strand. Vocabulary development is a key factor in reading comprehension. In Thorndike's (1917) seminal study on reading comprehension, correct reading was described as occurring when (a) the correct meaning of each word was known, (b) the importance of the word and contextual meaning within the sentence was understood, and (c) the purpose and comprehension of the passage was examined and validated for understanding and adjustments were made when there was a breakdown in understanding. Biemiller (2001) suggested that when readers understand less than 95% of the words in a text, they will probably have dif culty comprehending the meaning of the text.

There are various approaches to vocabulary instruction. These include (a) incidental, developed through wide reading by the reader; (b) implicit, developing vocabulary through use of contextual clues within text by the reader; and (c) explicit, direct instruction. The vocabulary activities in this module combine all three approaches, with an emphasis on explicit instruction through modeling of vocabulary activities that support individual inquiry into word study. The vocabulary activities in this module are designed so that vocabulary is not rote memorizing of a de nition. Rather, like the math activities, vocabulary should be through hands-on practice that is connected to students' experiential and knowledge backgrounds, which results in conceptual development.

Master Vocabulary List

Note: *some of these words are not expressly mentioned in the module, but they are all related to the concepts presented.*

Angle—a geometric gure formed when two lines, rays, or line segments meet at a point. The meeting point is called the vertex of the angle. An angle is measured in degrees of rotation between the two lines, rays, or line segments. An angle can measure between 0 and 360 degrees.

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth.

Area measurement—the number of square units needed to exactly cover a two-dimensional shape.

Base—a side of a polygon by which the gure is measured.

Basic repeating element—a shape or collection of shapes that, when repeatedly copied and moved, make up a pattern. Basic repeating element refers to the smallest set of shapes that can be used to generate an entire pattern. (See **linear pattern, two-dimensional pattern**.)

Center—a point that is equidistant from all points in a circle; a point that is the intersection of the diagonals of a square, rectangle, rhombus, or parallelogram.

Composition of shapes—combining smaller geometric shapes to make larger ones.

Congruent—geometric gures (or parts of gures) that are the same shape and size. Two shapes are congruent if one shape can be slid, ipped, and/or rotated so it ts exactly on top of the other one. Parts of a shape, for example, sides or angles may also be considered congruent.

Congruence—the property of two shapes that are congruent.

Conjecture—a statement about a mathematical fact, relationship, or generalization that is based on careful observation or experimentation, but which has not been proven (see **proof**).

Conservation of area—when a shape is divided into two or more parts, all of the parts cover the same total area as the original shape, no matter how the parts are separated or recombined to form new shapes.

Decomposition—dividing a shape into smaller shapes or pieces.

Design—a pleasing shape or combination of shapes. A design may be intentionally created by someone or may be a consequence of natural forces.

Diagonal—a line segment joining two nonconsecutive vertices of a polygon. For a quadrilateral, a line joining opposite vertices.

Equilateral triangle—a triangle with all three sides the same length and all three angles equal to 60 degrees.

Flip—A rigid geometric transformation; see **reflection**.

Glide reflection—a geometric transformation composed of a translation and a re ection. Glide re ections are frequently used in constructing linear patterns.

Growing pattern—a sequence of shapes or numbers such that each shape or number is related to the preceding one by a rule governing how the shape or number increases (or decreases) from one shape to the next.

Height–a measure of a polygon, taken as a perpendicular from the base of the gure.

Hexagon—a polygon with six sides.

Horizontal line—a line parallel to the ground, or the bottom of a gure, or a piece of paper. Horizontal and vertical lines meet at right angles.

Hypotenuse—the side opposite the right angle in a right triangle.

Isosceles triangle—a triangle with at least two sides the same length. The angles at the base of an isosceles triangle are also equal.

Linear pattern—a one-dimensional (potentially) in nite pattern that repeats a basic unit (geometric shape or design) over and over again, such as a band, border pattern, or strip pattern. These are sometimes called "frieze patterns" because of their use in architectural settings.

Line of symmetry—a line that is a property of a geometric gure (a shape, design, or pattern); it divides the gure into two equal parts such that when the gure is re ected about that line, the result is identical to the starting gure. For example, a square has four lines of symmetry: vertical, horizontal, and diagonal lines through the center of a square are all lines of symmetry. An informal test for a line of symmetry in a two-dimensional shape is to fold the shape along a line through its center. If both sides match exactly after folding, the line is a line of symmetry.

Line segment—a part of a line with two endpoints and a de nite length.

Linear measurement—a measure of length, width, height, or distance.

Mirror symmetry—a property of a geometric gure with one or more lines of symmetry. If you place a mirror along a line of symmetry, the visual effect is that the half of the object and its re ection look exactly like the complete object.

Orientation—the property of a shape that describes its location in relation to its rotation from a reference line.

Parallelogram—a quadrilateral with both pairs of opposite sides parallel and equal.

Pattern—a design that consists of a basic design element repeated over and over again; a pattern may be extended in one direction (linear or frieze pattern), two directions (two-dimensional or "wallpaper" pattern).

Pentagon—a polygon with ve sides.

Plane—a at surface that extends in nitely in all directions; mathematically, a plane is considered to have zero thickness.

Polygon—a simple closed curve made up of connected line segments or sides.

Proof—a mathematical argument—based on logical reasoning—that demonstrates that a particular fact or relationship is true.

Property—a trait or characteristic of an object. The parts of a geometric shape: sides, angles, symmetries, and their relationships, that de ne a particular shape as a unique shape. For example, the basic properties of a square are that it has four equal sides and four equal angles. From this stem other properties such as the fact that all the angles are 90 degrees, that a square has four lines of symmetry, and that it is rotationally symmetric when rotated 90 degrees about its center.

Quadrilateral—a four-sided polygon.

Rectangle—a quadrilateral with four equal angles, all equal to 90 degrees. Its opposite sides are parallel and equal to each other. Alternative de nition: a parallelogram with all angles equal.

Reflection—one of the three basic rigid transformations. A re ection of a shape about a line that transforms the shape to exactly where it would appear in a mirror placed along that line. Also known as a **flip**. Alternatively, a re ection takes a shape and moves it across a given line, in such a way that every point in the shape moves to a new point that is the same distance on the opposite side of the line as the original point. See also **line of symmetry, line symmetry** and **mirror symmetry**.

Rectangle—a quadrilateral with opposite sides equal and four right angles.

Regular polygon—a polygon whose sides and angles are all equal. A square is a regular quadrilateral.

Rhombus (pl. **rhombi**)—a quadrilateral with four equal sides. Alternative de nition: a parallelogram with four equal sides.

Right angle—an angle that measures 90 degrees, or one fourth of a full rotation. This is the angle found in squares and rectangles. It is sometimes called a "square angle."

Right isoceles triangle—a right triangle with two equal sides. A half-square triangle.

Right triangle—a triangle with one 90 degree angle.

Rigid transformation—moving a gure into another position in such a way as to make no change in its shape or size. The three rigid transformations are translation (sliding), rotation (turning) and re ection (ipping).

Rotation—one of the three basic rigid transformations. A rotation takes a shape and turns it through an angle about a point called the center of rotation. Also known as a turn. See also **rotational symmetry**.

Rotational symmetry—the quality of a geometric gure (shape, design or pattern) such that when the gure is rotated about a given point, through a particular angle, the result is identical to the original shape. For example, when a square is rotated about its center through exactly 90 degrees, the result is identical to the square before it was rotated.

Shape—the characteristic surface con guration of a thing; an outline or contour.

Side—a line segment joining two adjacent vertices of a polygon; part of the perimeter of a polygon.

Slide—one of three rigid transformations. See **translation**.

Square—a regular quadrilateral. All sides have the same length and all the angles are right angles. Alternative de nition: a rectangle with all sides the same length.

Square unit—the square unit usually means the one with coordinates (0, 0), (1, 0), (1, 1), (0, 1) in the real plane. A unit of measure in area.

Symmetrical linear pattern—a nite linear pattern with a line of symmetry through its center.

Symmetry—mathematically, symmetry is a quality of a shape such that it does not change when transformed by a rigid transformation. There are three types of symmetry: line symmetry (or mirror symmetry), rotational symmetry (or point symmetry) and translational symmetry.

Transformation—moving a shape using translation rotation and re ection, in such a way that the shape itself does not change its shape or size.

Translation—one of three rigid transformations. A translation moves a shape from one position to another without rotating or re ecting it. Also known as a slide.

Triangle—a polygon with three sides.

Turn—one of three rigid transformations. See **rotation**.

Two-dimensional pattern—a two-dimensional (potentially) in nite pattern which repeats a basic unit (geometric shape or design) over and over again; such as in a wallpaper design. Two-dimensional patterns are sometimes referred to as "wallpaper patterns."

Unit of area—a shape used repeatedly to measure the area of another (usually larger) shape.

Vertex, vertices—the point or points at which sides of a polygon, or lines of an angle, meet. Vertices of polygons are sometimes called corners.

Vertical line—a line perpendicular to the ground, or the bottom of a gure, or a piece of paper. Vertical and horizontal lines meet at right angles.

Yup'ik Mathematical Terms

Shape Names

Circle—*akagenqellria* or *uivenqellria* (one that goes around easily, one that rotates perfectly).

Line, slanted—*ciliggluni.*

Line, straight—*iqliluni.*

Lines, two slanted (balanced symmetrically)—*ciligglukek.*

Oval—*uivlugtellria* (one that's something like a circle but doesn't rotate properly).

Rectangle (with long side at)—*taksurenqellria yaassiik* (box that tends to be long).

Rectangle (with short side at)—*naparenqellria yaassiik* (box that is straight up).

Rhombus—*takuralria ciliggluni* (something stretched and tilted).

Right isosceles triangle—*ciligcimalria yaassiigenqellria* (cut from a square at an angle: same as tilted square).
Parallelogram—*ciligtellriik cetrek akiqliqellriit* (two that are put along side each other and properly aligned or parallel).
Square, standard orientation—*kangirenqellria* (one with perfect corners); also *yaassiik* (square or box, from the Russian *yashchik*); *yaassiigenqellria* (one that is a perfect box).
Square, tilted—*ciligtellria yaassiigenqellria* (a square turned at an angle—same as a diamond.
Triangle—*pingayunek kangiralek* (one with three corners).

Process and Pattern Terms

Some of these Yup'ik words are not in the module but they are useful terminology related to making patterns.

Airraq—string used for string stories
Airrartuq—she or he is telling a string story
Akunlengqerrluteng—pattern with something between the repeating shapes
Atkuut—overcoats
Ayuqlutek akiqliqellriik—pattern or design with opposite sides the same (symmetrical)
Ayuquralriit—repeating element in a pattern
Elirqun (shape used as a tool to cut something out)—shape used as a template to cut out other shapes
Elirqaaq—shape, cut to be used in a pattern (use *elirqun* to make *elirgaat*)
Elirqumalriit tumartarkat—shapes, a group to be sewn together
Ciligtaq—something cut at an angle
Miryaruak—white fur sewn onto the shoulder area of a parka
Kameksiik—boots
Nacarrluk—beaded headdress
Piluguut—warm boots
Qasperet—dresses
Quliraq (plural form *qulirat*)—a story that originates in the distant past, is passed down from generation to generation, and has the stamp and authority of collective tradition and authorship
*Qanemciq (*plural form *qanemcit*)—a personal and historical narrative that can be attributed to an individual author, even though he or she has been forgotten
Sugat—dolls
Tumaqcat—pieces, a set that can be copied to make a whole pattern
Tumaqcaq (singular form)—a piece
Tumaqcak (dual form)—two pieces
Tumaqluki—putting the pieces together (process)
Tumarciq—put pieces together to make patterns
Tumartat (plural form)—the gathering of pieces to put together to make a whole (product)
Tungliqu'urluteng ayuqellriit—pattern with repeating shapes
Ulruut—pants
Upterrlainarluta—we have to always be ready
Upterrlainaryaraq—way of always getting ready

Master Materials List

Teacher Provides

1-inch graph paper
1-foot rulers
8½ x 11 inch blank paper
Blue books (or equivalent) for student math journals
Butcher paper or chart paper
Clear blank transparencies to be used with Black Line Masters
Colored blank transparencies in two contrasting colors
Colored pencils or markers
Construction paper in two or more contrasting colors
Envelopes for saving cut-out shapes
Glue for felt
Index cards
Notebooks or three-ring binders (one per student)
Oak tag, manila folders, or cereal boxes
Overhead projector
Pencils
Scissors (1 pair per student)
String or yarn
4-inch long strips of elastic (one per student)
Transparency markers
Sheets of felt in contrasting colors
Optional: Needles and thread

Package Includes

DVD

Tumartat: Gathering the Pieces to make the Whole

CD

Yup'ik Glossary

Black Line Masters

Handout, Shortcut for the Area of a Rhombus
Handout, Area Recording Chart
Handout, Biographical Sketch of Winifred Beans
Handout, Headband Area Table
Handout, Properties Analysis Table
Handout, Related Shapes
Handout, Sample Headband Patterns
Handout, Shortcut for the Area of a Rectangle
Handout, Shortcut for the Area of a Rhombus (2)
Handout, Shortcut for the Area of a Triangle
Handout, Vocabulary Map
Transparency, 6 x 4 Inch Rectangle on 1-inch Grid
Transparency, Rhombus and Four Right Triangles Inscribed in a 6 x 4 Inch Rectangle, 1-inch Grid
Transparency, Comparing Areas in Two Puzzles
Transparency, Map of Southwest Alaska
Transparency, Sample Headband Patterns
Transparency, Shortcut for Area of a Rectangle
Transparency, Shortcut for Area of a Rhombus
Transparency, Shortcut for Area of a Triangle

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Internet Resources

http://www.pballew.net/rhomb.html

A historical essay on the origin and meaning of the term rhombus.

http://www.rpi.edu/~eglash/csdt.html

Culturally situated design tools: Teaching math through culture

Ethnomathematician Ron Eglash, of Rensselaer Polytechnic Institute (RPI), has created software to help students understand pattern-making activities from several indigenous cultures. A direct link to the prototype Yup'ik pattern tool is at http://www.ccd.rpi.edu/Eglash/csdt/na/yupik/yupik.html

Activity 1
Creating a Rhombus from a Rectangle: Two Folds, One Cut

In this activity, students are introduced to the rhombus module with examples of headband patterns, which will be their nal project. They will also learn some of the Yup'ik design principles such as balancing color (black and white typically) and symmetry. This activity challenges students to learn by observing an elder making a rhombus from a rectangle. They see the process modeled, rst in a DVD of Winifred Beans, a Yup'ik elder from St. Marys, and second by you, the teacher. Yup'ik elders have demonstrated to us a process of rst cutting out the pieces that they will use and then arranging the shapes in a pattern. This begins the process of developing the geometric tools for their project by folding and cutting. This activity sets the stage for all the other activities in this module. Students get an intuitive understanding of the geometrical relationships of the rectangle to the rhombus as well as of lines of symmetry. In later activities students explore these relationships more directly as they design and make a headband.

Goals

- Students will create a rhombus from a rectangle by folding and making one cut, by following a procedure modeled in a DVD, Tumartat: *Gathering the Pieces to Make the Whole.*
- Students will write a description of the procedure they used to make the rhombus.

Materials

- Transparency, Sample Headband Patterns
- Biographical sketch of Winifred Beans
- Wall map or transparency, Map of Southwest Alaska
- DVD of Winifred Beans, Tumartat: *Gathering the Pieces to Make the Whole*
- Plain paper cut into four rectangles—one sheet (or four rectangles) per student
- Scissors—one pair per student
- Butcher paper for a whole class chart
- Envelopes for saving shapes—one per student
- Math notebooks

Duration

One to two class periods.

Vocabulary

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth.

Congruence—two or more shapes having the same size and shape regardless of orientation

Design—a pleasing shape or combination of shapes. A design may be intentionally created by someone or may be a consequence of natural forces

Diagonal—a line segment joining two nonconsecutive vertices of a polygon. For a quadrilateral, a line joining opposite vertices

Isosceles triangle—a triangle with at least two sides the same length. The angles at the base of an isosceles triangle are also equal.

Line of symmetry—a line that is a property of a geometric gure (a shape, design, or pattern); it divides the gure into two equal parts such that when the gure is re ected about that line, the result is identical to the starting gure.

Parallelogram—a quadrilateral with opposite sides parallel and equal and opposite angles equal

Quadrilateral—a four-sided polygon

Rectangle—a quadrilateral with opposite sides equal and four right angles

Rhombus—a quadrilateral with four equal sides and opposite angles equal. (plural, rhombi)

Right triangle—a triangle with one 90-degree angle

Square—a quadrilateral with four equal sides and four right angles

Triangle—a polygon with three sides

Preparation

Have the clip of Winifred Beans ready to go. View this clip so that you are familiar with the process of making a rhombus by folding and then cutting a rectangle.

Explore different ways of cutting the folded rectangle and be prepared to model the procedure used by Winifred Beans. If possible, bring in a Yup'ik parka, headdress, or other objects that have border patterns.

Students will be using a math notebook throughout the module. Prepare their math notebooks by using a dedicated notebook, a three ring-binder, or your customary way.

Cultural Note

Yup'ik elders make their headbands using small pieces of black and white calfskin. Each piece is very carefully shaped and cut so that they line up precisely with all the other shapes. The entire headband is made by sewing together many smaller shapes. The animal skins are very precious, so that the elders plan their work carefully so they will waste as little material as possible. Students will learn to plan their shapes and patterns carefully so they can cut out the right number of each shape in the proper colors and then assemble the shapes into a headband.

Instructions

1. Explain to the students that during the next few math classes they will learn how to make several different types of shapes and learn how the shapes are related to each other. Later in the module they will make their own headbands.

2. Show the students the transparency, Sample Headband Patterns. Show the students a headdress if you can get one. Explain that later in the module they will be making their own headbands, similar to the ones shown in the transparencies. Ask them to describe the shapes they see in the patterns. (They may see squares, diamonds, triangles, parallelograms and rhombi. Don't worry if they don't know the technical names for all these shapes. They will learn them as they work through the module.) Next, ask them to describe the patterns they see. They may say that all the patterns seem to repeat the same shapes on both sides of the center.

3. Share the transparency, Map of Southwest Alaska (page 35) and point out the Southwest region and the village of St. Marys. Explain that this module is based on what we learned from Winifred Beans, an elder from St. Marys, as well as from elders in Togiak and Manokotak.

4. Read or tell the brief biographical sketch of Winifred Beans (page 36).

5. Introduce the DVD you are about to share of Winifred Beans. Explain that she learned to sew as a young girl by watching the elders at that time. She is now considered an expert seamstress in her community and surrounding villages. Among other things, she is known for the fancy shapes used in her border patterns. We are going to learn about her shapes and then build our own set of pattern pieces.

6. **Modeling.** Show the DVD of Winifred cutting out a rhombus from one piece of paper with only one cut. Ask students to watch carefully because they will be making the shape afterwards. Show it a second time if necessary. Ask students to describe what they saw in the DVD. The purpose of this is to help students clarify the procedure they are going to carry out (see Figure 1.1).

7. **Joint Activity.** Hand out several rectangles and a pair of scissors to each student. Ask them to make the same shape Winifred just made in the DVD. Model the process. Students may observe you or other students as they cut out the rhombus. Encourage students who have successfully cut out the rhombus to model for those who have not.

Math Note

Figure 1.1 illustrates one way that Winifred Beans cuts out a rhombus from a rectangle. In step 1, she creates a line of symmetry and folds her original rectangle to form two congruent halves. In step 2, she folds along another line of symmetry to form four congruent rectangles, each ¼ of the original. She cuts along the diagonal, and with one cut she creates one rhombus and four congruent right triangles. Later students will discover that the area of the four right triangles equals the area of the rhombus, and each is half the area of the original rectangle.

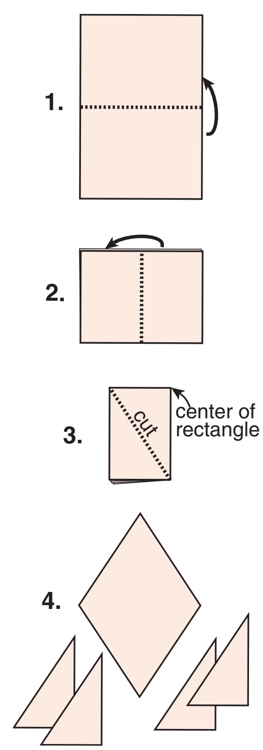


Fig. 1.1: Winifred's method of cutting a rhombus

Teacher Note

Students may or may not have a shape just like Winifred's, but they may say it is like hers anyway. Also, students may not be able to produce what Winifred made; encourage them by saying they will have more chances to learn this method and right now they are just practicing. This activity establishes an experiential base for students' understanding of the concepts of line of symmetry and congruence. Build students' intuitive knowledge by allowing them to try out methods to discover how the shapes they create may or may not be similar to what Winifred made. Encourage students to keep track of their methods, even when the shape didn't work out as they expected. Their mistakes can help them visualize the folded paper and anticipate what will happen when it is cut in different ways.

Teacher Note: Pedagogy

Joint activity can be an effective way for students to learn from the teacher and peers and vice versa. Joint activity is when the teacher solves the same problem and performs the same task simultaneously with the students. Students can observe the teacher to see how he or she makes decisions, makes mistakes, solves problems, etc. It is a form of cognitive apprenticeship. The project's qualitative research shows evidence that joint activity enhances student autonomy, responsibility, and learning. Joint activity appears to change the social relations in the classroom, changing it from a top-down authority to more open-ended explorations.

8. **Sharing.** Ask students to share if any of their shapes were similar to Winifred's shape and their methods for cutting out the shapes. Did anyone form two pairs of isosceles triangles instead of a rhombus?

   Ask the students, how do you know which corner to cut? (Winifred cuts her rectangle and four triangles from corner to corner, making sure *not* to cut through the center of the rectangle.) Ask students to predict what would happen if they cut from the corner formed at the center of the rectangle, to the opposite corner. Pass out two pieces of paper to each student and ask them if they can make both the rhombus and the isosceles triangles. Explain the difference in the procedure (see Figure 1.2).

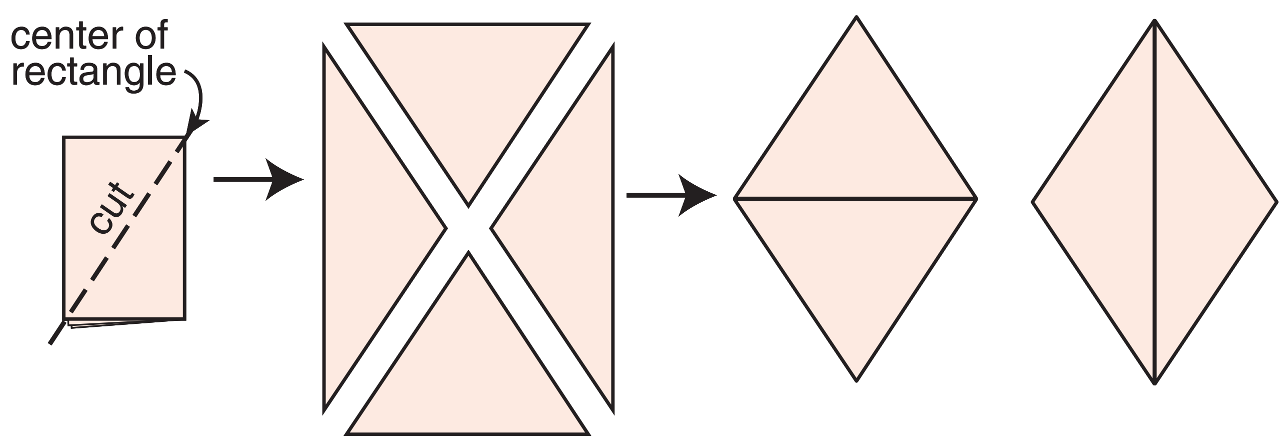


Fig. 1.2: Cutting the folded rectangle through the center produces two pairs of congruent isosceles triangles. This is not Winifred's method.

9. **Pair sharing and notebook writing.** Have students work in pairs to discuss speci cs of how to make the shape just like Winifred's. As the pair discusses the steps, they should write and/or draw in their notebooks the procedure they used to cut a rhombus from a rectangle by folding and cutting. The pairs should then create a set of written directions that may also include drawings for the procedure, so that someone who had not seen the DVD could follow their directions and create a rhombus. Ask pairs to test another pair's set of directions and provide feedback and revise if needed.

10. In groups or pairs, ask students to devise an informal name for the four-sided shape they have cut out. (Students typically call this shape a "diamond," and your students may have other names as well.) Encourage them to create a name that is descriptive of the shape. Have them write their name in their math notebooks and provide a drawing of a rhombus. Explain that this shape has a special mathematical name; it is called a *rhombus*. Ask students to describe the other shapes (four triangles) that were left over when they made the rhombus. These shapes are called *right triangles* because they include a square corner or *right angle*.

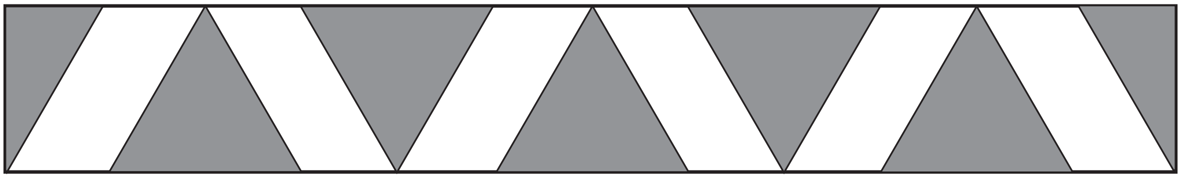
11. Ask students to save all their shapes, rhombus and triangles, in an envelope. Ask students to save all the shapes that they have created even if they were different from the shapes Winifred Beans made.

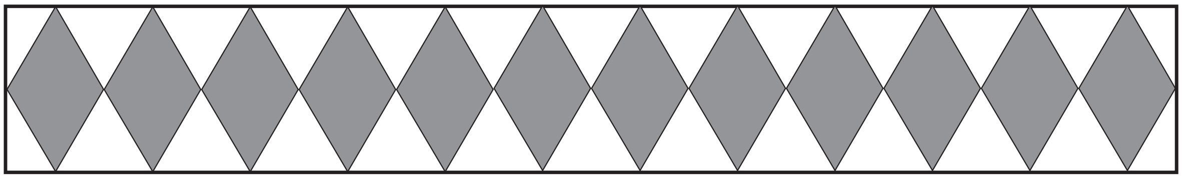
12. Have your class identify expert seamstresses in your village or area. If possible, have them come to your class and share their method of making parka pattern pieces. You can use this time as well to interview the elder and create your own biographical sketch similar to one for Winifred Beans.

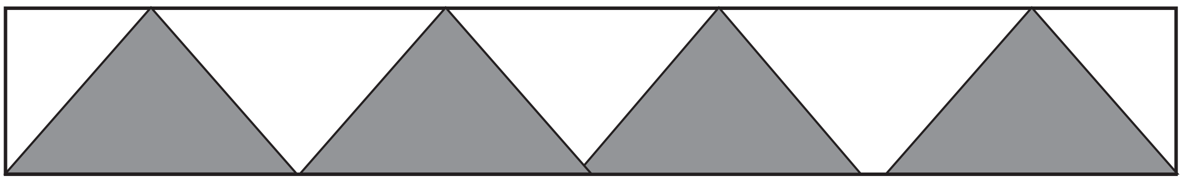
Teacher Note

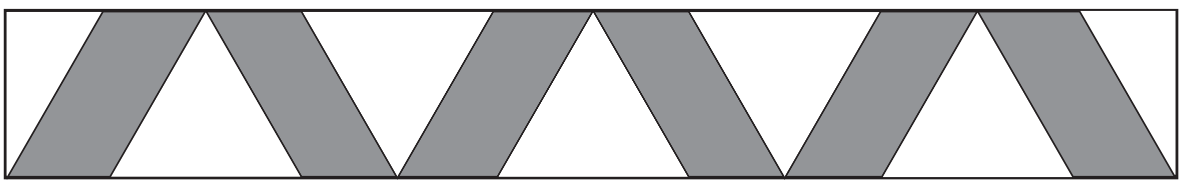
This activity introduces students to the use of their math notebooks, which will be used throughout the module as a way for students to describe and thereby consolidate their learning. Encourage students to write and draw pictures showing and describing what they have done. They can also use their notebooks to record vocabulary words and any questions they have about the geometry they are learning.

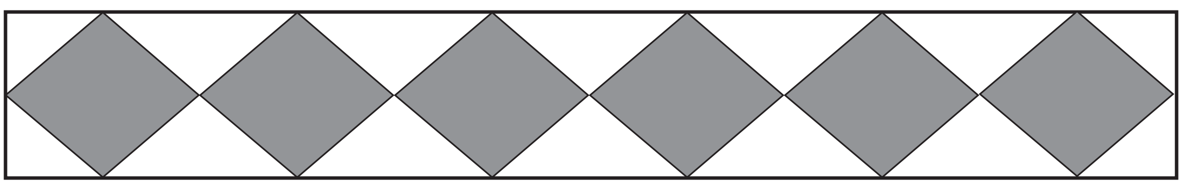
Sample Headband Patterns

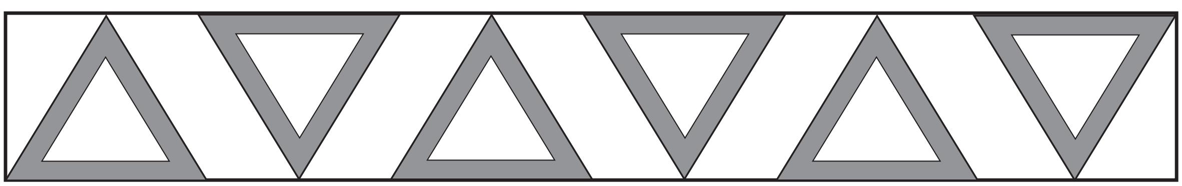
1.


2.


3.


4.


5.


6.


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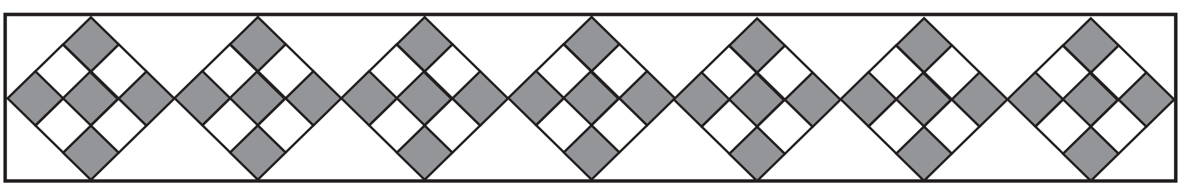

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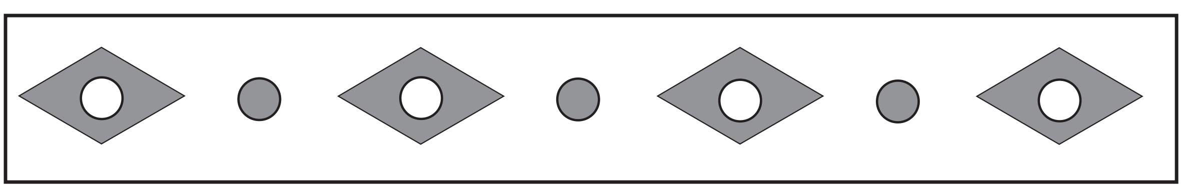
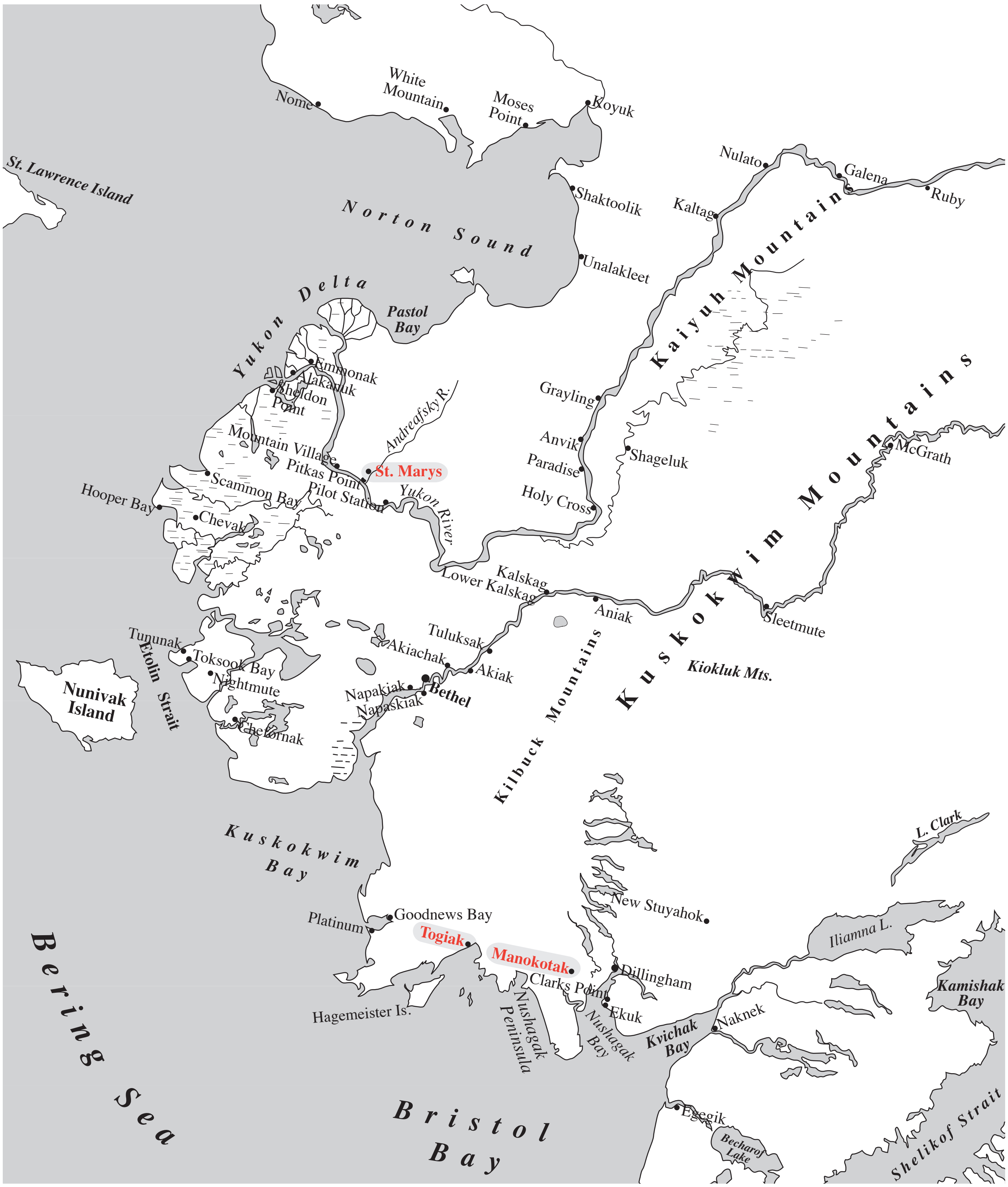


Map of Southwest Alaska



Biographical Sketch of Winifred Beans

Winifred Beans was born on August 10, 1917, in Black River, an old village below Qisilvak, which is a mountain across from Mountain Village. Winifred's parents were Anna and Joseph Joe. She and her family traveled to seasonal camps along the Yukon Delta and Black River while she was growing up.

Winifred is the oldest of nine children. She has been sewing for 80 years, since she was 8 years old. She was taught to sew by her grandmother and mother. She started learning by watching them. She made her first parka at the age of 18. She has sewn fur coats with inside and outside fur, mittens, hats, boots, fish skins, reindeer boots, and grass weaving baskets for her 12 children. She has more than 30 grandchildren, about 10 great-grandchildren, and 1 great-great-grandchild and sews for all of them.

In the Yukon region, Winifred is known for her skillful sewing. She likes to make all kinds of things. Recently she made a pair of sealskin slippers with beaver trimmings and moose soles. She's making a great-grandchild boots and a parka for his potlatch presentation. She made all of her children parkas and made three fancy parkas for her daughters. She also made a traditional squirrel parka for her great-grandchild. This parka has short sides and is long in the front and back, the kind that her grandmother made her when she was that age. Today these parkas are in Big Lake and Eagle River outside Anchorage and in St. Marys where her daughters are living.

Winifred has helped in the development of this module by sharing her knowledge and ways of making different parka pattern pieces. Winifred occasionally uses the rhombus as her starting piece. She makes fancy designs like flowers and circular pieces for her slipper patterns.

Activity 2
Lines of Symmetry and Congruence

Winifred Beans and other elders we have worked with use lines of symmetry as a way to create their pattern pieces. Similarly, in this activity, students explore the lines of symmetry of the rectangle and the cut-out rhombus; they also explore the congruence of the four right triangles that are also formed when cutting a rhombus. This activity builds on the previous one in which students learned how to make a rhombus from a rectangle. This time they identify and describe the mathematical properties of rectangles, rhombi, and right triangles and explore relationships among all three shapes. The mathematical principles that students learn become design tools that will enable them to produce symmetrical patterns for their headbands.

In this activity, students will also conduct word studies of math vocabulary terms using the Vocabulary Map. This approach to word study, using the Vocabulary Map, should continue through subsequent module activities.

Goals

Students will be able to:

- Identify lines of symmetry for a rectangle and a rhombus.
- Explain how lines of symmetry are used to cut a rhombus and four congruent triangles from a rectangle.
- Demonstrate whether shapes are congruent by laying one shape on top of another and seeing whether they match.
- Describe the properties of a rectangle and a rhombus and use them to distinguish between the two shapes.

Materials

- Plain paper cut into six rectangles—one sheet (or six rectangles) per student
- Scissors—one pair per student
- String or yarn
- Butcher paper for a whole class chart
- Math notebooks
- Handout, Vocabulary Map

Duration

One to two class periods.

Vocabulary

Angle—the figure formed by two lines or line segments that meet at a corner

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth.

Congruent/congruence—two or more shapes having the same size and shape regardless of orientation are said to be congruent to each other. Their relationship is described as congruence.

Conjecture—a statement about a mathematical fact, relationship, or generalization that is based on careful observation or experimentation, but which has not been proven

Diagonal—a line joining two opposite vertices (or corners) of a quadrilateral. For any polygon, a line joining any two nonadjacent vertices

Isosceles triangle—a triangle with at least two sides the same length. The angles at the base of an isosceles triangle are also equal.

Line of symmetry—a line through the center of a shape such that when you fold the shape about that line, the folded halves match exactly

Parallelogram—a quadrilateral with both pairs of opposite sides parallel and equal

Pentagon—a polygon with five sides

Polygon—a simple closed curve made up of connected line segments or sides

Property—a trait or characteristic of an object. Properties of polygons include number of sides and angles, relationships of sides to each other, types and relationships of angles, and number and location of lines of symmetry.

Quadrilateral—a four-sided polygon

Rectangle—a quadrilateral with four equal angles, all equal to 90-degrees. Its opposite sides are parallel and equal to each other. Alternative definition: a parallelogram with all angles equal

Rhombus (pl. rhombi)—a rhombus is a quadrilateral with four equal sides. Alternative definition: a parallelogram with four equal sides

Right triangle—A triangle with one right angle

Side—a line segment joining two adjacent vertices of a polygon; part of the perimeter of a polygon

Square—a regular quadrilateral. All sides have the same length and all the angles are right angles. Alternative definition: a rectangle with all sides the same length

Symmetry—mathematically, a quality of a shape such that it does not change when transformed by a rigid transformation. There are three types of symmetry: line symmetry (or mirror symmetry), rotational symmetry (or point symmetry) and translational symmetry

Triangle—a polygon with three sides

Vertex—the point at which two lines or edges of a polygon meet (pl. vertices). Students may also call them corners.

Preparation

For Extension 1, practice other methods of cutting the folded rectangle. Familiarize yourself with the cut-out pieces and their relationships. Re-create the ndings shown in in the Math Note “Other Ways of Cutting” (page 43).

In preparation for Extensions 2 and 3, get string or yarn, tie it together into a loop, and practice making string gures. Notice the different shapes that are made. Prepare tied loops of string for your students. View the accompanying DVD Tumartat: *Gathering the Pieces to make the Whole* to more fully understand how different elders use lines of symmetry and midpoints as a way of making their pieces.

Instructions

1. **Model.** Gather the students around. Connect the students’ name for the rhombus shape to the technical term rhombus. Remind them of Winifred Beans’ process for cutting a rhombus from a rectangle with one cut. Fold the rectangle in one direction. Ask them how you knew where to make the rst fold. (They may say, “down the middle.”) Open the fold and ask them what they can observe about the two halves. The two halves are congruent (identical) and symmetrical. Point out the line of symmetry if they don’t identify it.

   Fold the rectangle in the other direction. Ask students how you knew where to fold it. (They’ll probably say in the middle.) Ask them whether both sides of the rectangle match. How can they tell? Ask what would have happened if they had folded their rectangles along any other lines.

   Introduce the vocabulary term “line of symmetry.” Explain that one way to test whether a line is or is not a line of symmetry is to fold a shape along a particular line and see whether the sides match each other.

2. **Explore.** Have the students work in pairs. Hand out six paper rectangles and a pair of scissors to each student. Now ask the students to take one of their rectangles and nd out how many lines of symmetry it has. Ask them to also try folding along other lines to prove that they are not lines of symmetry. Ask if they can create a fold that is not a line of symmetry but has two congruent shapes. Ask for volunteers to show you what they have discovered.

3. **Model.** Now refold your rectangle into quarters and ask students to tell you where to cut to make a rhombus. Ask them how they know where you should cut. If they say “corner to corner,” model folding through the center to the opposite corner. (This cut does not produce a rhombus.)

Teacher Note

Some students use the term “hot-dog-wise” to describe folding a rectangle along its longer line of symmetry. They use the term “hamburger-wise” to describe folding a rectangle in half about its shorter line of symmetry.

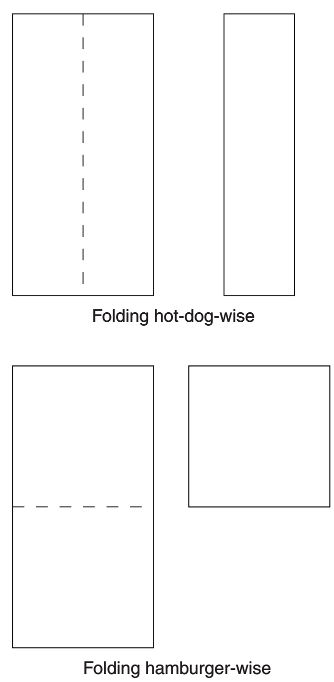


Fig. 2.1: Folding a rectangle hot-dog-wise and hamburger-wise

Math Note

A rectangle is usually defined as a quadrilateral with four 90-degree angles. (All its other properties follow from this basic definition.) In general, the opposite sides of a rectangle are equal, with one pair of sides longer than the other. A square can be thought of as a "special rectangle"—a rectangle with all four sides equal.

A rhombus is usually defined as a quadrilateral with four equal sides. (All its other properties follow from the basic definition.) In general the opposite angles of a rhombus are equal, one pair being acute angles (less than 90 degrees) and the other pair being obtuse angles (between 90 and 180 degrees). A square can be thought of as a "special rhombus"—a rhombus with all four angles equal.

Help them remember that they have to cut from corner to corner without cutting the center.

Math and Cultural Note

Winifred Beans' method of cutting a rhombus from a rectangle illustrates three important geometric principles used in Yup'ik mathematical designs:

1. Take advantage of symmetry. By folding her rectangle into quarters, Winifred ensures that the resulting rhombus will be symmetrical and that the four right triangles will be congruent.
2. Create shapes whose edges match. The edges of the right triangles and rhombi produced by this method will match up exactly when they are formed into patterns.
3. Divide shapes in a way that preserves geometric relationships. The four right triangles can be rearranged to form a rhombus. The areas of the eight triangles form a ratio of 1 to 4. Yup'ik designs frequently contain shapes whose areas are related by simple ratios.

4. **Explore.** Have students work in pairs or small groups to cut a rhombus from one of their rectangles. Ask them to use what they have learned about lines of symmetry and congruence. Have the students use what they learned through your modeling (lines of symmetry and congruence) to explore the rhombus and the rectangle further and make other discoveries about the shapes. (See Math Note, left.) Have them record their results in their math notebooks as they discover them.

Teacher Note

Have students keep copies of their paper folding before cutting and place them in their math notebooks. The folded paper provides a good record of their work and mathematical understanding. If students anticipate what the lines on their rectangle will look like before they unfold their paper, it will strengthen their ability to visualize.

5. Have students share their discoveries. Here are some questions you can use to get at ideas not brought up by students:
   - How many sides does your shape have? Are any of the sides equal to each other? How do you know?
   - How many angles does your shape have? Are any of the angles equal to each other? How do you know?
   - How many lines of symmetry does each shape have? How do you know?

- Are there other ways to fold a rectangle or a rhombus in half that are not lines of symmetry? If they have not found these, ask them to try and show what they have found. (See Math Note, below.)

6. Use the class discussion in step 5 to create a list of properties, key vocabulary, and illustrations if necessary of a rectangle and a rhombus on the board or a sheet of butcher paper. From this create a class chart. **Teacher Note:** Figure 2.3 shows some of the properties they might come up with, not necessarily in this order or with this exact wording. See the Math Note on the next page for suggested math vocabulary. As the class list of properties is created have students record them in their math notebooks. This is an ongoing process: as students explore and discover other properties, record them on the poster and have them

Math Vocabulary: Build students' math vocabulary throughout the module. Use the idea of creating posters, illustrations, properties, and definitions.

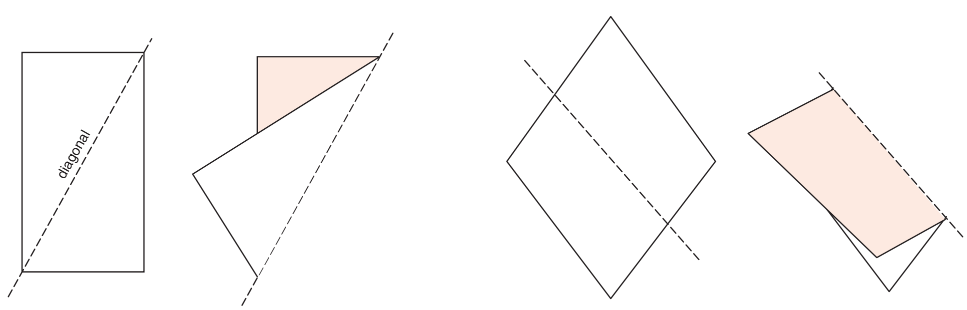
Math Note

The lines of symmetry of a rectangle become the lines of symmetry of the rhombus after it is cut out. However, the lines of symmetry of a rectangle pass through the midpoints of its sides, while the lines of symmetry of a rhombus pass through its vertices.

If you fold a rectangle along a diagonal (from vertex to vertex), you divide it into two congruent halves (triangles). This is not a line of symmetry because the folded shapes do not match. But they are congruent. In order to prove they are congruent, you would have to cut them apart and place one triangle on top of the other.

You can also fold a rhombus into two congruent parallelograms along a line passing through two opposite sides. Like the diagonal of a rectangle, this line is not a line of symmetry, because the two halves don't match when folded. You have to cut them apart and place one parallelogram on top of the other to prove they are congruent.

Investigating shape properties in this way can be an important step in mathematical thinking; it helps students make mathematical distinctions (the two halves are congruent but not symmetrical) and leads to the idea of what it means to prove a mathematical conjecture. In addition, labeling key math vocabulary can help students build their math vocabulary and concepts.



Congruent but not symmetrical

Congruent and symmetrical

Fig. 2.2: Dividing a rectangle and a rhombus into congruent but nonsymmetrical halves

record them in their notebooks. Leave room for another column that will be added in Activity 3 when students explore squares and rhombi.

Properties of a Rectangle	Properties of a Rhombus
• Four sides	• Four sides
• Opposite sides equal	• All sides equal
• Opposite sides parallel	• Opposite sides parallel
• All angles equal to 90 degrees	• Opposite sides equal
• Two lines of symmetry (passing through middle of sides)	• Two lines of symmetry (passing through corners)

Fig. 2.3: Chart of properties of a rectangle and a rhombus

7. Ask the students to explore and describe the four triangles created by cutting out the rhombus using what they have learned about symmetry and congruence. Students should recognize that the triangles are congruent to each other and that they have no lines of symmetry.

8. **Discuss.** Ask students to explain why they always get a rhombus and four congruent triangles when they use Winifred Beans' method. This may be dif cult to explain, but it will be helpful for students to try. You may revisit this question again when they have had more experience. You may prompt their thinking by asking why they think Winifred begins by folding a rectangle into a smaller rectangle using its lines of symmetry.

   The critical idea is that Winfred has already made four equal rectangles before she cuts. By cutting all four small rectangles at the same time, she ensures that the central shape will have four identical sides (a rhombus) and that the cut-off shapes will all be congruent to each other.

9. Model the Vocabulary Map using one of the vocabulary words. Ask students to buddy study one word together. This will promote discussion of the word before writing. Figure 2.4 shows an example of the Vocabulary Map lled in. Have students work in pairs or small groups. Have each student per group choose a math vocabulary word. Remind students to write in their math notebooks (and draw diagrams) to show what they learned about the shapes they have been studying, including math vocabulary. Hand out the Vocabulary Map to help students in de ning math vocabulary and concepts.

Teacher Note

The vocabulary map is another way to build students' math concepts and vocabulary. Use this approach as a template for later math activities. Math vocabulary development should be ongoing throughout the module. Repeat the Vocabulary Map as a word study of important math vocabulary in subsequent activities. Use other modalities to develop vocabulary. For example, have students act out math vocabulary words and have other students guess the term.

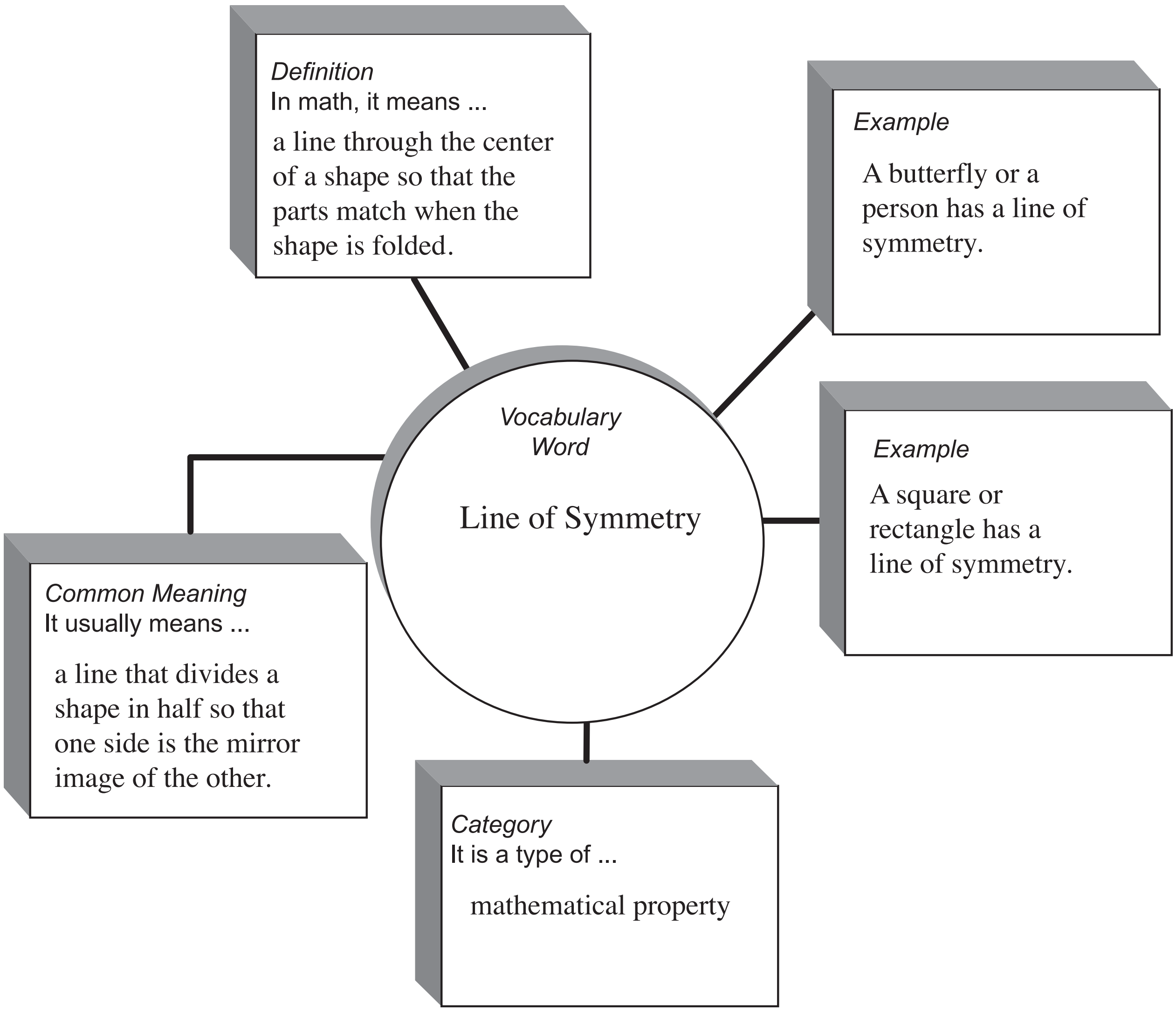


Fig. 2.4: A filled-in Vocabulary Map

Extension Activities

Extension 1: Cutting Without Folds

Ask students to try to cut out a rhombus from a rectangle without folding it. Ask the students how they decided where to cut and how they can be sure the piece they cut is a rhombus. Ask them to discuss which method is easier, folding or not folding. See Figure 2.4 and the Math Note, next page.

Extension 2: Finding Shapes in String Figures

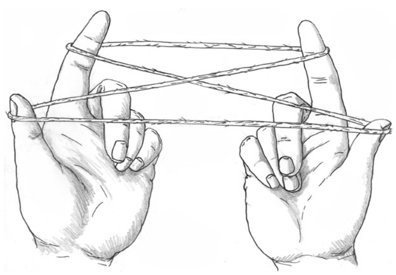


Fig. 2.5: An example of making shapes with string

Your students may enjoy exploring another way of making rhombi, rectangles, triangles, and other shapes using string games. These games, sometimes called "cat's cradle," are part of Yup'ik culture and are known in many cultures worldwide. String games can be played by two or more people taking turns or by individuals working alone. (The Yup'ik word or concept for these games is *airraq,* string used for string stories, or *airrartuq*—she/he is telling a string story.)

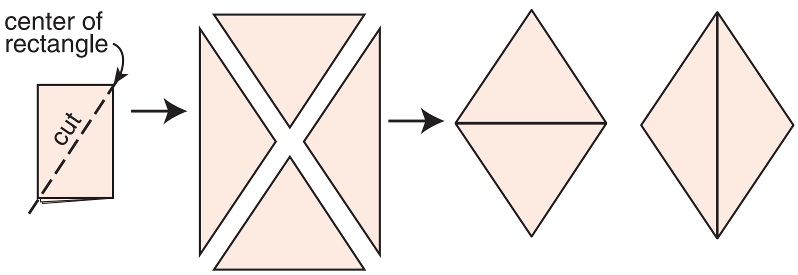
Model making string gures and ask students to identify the shapes they see. Then distribute string loops to your students. Invite students who know some string gures to demonstrate them for the others. As the students play, ask them to informally name the shapes they see. Ask them if they can see rectangles, rhombi, triangles, and any other shapes they recognize.

Extension 3: Cross-age Sharing

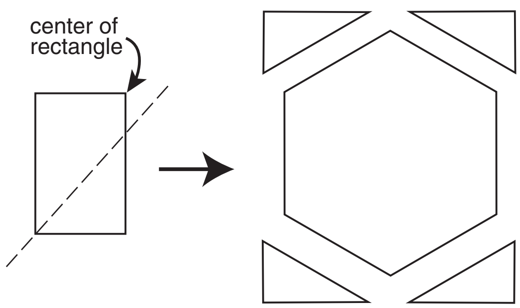
Tell the students that they can share this method of making a rhombus from a rectangle with rst or second graders.

Math Note: Other Ways of Cutting

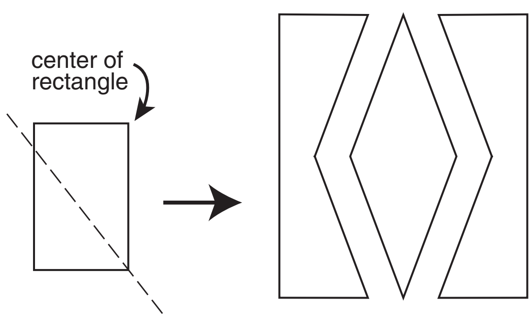
Ask students to explore what happens when they fold a rectangle into four equal parts and cut it in a different way (not corner to corner). Ask them to experiment with different ways of making one cut through the folded rectangle. What shapes can they get? What are the properties of these shapes? Make a class chart of the different shapes that they make by cutting a folded rectangle. Be sure to include all the pieces cut from each rectangle. Here are few examples of shapes students might create as they work through this.



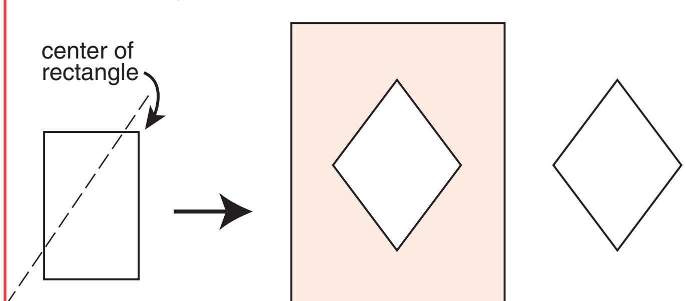
a. Cutting through the center to the opposite corner produces two pairs of isosceles triangles. Either pair can be put together to form a rhombus or cut in half to form two right triangles.



b. Cutting across the center from one corner to the side opposite the center produces a hexagon and four (narrow) congruent right triangles.



c. Cutting across the center from one corner to the side next to the center produces a narrow rhombus and two concave pentagons (five-sided shapes).

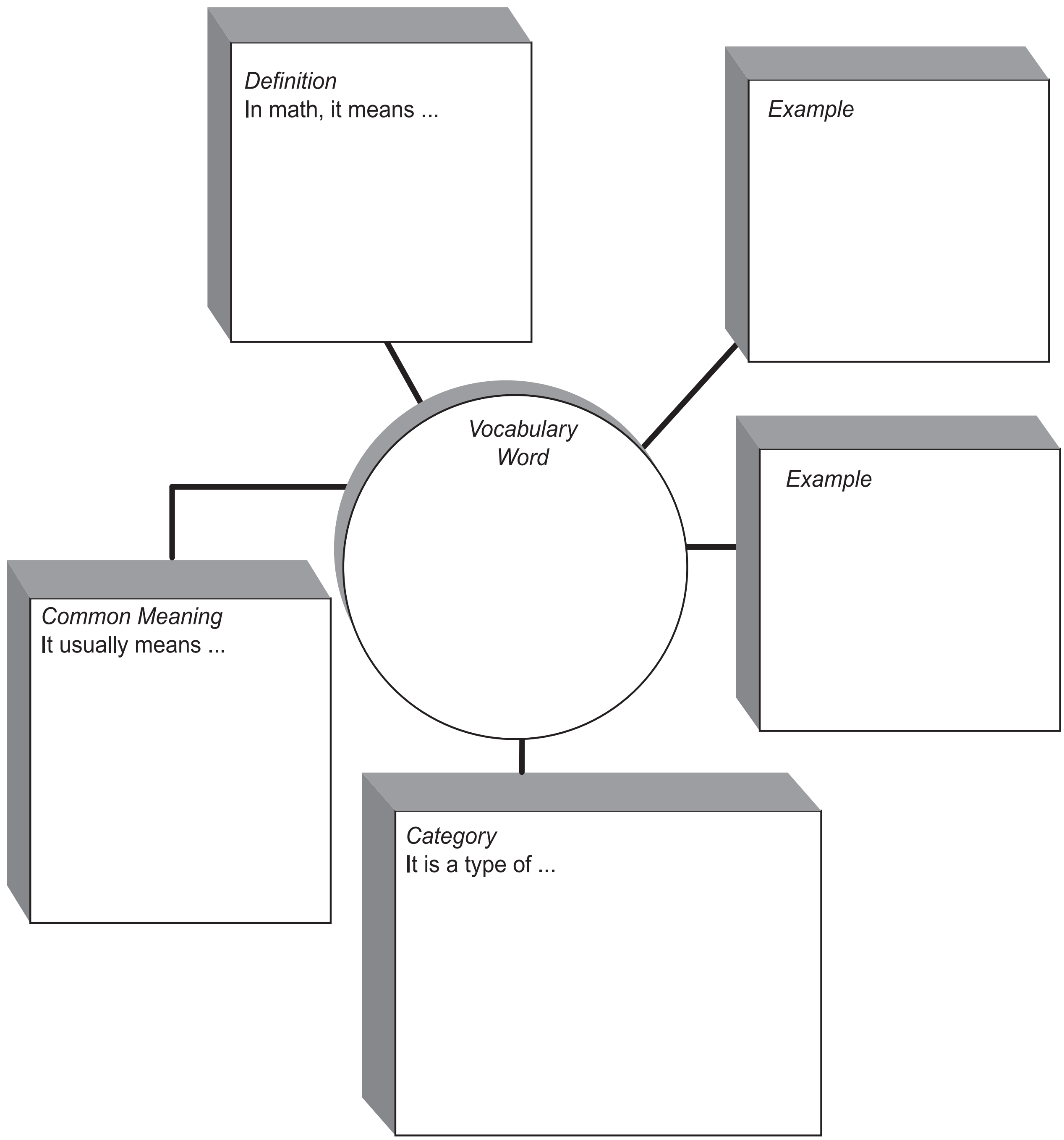


d. Cutting across the center, inside the corners produces a small rhombus and a rectangle with a rhombus-shaped hole in it.

A few other variations are also possible depending on the location and direction of the cut.

Fig. 2.6 (a, b, c, and d): Different ways of making one cut in a rectangle folded into quarters

Vocabulary Map



Activity 3
Starting with Square: An Exploration

In this activity, students start with a square. Using symmetry as a tool, they follow Winifred Beans' procedure for making a rhombus from a rectangle. The students compare their results (a smaller square and four right isosceles triangles) with the rhombus and triangles they got when starting from a rectangle. They explore the properties of a square and compare them with those of a rectangle and a rhombus. Interestingly, the Yup'ik word for rectangle is *taksurenqellria yaassiik,* which literally means "stretched out." This focuses our attention on the difference between rectangles and squares. Students also compare the right isosceles triangles with the right triangles they got starting from a rectangle. All of this will help students develop an understanding that a square is a special kind of rectangle and a special kind of rhombus. When folding and cutting the square, which is a special rectangle, students will continue to get smaller squares, which are special rhombi. Squares and right isosceles triangles are additional shapes that students can use to design their headbands. This activity gives them a greater variety of shapes to use when they make their designs, while reinforcing their intuitive understanding of Yup'ik design tools.

Math Note

The quadrilateral family tree shows the "family relationship" among six types of quadrilaterals. In particular, it illustrates that a rhombus and a rectangle are special cases of parallelograms and that a square is both a special case of a rhombus and a special case of a rectangle. A square is also a special case of a parallelogram.

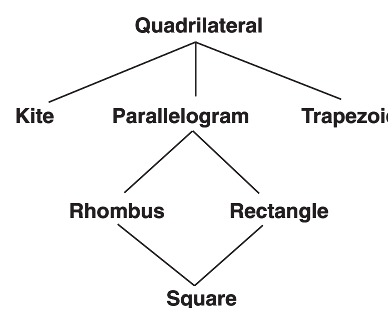


Fig. 3.1: A quadrilateral family tree

Goals

Students will be able to:

- Perform the same cutting and folding procedure on a square that they did on a rectangle.
- Compare the properties of a square with those of a rectangle and a rhombus.
- Compare the properties of a right isosceles triangle with those of a right triangle.

Materials

- Plain paper cut into squares—four squares per student
- Scissors—one pair per student
- Class chart of shape properties from Activity 2
- Handout, Properties Analysis Table—one per student
- Math notebooks

Duration

One class period.

Vocabulary

Angle—a geometric gure formed when two lines, rays, or line segments meet at a point. The meeting point is called the vertex of the angle. An angle is measured in degrees of rotation between the two lines, rays, or line segments. An angle can measure between 0 and 360 degrees.

Congruent—geometric gures (or parts of gures) that are the same shape and size. Two shapes are congruent if one shape can be slid, ipped, and/or rotated so it ts exactly on top of the other one. Parts of a shape, for example, sides or angles may also be considered congruent.

Design—a pleasing shape or combination of shapes. A design may be intentionally created by someone or may be a consequence of natural forces.

Diagonal—a line segment joining two nonconsecutive vertices of a polygon. For a quadrilateral, a line joining opposite vertices.

Isosceles triangle—a triangle with two equal sides

Line of symmetry—a line that is a property of a geometric gure (a shape, design, or pattern); it divides the gure into two equal parts such that when the gure is re ected about that line, the result is identical to the starting gure.

Parallelogram—a quadrilateral with both pairs of opposite sides parallel and equal

Properties—the parts of a geometric shape: sides, angles, symmetries and their relationships, that de ne a particular shape as a unique shape.

Quadrilateral—a four-sided polygon

Rectangle—a quadrilateral with four equal angles, all equal to 90-degrees. Its opposite sides are parallel and equal to each other. Alternative de nition: a parallelogram with all angles equal.

Rhombus (pl. rhombi)—a rhombus is a quadrilateral with four equal sides. Alternative de nition: a parallelogram with four equal sides.

Right angle—an angle that measures 90 degrees, or one fourth of a full rotation. This is the angle found in squares and rectangles. It is sometimes called a “square angle.”

Right isosceles triangle—a right triangle with two equal sides. A half-square triangle

Side—a line segment joining two adjacent vertices of a polygon; part of the perimeter of a polygon

Square—a regular quadrilateral. All sides have the same length and all the angles are right angles.

Symmetry—a quality of a shape such that it does not change when transformed by a rigid transformation. There are three types of symmetry: line symmetry (or mirror symmetry), rotational symmetry (or point symmetry) and translational symmetry.

Triangle—a polygon with three sides

Vertices—the points at which sides of a polygon, or lines of an angle meet. Vertices of polygons are sometimes called corners.

Preparation

Practice cutting a rhombus and four right triangles starting from a square. Familiarize yourself with the Quadrilaterals Family Tree in Figure 3.1. Extension 3 is both culturally and mathematically rich. Adapt your instruction accordingly.

Math Note

If you start with a square, fold it in quarters and cut it from corner to corner, you will end up with a smaller square and four right isosceles triangles. This is similar to starting with a rectangle (a square is a specialized type of rectangle) and ending with a rhombus (a square is also a specialized type of rhombus, one with sides and angles equal). Later in the lesson you can challenge students to think more deeply about shape relationships. Ask them how a square can be both a special kind of rhombus and a special kind of rectangle.

Instructions

1. As a way to increase the types of shapes we will use in our project, model cutting out a rhombus from a square. Follow Winifred Beans' method of making a rhombus, as the students did in previous activities.

2. Challenge: what shapes do you get? Hand out square sheets of paper to each student and have the students work in small groups as they try to cut out a rhombus from the square.

3. Ask students to discuss and show what they found. Ask, is the quadrilateral that you got a rhombus? Have students explore the shape they made in order to prove if it is or isn't a rhombus by using the properties chart. How is this result like what you made before? How is it different? Students should recognize that the shape is a rhombus because all four sides are equal. They should also recognize it as a square because (if they folded and cut their rst square carefully) all four angles are equal as well.

4. **Discuss.** Add a new column, Properties of a Square, to the class chart and ll in what they discovered. Some possibilities are shown in Figure 3.2.

Properties of a Square	Properties of a Rectangle	Properties of a Rhombus
• Four sides • All sides equal • All angles equal • All angles 90 degrees • Four lines of symmetry passing through sides and vertices • Diagonals are equal and are each a line of symmetry. There are four lines of symmetry and two of those lines are the diagonals.	• Four sides • Opposite sides equal • All angles equal • All angles are 90 degrees • Two lines of symmetry passing through sides	• Four sides • All sides equal • Opposite angles equal • Two lines of symmetry passing through vertices

Fig. 3.2: Properties of a square, a rectangle, and a rhombus

5. Ask students to respond to the following in their math notebooks.
   - What are the similarities and differences between a rectangle and a square?

Teacher Note

These questions are a way to get at a form of logical reasoning called class inclusion. Logically speaking, a square has all the properties of a rectangle: opposite sides equal and parallel, all angles equal to 90 degrees, two lines of symmetry passing through the midpoints of the sides. Therefore a square is a rectangle. However, since it is a special rectangle with all sides equal, it is given a special name, square. Review the properties of a rhombus. A square has all the properties of a rhombus, so it is a rhombus. It is, however, a special kind of rhombus, one with all four angles equal.

Students who are abstract thinkers will be able to grasp this logical relationship. However, concrete thinkers may have difficulty accepting it. For example, they may argue that a shape with four equal sides can never be called a rectangle, because they only have experience with certain types of rectangles. One way that students develop their reasoning abilities is by discussing logical questions with those who have different ways of thinking. If you discuss this with your class, ask them to justify their statements with examples from the shapes they have been working with. Try to leave these as an open question. Forcing students who are not ready to memorize the class inclusion relationships will not help them advance their learning in the long run. Exposing them to the ideas and letting them percolate will help them in the long run.

Math Note

Right isosceles triangles can be put together in three different ways, as shown in Figure 3.3(a). The parallelogram can be difficult for some students to find. When a right isosceles triangle is divided in half along its line of symmetry, it produces two congruent right isosceles triangles, shown in Figure 3.3(b).

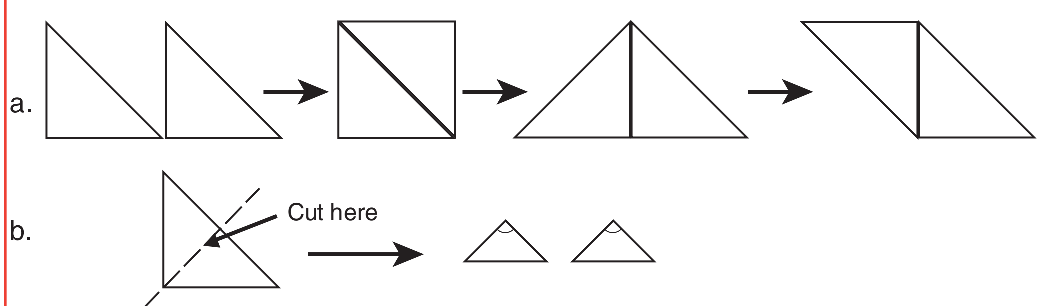


Fig. 3.3: (a) Two right isosceles triangles can be rearranged to form a square, a larger right isosceles triangle, or a parallelogram. (b) If you cut a right isosceles triangle along its line of symmetry, you get two smaller congruent right isosceles triangles.

- Some people say that a square is a special kind of rectangle. Do you agree? Why or why not?
- What are the similarities and differences between a rhombus and a square?
- Some people say that a square is a special kind of rhombus. Do you agree? Why or why not?

6. Hand out the Properties Analysis Table and students' squares, rhombi, and rectangles. Fill in the appropriate properties for each shape. When the properties list is complete, indicate whether the square, rhombus or

rectangle has the property by recording either a plus (+) or minus (–). This way of organizing information about shape properties makes it easy to compare different shapes property by property.

	Four sides	All sides are equal	2 lines of symmetry	4 lines of symmetry	All angles = 90°	Diagonals are =
Rectangle	+	–	+	–	+	+
Square	+	+	+	+	+	+
Rhombus	+	+	+	–	–	–

Fig. 3.4: Properties analysis table

7. **Explore.** Students use their cut-out shapes from previous activities: triangles from squares and from rectangles derived from cutting out rhombi. Ask them to find as many properties of these shapes as they can and compare them to the triangles they made when they folded and cut a rectangle. Have students create a similar table as they did for step 6. Here are some of the properties they might come up with:
   - All four triangles are congruent to each other.
   - Each triangle includes a right angle.
   - The triangles have two equal sides and two equal angles.
   - Each triangle has one line of symmetry.

   The first two properties are the same for the triangles they got when cutting a rectangle. The second two properties are different.

8. Explore designs by putting right triangles together. Point out that a right triangle with two equal sides has a special name, right isosceles triangle. These triangles have some interesting properties. Ask students to explore the shapes they get if they put two right isosceles triangles together so that two sides match exactly. They should find three different ways to do this. Then ask them to cut one of their right isosceles triangles in half. What shapes do they get?

9. **Recording.** Ask students to record what they have learned about squares and right isosceles triangles in their math notebooks. Encourage them to draw sketches of the shapes and/or paste shapes into their notebooks. This helps students consolidate their learning and gives you an opportunity to assess their understandings.

10. Remind them to save their cutouts in their envelopes for use in Activity 4 and as possible pieces for their patterns.

Literacy Counts Teacher Note

The Properties Analysis table is similar to a vocabulary strategy called semantic feature analysis. For example, a semantic feature analysis for fish might include gills, dorsal fin, scales, two eyes on one side of head, eyes on both sides of head, pink flesh, white flesh, etc. Different kinds of fish could include king salmon, coho salmon, ling cod, halibut, etc.

Math Vocabulary: Build math vocabulary, concepts, and properties of shapes.

Assessment: Use students' recordings of what they have learned in their math notebooks.

Extension Activities

Teacher Note

If you don't cut from corner to corner you can get a smaller square or a rhombus that is not a square. If you do so, the "leftover" shapes will not be triangles. The results are similar to those described in the Math Note for Activity 2, Extension 1 (see page 45).

Extension 1: Other Ways of Cutting a Square

Ask students, "Can you cut out more than one type of rhombus from the square?" Encourage them to explore different ways of cutting a square folded into quarters and to record the results they get and the properties of the resulting shapes.

Math Note

Here is how Winifred Beans cuts a square from a rectangle.

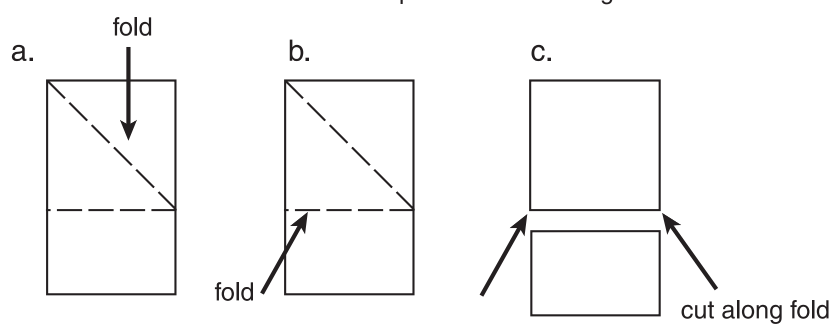


Fig. 3.5: How to cut a square from a rectangle

Extension 2: Cutting a Square from a Rectangle

Your students may be interested in trying to cut a square from a rectangle. Winifred Beans says that "There is a square in every rectangle." Ask students if they can nd a way to fold a rectangle so they can cut out a perfect square. This is another way for them to become familiar with the properties of a square. Depending on your students' con dence level, you can let them explore this on their own before modeling it, or you can model Winifred's method rst and ask them to reproduce it.

"It's discovery for them, discovery for me. I've learned more about how my students think than in any other thing I've done in math."
Terri Chapdelaine
Buckland, Alaska

Extension 3: There is a Circle in Every Square

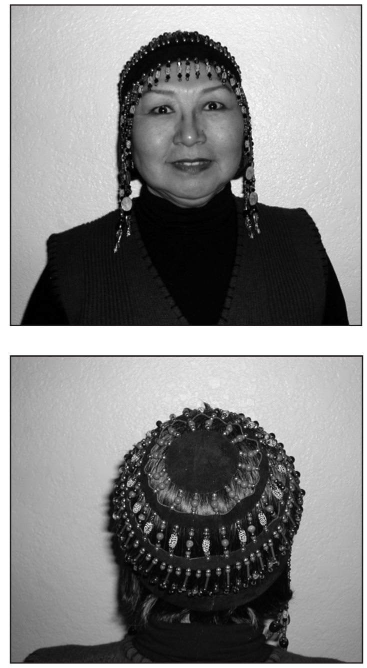


Fig. 3.6: Dora wearing her headdress. Photo by Pamela VanWechel.

Dora Andrew-Ihrke made a headdress using circle patterns (Figure 3.6). Here's how she made the pattern for the circle, called the Nuna or the Earth.

Begin with a square, using the body measure of the space between the first and second knuckle of the index finger. Measure two of these for the length and two for the width. (See Figure 3.7.) Cut a square of this size.

Next, locate the centerpoint of the square by folding it into quarters to form a smaller square. Cut a strip of paper the length of this smaller square. This becomes the template for the radius of your circle. See Figure 3.8.

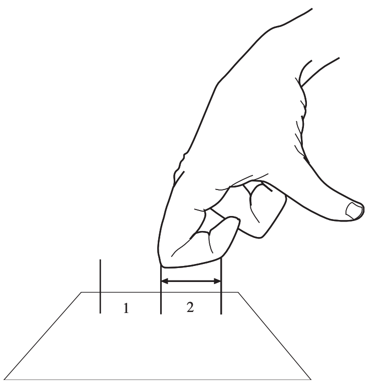


Fig. 3.7: Measuring a square with your finger

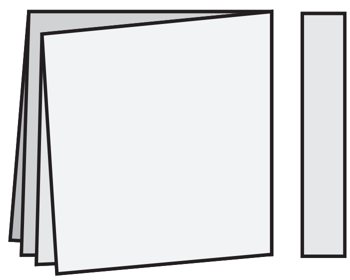


Fig. 3.8: A square folded into quarters, and a strip cut to the length of the folded square

Open up the square. The vertical and horizontal folds represent the wind directions: north, south, east and west (Figure 3.9).

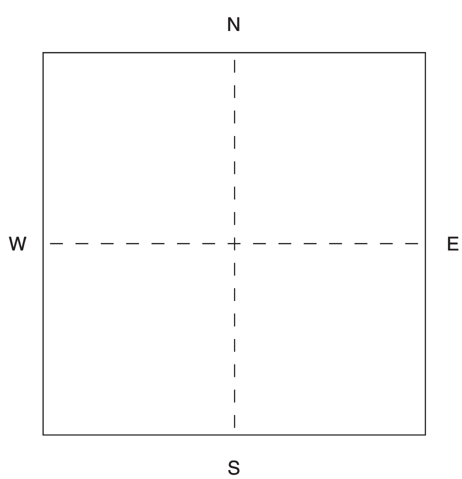


Fig. 3.9: The folds in the open square represent the four wind directions.

Fold the square along its diagonals and open it up. Use your template to mark the distance from the center along each diagonal fold. Draw a straight line at each mark, perpendicular to the fold. Cut along each line to form an octagon (Figure 3.10). Use your scissors to round the edges (Figure 3.11).

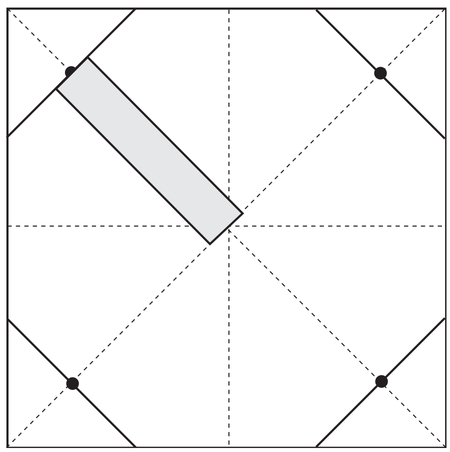


Fig. 3.10: Using the template to mark the distance from the center

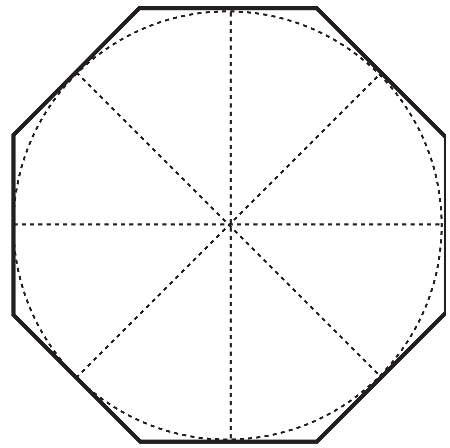


Fig. 3.11: Rounding the edges

Cultural Note

When Dora Andrew-Ihrke worked for Dillingham City Schools, she asked the elders if she could use their knowledge for teaching and learning. The following was given to her from Elena Pat. Dora was the only one she allowed to do the louse dance. Elena told Dora about the beaded headdress, *nacarrluk* (see Figure 3.12).

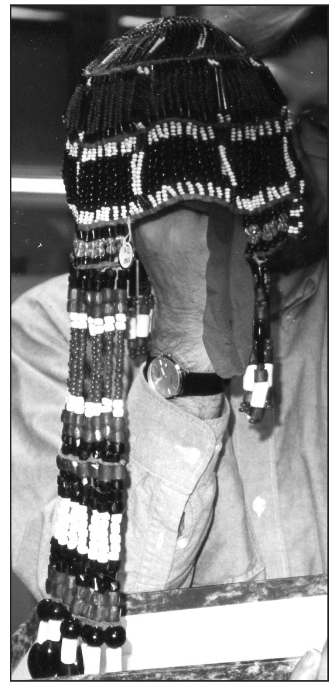


Fig. 3.12: A traditional beaded headdress (nacarrluk)*. Photo taken by Dora Andrew-Ihrke at the Museum Fur Volkerkunde, Berlin, Germany.*

To make the *nacarrluk*, you begin with the circle, which is at the top or crown of the head. This represents the earth, which is connected to the universe. Strips are then connected to the circle. These strips represent rings that encompass the stars, constellations and planets of the universe. To make a strip, you measure the width of the fingernail of the index finger (see Figure 3.13). The number of the strips depends on the size of the beads. For example, if you have small beads you might have three strips (see Figure 3.14). And if you have larger beads you might have two strips. The space between the strips is measured by two fingers (index and middle finger).

Fig. 3.13

The bottom strip is measured right over the eyebrow, over the top of the ear around the back of the head. When Elena Pat measured Dora's headress with string, she started the measurement at the center of the forehead, positioned right over the eyebrow and top of the ear. She then proceeded measuring with the string in a right-to-left direction around the head, until the string met again at the center. This formed the bottom strip. The second strip was measured the same way, but three fingers apart from the bottom strip. The third strip is measured three fingers from the second.

At the top of the *nacarrluk* is a Nuna or Earth, made according to the instructions given for making a circle above. You now have a circle surrounded by three rings (Figure 3.15). Now you can finish the *nacarrluk*.

1. Locate the four winds. The lines of symmetry become the four wind directions north, south, east, and west. The diagonals will form the directions of northwest, northeast, southwest, and southeast. See Figure 3.16.
2. Use these wind directions as connection points to the first strip with beads.
3. Long dangling beads are sewn at the northwest and northeast points.
4. Then the beads are arranged in an arc from longest to shortest beads with the shortest beads meeting at the center point of the forehead.

After the circle and strips are made, you connect them by adding the beads from the north/south/east/west. (See Figure 3.12.)

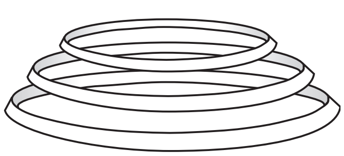


Fig. 3.14

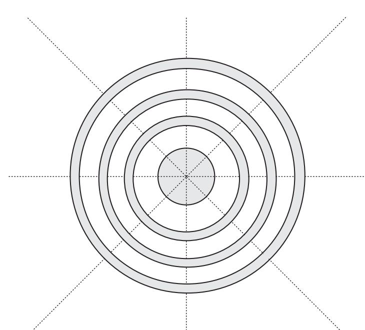


Fig. 3.15

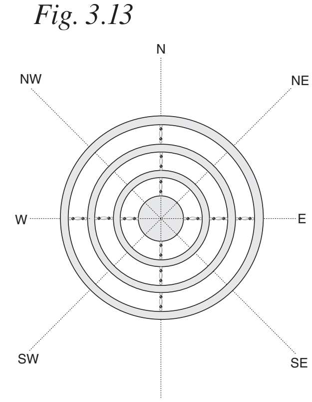


Fig. 3.16

Properties Analysis Table

	Four sides	All sides are equal	2 lines of symmetry	4 lines of symmetry	All angles are 90˚	Diagonals are equal
Rectangle						
Square						
Rhombus						

Activity 4
Exploring Shape Relationships: Geometrical Composition and Decomposition

In this activity, students explore the shapes they created by cutting and folding a rectangle. They will decompose and recompose the shapes to get an understanding of the area relationships between the rectangle and the cut-out pieces and among the cut-out pieces. They repeat this process with the shapes they cut from a folded square. In this way students explore part-to-whole and part-to-part relationships and conservation of area. Note that elders have told us that they usually have the final product in mind when creating a design. So far in the module, students have only focused on shapes. This lesson begins to focus students' attention on design, as they rearrange the cut-out pieces into other shapes that could become part of their final projects. As they find pleasing ways in which pieces fit together they should copy them into their notebooks. These shapes may be used when students make their headbands or final project.

Goals

Students will be able to:

- Describe the area relationships among the rhombus, the four triangles, and the original rectangle they were cut from.
- Compare the area relationships that result from cutting a square with those from cutting a rectangle.

Materials

- 1-inch graph paper—one sheet for every two students
- Shapes students have cut out previously
- Handout, Related Shapes—one per student
- Math notebooks
- Index cards or stock paper (optional)
- Colored pencils or markers (optional)
- Colored transparency paper or plain transparency paper and transparency markers to color

Duration

One to two class periods.

Vocabulary

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth.

Composition of shapes—putting smaller shapes together to form a larger shape

Conservation of area—When a shape is divided into two or more parts, all of the parts cover the same total area as the original shape, no matter how the parts are separated or recombined to form new shapes.

Decomposition of shapes—dividing a shape into smaller shapes

Design—a pleasing shape or combination of shapes. A design may be intentionally created by someone or may be a consequence of natural forces.

Isosceles triangle—a triangle with at least two sides the same length. The angles at the base of an isosceles triangle are also equal.

Parallelogram—a quadrilateral with both pairs of opposite sides parallel and equal

Properties—the parts of a geometric shape: sides, angles, symmetries and their relationships, that de ne a particular shape as a unique shape. For example, the basic properties of a square are that it has four equal sides and four equal angles. From this stem other properties such as the fact that all the angles are 90 degrees, that a square has four lines of symmetry, and that it is rotationally symmetric when rotated 90 degrees about its center.

Rectangle—a quadrilateral with four equal angles, all equal to 90 degrees. Its opposite sides are parallel and equal to each other. Alternative de nition: a parallelogram with all angles equal.

Rhombus (pl. rhombi)—a quadrilateral with four equal sides. Alternative de nition: a parallelogram with four equal sides.

Shape—the characteristic surface con guration of a thing; an outline or contour.

Square—a regular quadrilateral. All sides have the same length and all the angles are right angles. Alternative de nition: a rectangle with all sides the same length.

Triangle—a polygon with three sides

Preparation

Use colored transparencies to make a set of related shapes: an “original” size rectangle, a rhombus cut from that size rectangle, and four triangles cut out from the rectangle.

Use colored transparencies to make a similar set of shapes starting from a square. If color transparencies are not available, color the shapes with transparency markers.

Instructions

1. Have students take out their cut-outs from the previous activities.

2. **Model.** Use the overhead projector and colored transparencies to illustrate that the four triangles can be placed exactly on top of the rhombus. Explain that this is one way to demonstrate that the area, the amount of space covered by the four triangles, is equal to the area of the rhombus. Later, we will see that we can make different designs by rearranging shapes to form new ones with the same area. When we make our headbands, we will all have the same amount (area) of material, but through creativity there are many different possibilities for making unique designs.

3. **Explore.** Ask students to work with their shapes to see if they can determine what fraction of the area of the original rectangle is covered by the rhombus. If students have dif culty with this, ask them to determine how many triangles would be needed to cover the rectangle.

4. **Explore.** Ask students to work in pairs to see what other relationships they can work out among the rhombus, the four triangles, and the rectangle. You can encourage pairs by asking these questions if necessary: What is the relationship between the area of one triangle and the area of a rhombus? What is the relationship between the area of a triangle and the area of a rectangle? How can they demonstrate these relationships using the cut-out shapes? Encourage students to make more copies of their shapes if necessary so that they can physically show the different geometrical relationships between the parts and the parts and the whole.

5. Ask students to represent their ndings by drawing pictures on graph paper and writing sentences to prove that their ndings are correct.

Teacher Note

Students could explain by showing paper folding and cutting or you could provide crayons or markers if they would prefer to color in other shapes and how they fit into the starting rectangle as well as how each shape relates to the others.

Teacher Note

Students should be able to discover and explain the following area relationships:

- The area of a cut-out rhombus is ½ the area of the starting rectangle.
- The area of all four cut-out triangles is equal to the area of the cut-out rhombus.
- The area of each cut-out triangle is ⅛ the area of the starting rectangle.
- The area of each cut-out triangle is ¼ the area of a cut-out rhombus.

In step 6, students should find that the same relationships hold true for the smaller square and four right isosceles triangles cut from a larger square. It is important for students to be able to explain their reasoning. This is the kind of reasoning they will need in order to draw conclusions about other relationships.

6. **Explore.** Have students explore the same relationships starting from a square, using the cut-out rhombus (smaller square) from the square and the four right isosceles triangles. They should find the same area relationships among these shapes that they did for shapes cut from a rectangle. Ask students to record these findings on another sheet of graph paper.

7. **Further Explorations:** Encourage students to further explore geometric and area relationships, or distribute the handout, Related Shapes (see Figure 4.1). Ask students to explain how each shape is related to every other one. Ask if they found additional shapes and relationships based on this set of material (for example, making a parallelogram out of two of Triangle 1, etc.).

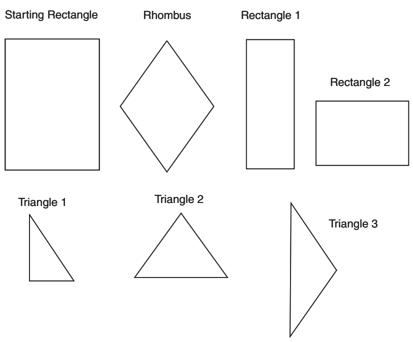


Fig. 4.1. Related shapes

8. **Explore.** Ask students to use one set of pieces (either those they cut from a rectangle or those cut from a square), and to see how many different shapes they can make by rearranging the pieces. Shapes should be rearranged so their sides match, edge to edge. They can use different numbers of pieces: two triangles, three triangles, or four triangles to form a new shape. Or they can use the triangles in combination with the rhombus.

   Suggest that students record their new shapes by drawing them on graph paper and placing in their math notebooks. As students recompose their shapes, ask them to note any pleasing arrangements that could be used later in their headband project. Ask students to name and describe the properties of the new shapes they create.

Continue to build math vocabulary. Use Vocabulary Maps.

9. **Make Tangram-like Puzzles.** Ask students to use one of their combined shapes (either from the square or the rectangle) and encourage them to make puzzles by drawing around the outsides of their combined shapes. They can use any of their shapes as long as they combine them so that their edges match. Then pass the outlines to another student along with a set of shapes. Students now try to find the shapes that fill the outline. (See Math Note, next page, for examples.) Students can create their puzzles on index cards, stock paper, or in their math notebooks as a way to record their designs for possible use for their headbands.

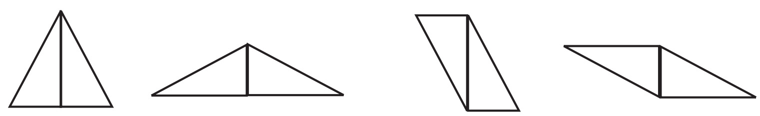
   You may want to make a set of puzzles for the class on index cards, with each student contributing one or two. (Of course all the shapes must be cut from the same-sized rectangle so the puzzles will work with the shapes.) This activity helps reinforce understanding of shape relationships and helps students share design ideas. **Teacher Note:** This is a good activity for your students to use with younger students (cross-age tutoring).

Assessment: Use students' artifacts to determine their mathematical understanding.

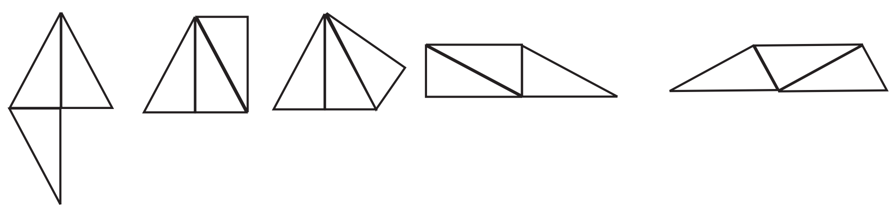
Math Note

Here are some of the shapes students might come up with. Students may find other arrangements. Additional arrangements will be possible using the rhombus and one or more triangles. A similar set of shapes can be made by using the squares and right isosceles triangles.

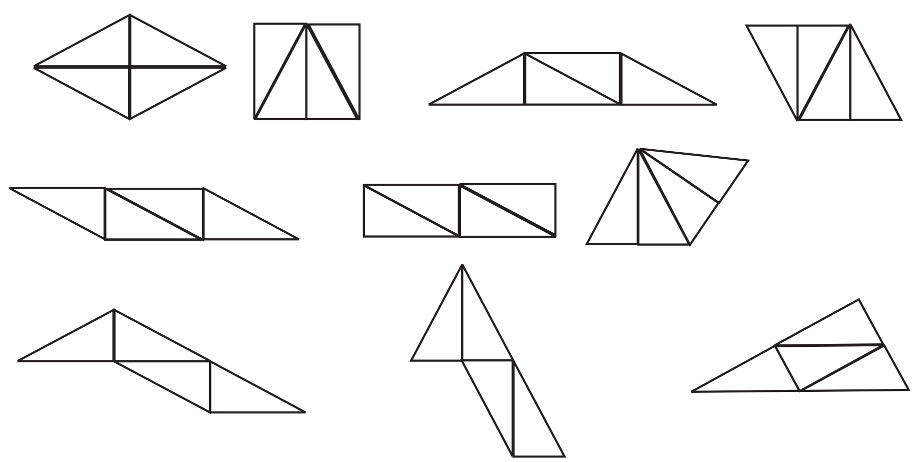
Two triangles:



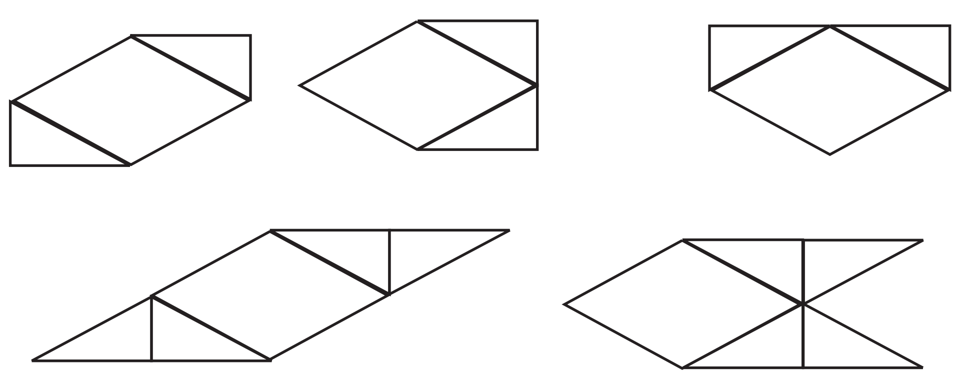
Three triangles:



Four triangles:

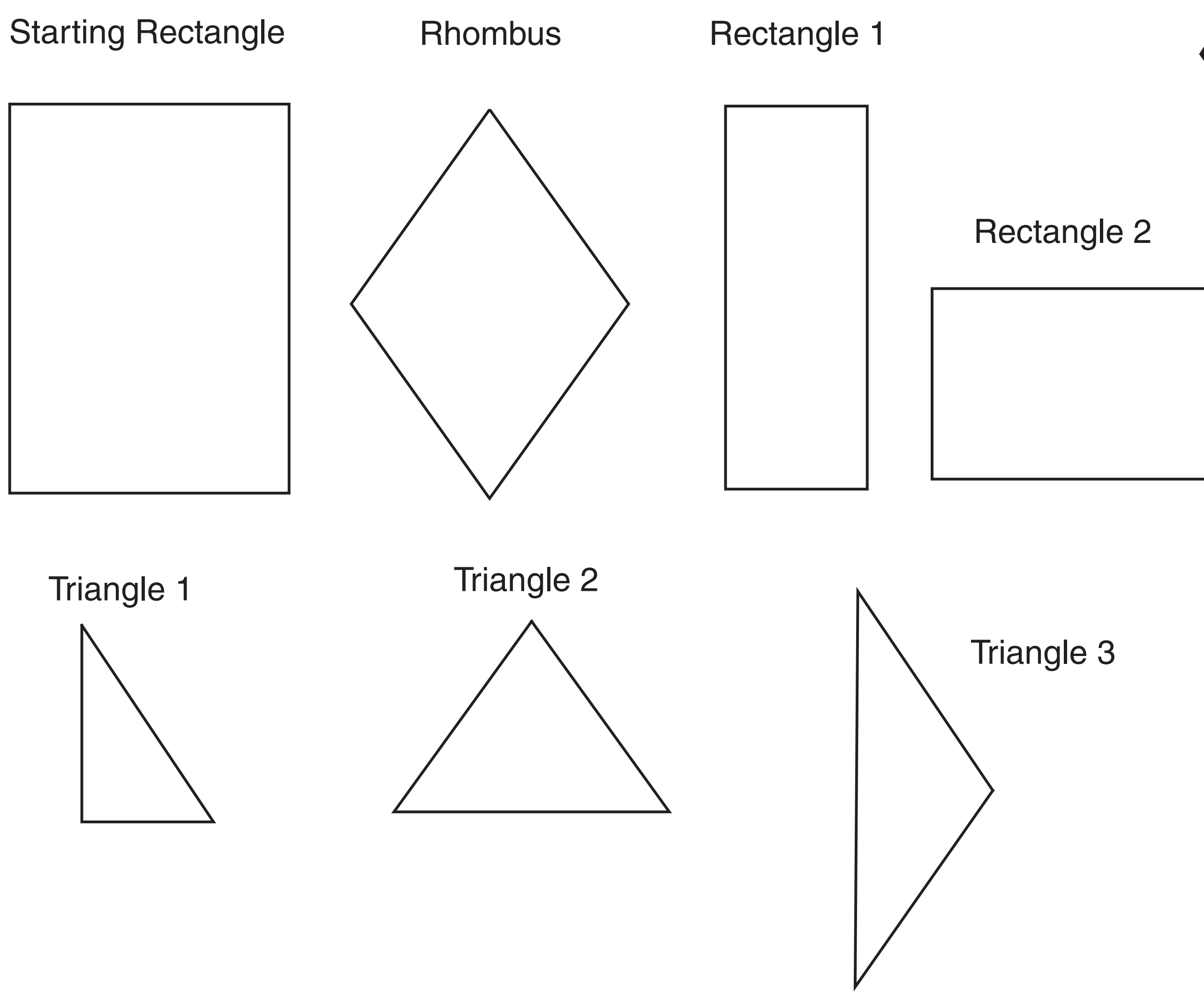


Triangles can also be combined with a rhombus in several different ways. Here are a few examples:



Related Shapes

How many ways is each shape related to the others?



1-Inch Grid Paper

Activity 5
Constructing a Rhombus Pattern Puzzle

This is a cornerstone activity. It lays the groundwork for students to begin constructing their headband patterns and for the more detailed study of area. They start from 6 x 4 inch rectangles and create two-color patterns using pieces cut out of the rectangle. Starting with a rhombus and four right triangles, they subdivide these into smaller pieces in two different colors. They create a pattern puzzle by using the smaller pieces to fill their original rectangle. Students learn Yup'ik pattern design principles of symmetry and color balance while creating their own patterns. They reinforce their understandings of the geometrical relationships among the pieces of the decomposed rectangle. This gives them an intuitive sense of conservation of area and part-whole relationships that will be developed more formally later in the module.

Goals

Students will be able to:

- Create a pattern in two contrasting colors by decomposing a rectangle and rhombus into smaller pieces.
- Create a drawing showing all their puzzle pieces assembled into a rectangle.
- Recognize that rearranging pattern pieces with the same area into different configurations will not change the total area.

Materials

- 1 inch graph paper—one sheet for every two students (see page 63)
- Construction paper in two colors, one dark and one light, precut into 6 x 4 inch rectangles
- Colored transparency paper in contrasting colors for cutting out rhombi, triangles, and puzzle pieces
- Colored pencils or markers
- Transparency, Sample Headband Patterns (Activity 1, page 34)
- Scissors—one per pair of students
- Puzzles and shapes from Activity 4
- Math notebooks

Duration

One to two class periods.

Vocabulary

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth

Center—a point that is equidistant from all points in a circle; a point that is the intersection of the diagonals of a square, rectangle, rhombus, or parallelogram

Congruent—geometric figures (or parts of figures) that are the same shape and size. Two shapes are congruent if one shape can be slid, flipped, and/or rotated so it fits exactly on top of the other one.

Conservation of area—when a shape is divided into two or more parts, all of the parts cover the same total area as the original shape, no matter how the parts are separated or recombined to form new shapes

Decomposition—breaking down into smaller shapes or pieces

Design—a pleasing shape or combination of shapes. A design may be intentionally created by someone or may be a consequence of natural forces

Line of symmetry—a line that is a property of a geometric figure (a shape, design, or pattern); it divides the figure into two equal parts such that when the figure is reflected about that line, the result is identical to the starting figure

Pattern—a design that consists of a basic design element repeated over and over again; a pattern may be extended in one direction (linear or frieze pattern) or two directions (two-dimensional or "wallpaper" pattern)

Properties—the parts of a geometric shape: sides, angles, symmetries and their relationships, that define a particular shape as a unique shape

Rectangle—a quadrilateral with opposite sides equal and four right angles

Right triangle—a triangle with one 90 degree angle

Symmetry—mathematically, a quality of a shape such that it does not change when transformed by a rigid transformation. There are three types of symmetry: line symmetry (or mirror symmetry), rotational symmetry (or point symmetry) and translational symmetry.

Triangle—a polygon with three sides

Preparation

Explore the various shapes that can be created from the rectangle and the rhombus by folding and then cutting. Pay attention to the geometrical relationships and geometrical properties that allow you to produce these shapes.

Cut out one set of puzzle pieces from one piece of colored transparency paper rectangle of 6 x 4 inches for a sample puzzle. Save them to combine with the pieces you'll cut out from another color during modeling. Read the Teacher Note on page 101, which suggests an alternative order to Activities 5 through 9, in particular how to integrate these activities.

Instructions

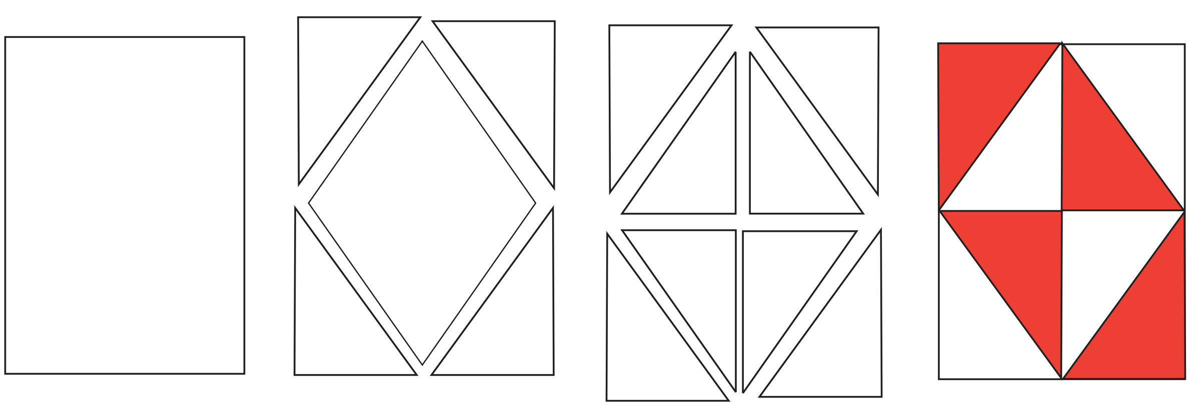
1. Explain to students that they are now going to start working on making their own pattern designs. Display sample patterns. Show the transparency, Sample Headband Patterns, and point out that Yup'ik parka patterns are usually constructed using two contrasting colors of fur, to make the patterns interesting. In order to make their patterns they will start by creating a pattern puzzle by dividing their original rectangle and rhombus into even smaller pieces, cutting out the pieces in two different colors. In this activity all the patterns will t within a 6 inch x 4 inch rectangle. When students design their headbands, they will make linear symmetrical patterns that ll a long strip rather than a 6 x 4 inch rectangle.

2. Using the colored transparencies, model the process of cutting and folding shapes to make a pattern puzzle. Cut and fold one color; have the other color already cut. The process is illustrated in Figure 5.1.

   Ask students what they can tell you about the relationships among the eight triangles. They should recognize that all eight triangles are congruent.

   As you arrange the pieces into a pattern, explain to students that Yup'ik pattern makers usually make their patterns in two colors and that they like to alternate the colors. After you have made the pattern in Figure 5.1, ask students if they can arrange the eight triangles into a rectangle puzzle in any other ways. Some additional possibilities are shown in Figure 5.2.

Math Note



A
Start with a rectangle.

B
Cut a rhombus and four triangles.

C
Subdivide the rhombus into four more congruent triangles.

D
Copy the same pieces in a contrasting color, and reassemble into a puzzle.

Fig. 5.1: Cutting and folding a rectangle to make a two-color pattern puzzle

Math Note

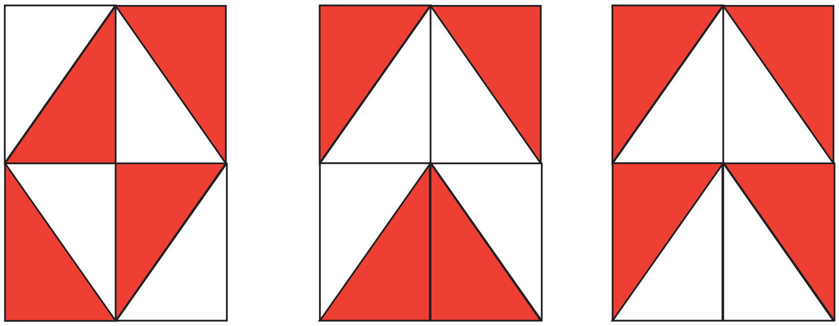


Fig. 5.2: Some additional ways to arrange four dark and four light right triangles into a rectangular pattern puzzle

3. Have students work in pairs. Hand out the supplies (construction paper in two contrasting colors, cut into 6 inch x 4 inch rectangles; also graph paper, scissors, etc.). Each group decides how they will cut their pieces. Ask students to follow these simple rules when cutting their pieces: All their folds and cuts should be along a line of symmetry or should pass through the midpoint of one of the sides or angles of the shape they are dividing.

   Other ways for cutting a rectangle into a pattern puzzle are shown in the Teacher Note on page 67 and Figures 5.3–5.5. There are several additional possibilities depending on how students divide their main pieces.

4. **Joint Activity.** Begin making your pieces for your own pattern puzzle. Some students may choose to work with you as well. Allow students suf cient time to create their pattern set. Emphasize creativity and remind them to make duplicate sets in two colors so they can arrange them into a pattern puzzle. Each group should be able to form two puzzles, in opposite color combinations.

5. **Recording.** Ask students to record their completed puzzles by laying them out on graph paper, outlining the pieces and coloring them in using two colors or paste into math notebooks.

6. **Sharing.** Have the students share and show the unique ways they made their pieces. Help them describe the new shapes they have made and their relationships to the original shapes and each other. Ask them to show the fold lines they used, describe the relationships among their new shapes, and describe how these shapes relate to the original rectangle, the large rhombus, and the right triangles. Ask students to demonstrate the relationships by laying pieces next to or on top of other pieces. Try to get students to use their own words, but it's very important that

Assessment: Use students' work as an indicator of their mathematical understanding.

Teacher Note

Encourage students to make even smaller pieces than you did for your example. Remind them to use lines of symmetry and lines through midpoints to divide their rectangles, rhombi, and triangles into smaller pieces. This is another important Yup'ik design principle that ensures that the cut pieces will have a pleasing relationship to each other. It also ensures that the areas will be related to each other in simple ways. This will be important when students compare areas in Activity 7.

"The word fun comes up in their math notebooks an awful lot. The level of enthusiasm was really high."
Terri Chapdelaine
Buckland, Alaska

Teacher Note (Do Not Distribute)

Here are some other ways that a rhombus puzzle could be made.

METHOD A (Figure 5.3)

a. Fold and cut the rhombus and four triangles in the usual way.
b. Carefully fold the four corners of the rhombus into the center.
c. Unfold and cut along the fold lines. You will have created a smaller rectangle and four smaller triangles.
d. Combine these four triangles with the large triangles and small rectangle pieces of the other color to create a second rectangle puzzle. This one has a smaller rectangle inside the rhombus which is inside the original rectangle.

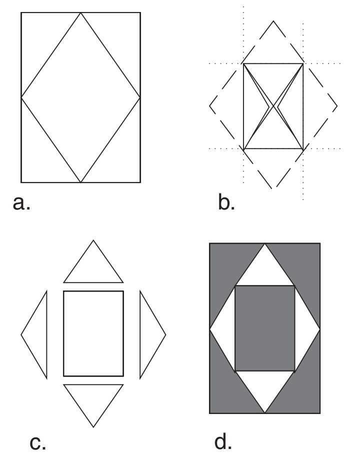


Fig. 5.3: Method A

METHOD B (Figure 5.4)

a. Fold and cut the rhombus and four triangles in the usual way.
b. Fold the rhombus along a line passing through the center and the middle of each edge.
c. Now unfold the rhombus and fold it again along the opposite midline.
d. Cut along both midlines into four smaller rhombi.
e. Combine two of the rhombi and two of the triangles with rhombi and triangles of the other color to make a puzzle. Ask students what they can tell you about the relationship of the smaller rhombi to the larger one. (Each one is ¼ the area, and their sides are half as long as the original rhombus.)

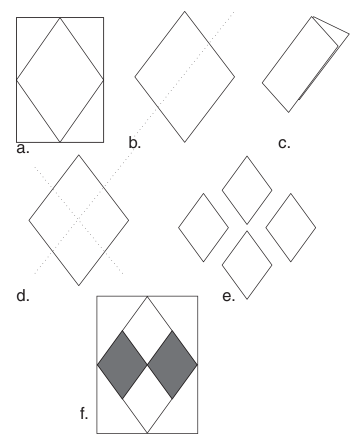


Fig. 5.4: Method B

METHOD C (Figure 5.5)

Note that you could also cut out four rhombi from a rhombus as shown in Figure 5.5.

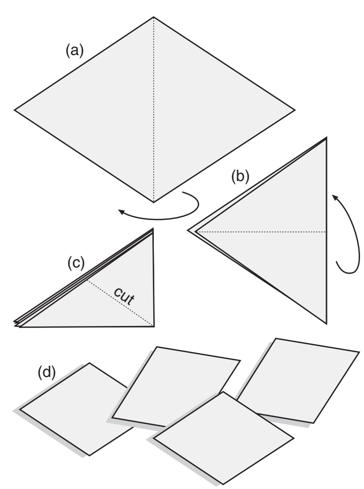
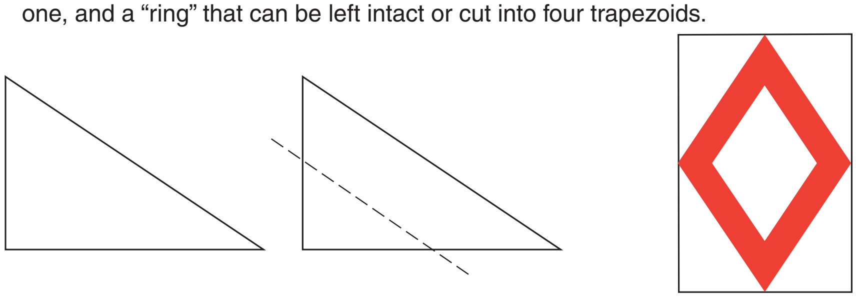


Fig. 5.5: Another way to cut four rhombi from one

METHOD D (Figure 5.6)

Yet another possibility for creating the puzzle is shown in Figure 5.6. After cutting out an original rhombus and four congruent triangles, fold the rhombus along its lines of symmetry to form a triangle as shown in Figures 5.5 a–c. Then instead of cutting through the center, cut through the midpoints of the sides, parallel to the original cut. The result will be a small rhombus, one quarter the size of the large one, and a "ring" that can be left intact or cut into four trapezoids.



a. Fold rhombus into quarters.

b. Cut along a line through the midpoints parallel to the sides.

c. The result is a small rhombus with a "ring" around it. This can be further divided to make a puzzle.

Fig. 5.6: Yet another possibility for creating a rhombus puzzle

these relationships and the reasoning behind them are stated and shown explicitly.

7. Ask students to get out their puzzles and shapes from Activity 4. In that activity, students used their cut-out shapes and rearranged them in different combinations to make new shapes. Invite students to make new shapes by rearranging their puzzle pieces. They can use different numbers and combinations of pieces. Shapes should be rearranged so their sides match, edge to edge.

   Suggest that students record their new shapes by drawing them on graph paper and copying them in their math notebooks. Ask students to name and describe the properties of the new shapes they create.

8. Have the students save their pattern pieces in their storage envelopes. They will use them again for Activity 7.

Extension Activity

Cross-age Sharing

Tell the students that they are going to share their puzzle with rst or second graders. Their task is to:

- Give a title (name) to their puzzle.
- Teach the different shapes they have used in the puzzle to rst or second grade students.
- Have the rst or second grade students try to reassemble the puzzle.

Teacher Note

Peer and cross-age tutoring is an effective way for students to learn. This is because tutors and their students (tutees) often speak a more similar language than do teachers and students (Cazden, 1986), and as a result are able to relate to one another better.

This extension activity provides another way for your students to consolidate their learning. Observing your students as they share also provides you with an excellent opportunity for informal assessment of their learning.

Activity 6
Exploring Area Relationships Using Nonstandard Units

This is the first of three area investigations. The study of area relates to the Yup'ik cultural practice of not wasting materials and carefully estimating the materials needed for any project. Yup'ik pattern makers are careful to adapt their patterns to the amount of material available and to save any extra material for later use. This type of estimation corresponds to the mathematical concept of area.

Activity 6 introduces nonstandard units of area, using puzzle pieces to measure the area of the entire puzzle. Activity 7 introduces standard units of area, asking students to measure areas by counting square units. Activity 8, which is optional, uses what students have already learned about area to introduce shortcuts (formulas) for calculating the areas of rectangles, triangles, and rhombi. You may want to include Activity 8 if area formulas are part of the learning standards for your grade level.

The actual design and construction of headbands starts in Activity 9. Activities 6, 7, and 8 could be used while students are working on making pieces and assembling their headbands. You may choose to alternate headband pattern-making activities and area activities, or complete the area activities before students start their patterns. If you choose to start the pattern-making (Activity 9) before completing the area activities, look for opportunities to include area explorations while students are working on their headband patterns.

Students use the pattern puzzles they created in Activity 5 to explore area relationships and nonstandard area units. Starting with the rhombus and four right triangles cut out from the original rectangle, they review the area relationships to determine that eight right triangle units will cover the rectangle; four of the same units will cover the rhombus, and two rhombus units cover the rectangle. This review serves as a model for the investigations that students undertake with their puzzle pieces. Their goal is to use one of their small pieces and work out how many of those pieces will cover the original rectangle. This gives them a nonstandard area unit that they use to find the areas of their remaining puzzle pieces. Since all students started from the same 6 inch x 4 inch rectangle, they can easily compare units from each other's puzzles. This paves the way for transferring to standard area units in the next investigation.

The open-ended design of this module allows for tailor-made adaptations to t your class. For example, an extension activity shows how the same area relationships can be represented as fractions. This provides a template for students to explore part-to-whole relationships—fractions. The students' puzzles can be viewed as a set of fractional pieces.

Culturally, different elders have their own pattern sets derived from different ways of partitioning their starting piece through folding or nding midpoint bisectors or through the use of diagonals. In other words, they follow certain design principles, which allow for a variety of designs. See the Teacher Note on the next page for additional cultural information.

Likewise, in the classroom we suggest certain pedagogical guidelines; however, we expect different teachers in different circumstances to achieve their goals through a variety of alternative ways. For example, Activities 6 through 9 could be accomplished in linear fashion or they could be done in an integrated way. The Teacher Note on page 101 provides additional suggestions.

Goals

Students will be able to:

- Use their smallest puzzle piece as a nonstandard area unit for measuring the area of their original rectangle.
- Use their nonstandard unit to measure the area of other puzzle pieces.
- Recognize that using different pattern pieces, or rearranging pieces within a puzzle does not change the total area.
- Gain a basic understanding of the fractional (part-to-whole) relationship of their pattern pieces to the starting 6 x 4 inch rectangle.

Materials

- Students' puzzle pattern pieces from Activity 5
- Colored transparency paper for cutting out rhombi and triangles
- Handout, 6 by 4 Inch Rectangle on 1-inch Grid
- Plain transparencies
- Overhead markers
- Transparency, 6 x 4 Inch Rectangle on 1-inch Grid
- Handout, 1-Inch Grid Paper (see page 63)
- Handout, Area Recording Chart
- Math notebooks

Duration

One to two class periods.

Vocabulary

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth.

Area measurement—the number of square units needed to exactly cover a two-dimensional shape

Design—a pleasing shape or combination of shapes. A design may be intentionally created by someone or may be a consequence of natural forces.

Diagonal—a line segment joining two nonconsecutive vertices of a polygon. For a quadrilateral, a line joining opposite vertices.

Linear measurement—a measure of length, width, height, or distance

Proof—a mathematical argument—based on logical reasoning—that demonstrates that a particular fact or relationship is true

Unit of Area—a shape used repeatedly to measure the area of another (usually larger) shape

Preparation

Use one of the puzzle sets already made in colored transparencies to work out the area relationships.

In addition use two transparent 6 x 4 inch rectangles and cut a rhombus and four right triangles from each. This will give you two identical rhombi and a total of eight right triangles. Also, prepare a transparency of the graph paper with a 6 x 4 rectangle outlined on it.

Take some time to practice the area comparison activities in instruction steps 3 and 4. Try these for yourself to better understand the experiences students will go through before reading the Math Note (page 74) that explains them. By going through the activities and the extension, you will be able to guide the students' work in directions appropriate for your class, for example, more into part-to-whole relations or more into area relations.

Instructions

Math Vocabulary: Continue to extend students' math vocabulary.

1. Ask for volunteers to show and explain the mathematical meaning of the term "area." Include the students' insights and explain that area is way of measuring the amount of surface covered by a particular shape. Compare area measurement with linear measurement. We make linear measurements using a unit such as an inch, a foot, a meter, and so forth. In this activity, students will use their pattern puzzle pieces to nd their own units for measuring the area of their original rectangle and other shapes.

2. **Model.** Show students the transparencies of the rhombus and four right triangles. Start with the smaller shape, the triangle. Explain that you'd

like to use the smallest shape as a unit to measure the area of the original rectangle. (You may want to ask students which shape is smallest.) Ask students if they can tell how many of the small right triangles could be used to cover the entire rectangle. Place the rectangle outline below the triangle pieces. Invite students to come up to the overhead and show you their reasoning.

Students may recall that the rhombus can be cut into four additional triangles and that therefore eight right triangles cover the rectangle. Others may show that four right triangles can be rearranged to cover half the rectangle, and reason that four more would be needed to cover the other half.

If students have dif culty following these arguments, pass out rectangles to groups of students and ask them to explore this question themselves.

Explain that they have just determined that the area of the rectangle is eight right triangle units. Ask, "How many triangle units will cover the rhombus?" (4). Therefore, the area of the rhombus is four right triangle units. Now ask, How many rhombi will be needed to cover the rectangle? If students say two, ask them to demonstrate this on the overhead. Two rhombi cannot exactly cover the rectangle unless one of them is cut into four right triangles. However, it is still correct to say that the area of the rectangle is two rhombus units.

Copy this information onto a classroom chart like the one below. As students explore different ways of dividing the rectangle into smaller pieces they can use this chart as a model for recording the area relationships they discover.

Unit	**Shape being measured**	**Area of the shape**
Large right triangles	Original rectangle	8 large right triangle units
Large right triangles	Large rhombus	4 large right triangle units
Large rhombus	Original rectangle	2 large rhombus units

Fig. 6.1: Area recording chart

3. **Explore.** Distribute a handout of the 6 x 4 inch rectangle to each pair of students. Tell each one to cut out a rhombus and four right triangles in the usual way. Then ask them if they can divide the rhombus into four smaller congruent rhombi. (This can be a dif cult construction for some students to carry out accurately. Two ways of doing this are illustrated

Teacher Note

This work is, in part, a review of work students did in Activity 4. The main difference is that in this case you have introduced the concepts of area and area units. You and the students are reviewing a way of reasoning, showing that if shapes can be moved on top of other shapes, or if a shape can be folded or cut apart, it may be possible to draw conclusions about area relationships. This is the kind of reasoning students should use to draw their own conclusions about area relationships within their puzzles.

Proof is a fundamental concept in geometry. Students learn proof by manipulating geometrical pieces to show geometrical relationships

Assessment: Students' math notebooks and writing and refining definitions is a way to assess students' mathematical understandings.

in Activity 5, Figures 5.4 and 5.5.) Ask them to figure out how many of the small rhombi cover the rectangle. (Since two large rhombi cover the rectangle, and there are four small rhombi in a large rhombus, the area of the rectangle is equal to eight small rhombus units.) Distribute copies of the Area Recording Chart handout and ask students to record their findings.

4. **Challenge.** Students from another class stated that the area of the small rhombi was equal to the area of the right triangle. Prove whether this is true or not. How many of each cover the original rectangle? See Math Note, next page, for sample proofs.

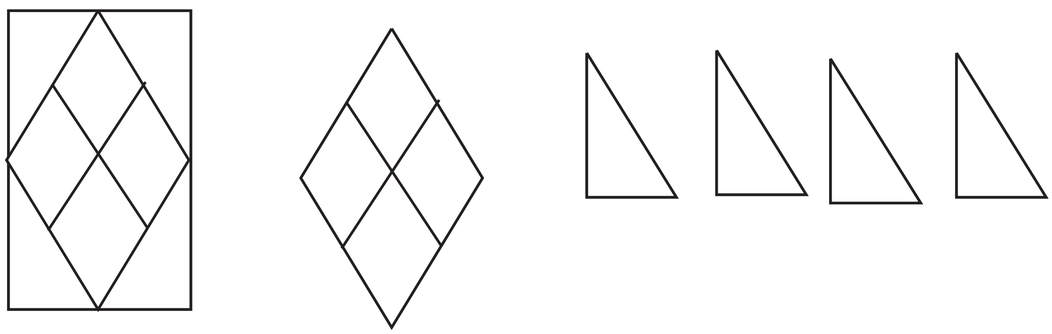


Fig. 6.2: Can you prove that a small rhombus and a right triangle have equal areas?

5. **Explore.** Pass out another Area Recording Sheet to each student. Have the students take out their "puzzle" and have them determine how many of each type of piece it would take to cover the same area as the original 6 x 4 inch rectangle. This is their own area unit. They can then determine the area of the rectangle in terms of that unit. Students should record their results on the Area Recording Sheet. Ask students to determine the area of the large rhombus using their own unit, and record that on the Area Recording Sheet as well.

6. Students may refine their definition of area in their math notebooks at this time.

Math Note: Comparing a Small Rhombus with a Right Triangle

Here are three out of several ways that people have proven that the areas of the rhombus and triangle are equal.

1. Folding the rhombus in different directions. (See Figure 6.3.)

a. Fold the small rhombus in half, hamburger-wise.

b. Lay the folded shape on top of the triangle. Draw a line showing where the folded rhombus overlaps the right triangle.

c. Unfold the rhombus, and refold it hot-dog-wise.

d. Lay the folded rhombus on top of the triangle so that one of its edges lies along the line already drawn.

e. Since ½ the rhombus covers ½ the triangle when folded one way, and ½ the rhombus covers the rest of the triangle when folded the other way, ½ + ½ = 1 whole, and the two areas are equal.

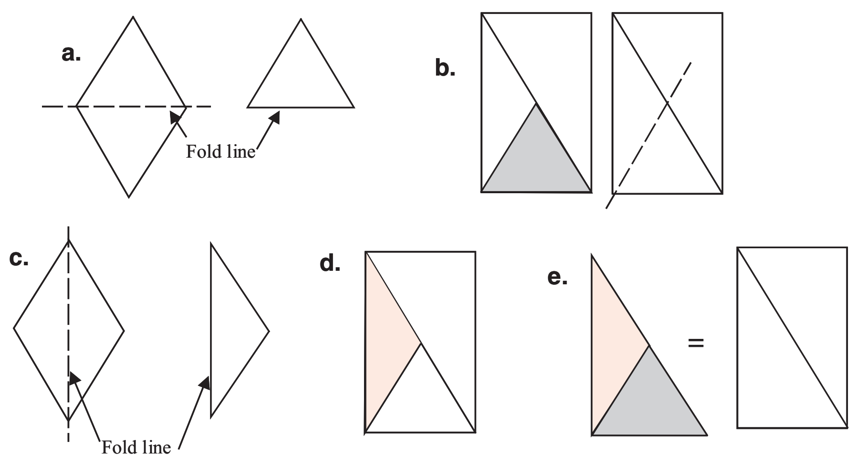


Fig. 6.3: One way to prove that the small rhombus has the same area as the right triangle

2. Folding the rhombus into quarters and cutting the right triangle (see Figure 6.4).

a. Carefully fold the small rhombus into quarters along both of its lines of symmetry to form a small triangle.

b. Lay the small triangle on top of the larger one, draw a line, and cut one small triangle out of the large one.

c. Lay the folded rhombus over the remaining trapezoid, draw a line, and cut out a second triangle.

d. Now lay the folded rhombus on the remaining rectangle, draw a line, and cut it into two triangles.

e. Unfold the rhombus and lay the four small triangles one at a time on top of it to prove the areas are equal.

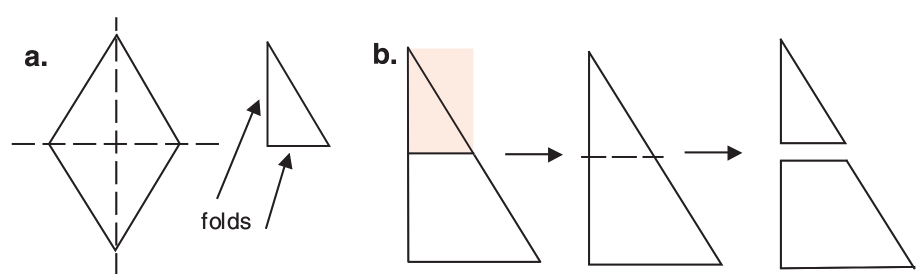
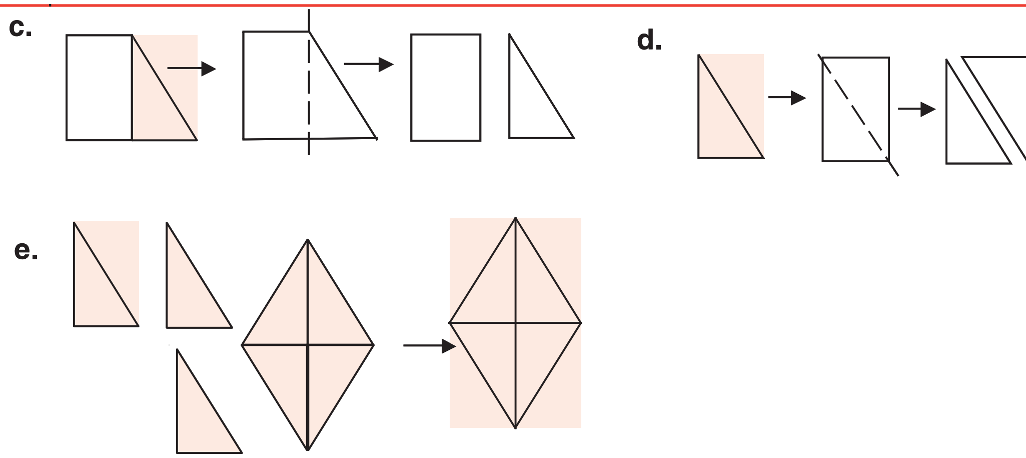


Fig. 6.4: A second way to prove that the small rhombus has the same area as the right triangle



3. Area relationships of a puzzle containing a small central rectangle.

Some groups may construct their puzzles differently. For example, folding all four corners of the rhombus to its center, creates a puzzle with a small rectangle at the center, surrounded by four isosceles triangles.

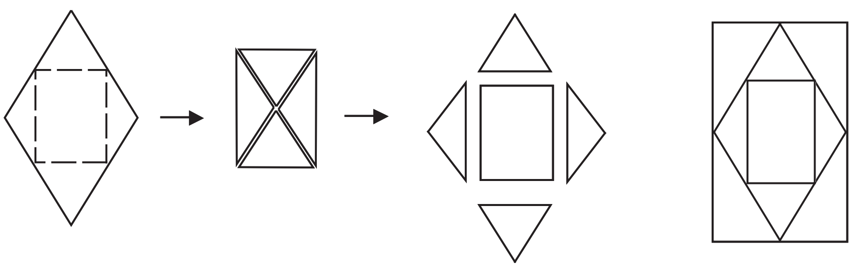


Fig. 6.5: Dividing the rhombus into a small rectangle and four isosceles triangles is another way to create a puzzle.

To find the area of the small rectangle, recall that the rhombus is ½ of the original rectangle. The four small triangles cover the small rectangle, so that the small rectangle must be ½ the area of the rhombus, or ¼ the area of the original rectangle. Using the small rectangle as a unit, the area of the large rectangle is 4 small rectangle units.

Finding the areas of the four isosceles triangles takes another step. The triangles are not all congruent, so we can't immediately conclude that their areas are equal to ¼ of the small rectangle. However, opposing triangles are congruent, so all we need to do is compare the areas of two of them. Folding triangles A and B about their lines of symmetry, we can determine that the half triangles are congruent; therefore, their areas are the same. If we use either small triangle as a unit, the areas of the small rectangle, the rhombus, and the original rectangle are 4, 8, and 16 small triangle units, respectively.

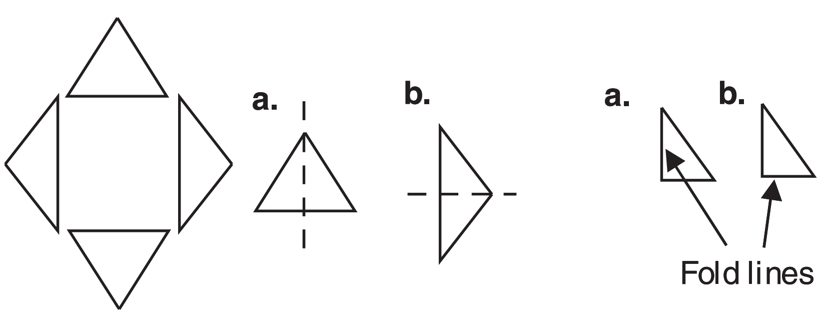


Fig. 6.6: The four small isosceles triangles all have the same area, ¼ that of the small rectangle.

Extension Activities

Extension 1: Part-whole Relationships and Fraction Concepts

Students' pattern puzzles can be used to illustrate fraction relationships, using an area model for fractions. These related shapes can be adapted like Cuisennaire rods by varying which is the unit and which is the fractional part.

1. For each shape students have made, ask how many of each shape would cover the original rectangle. For example in the puzzle shown to the right (Figure 6.7), eight red and white triangles (four each) cover the rectangle. Therefore each triangle represents ⅛ of the rectangle.

2. What fraction of the rectangle is colored red? Students may answer ½ or 4⁄8. Both are correct. Ask students if they can rearrange their pieces to prove that the red triangles cover exactly half the rectangle (provide an obvious visual representation).

3. Ask students to answer the same questions for their own puzzles. For example for this puzzle (Fig. 6.8):
   - The red rectangle can be covered by two red triangles, so the red rectangle represents 2⁄8 or ¼ of the whole rectangle.
   - The four white triangles also cover the red rectangle. (The white triangles are not congruent, but all four have the same area.) So 4 x 4 or 16 of them would cover the entire rectangle. Each white triangle is 1⁄16 of the rectangle.
   - White triangles cover ¼ of the rectangle, the red rectangle covers another ¼, and the red triangles cover 4⁄8 or ½ the rectangle.
   - Students can rearrange the shapes to show this more clearly, as shown in Figure 6.9.

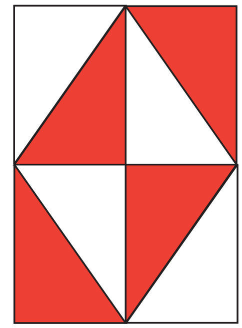


Fig. 6.7: Using puzzles to teach fractions—triangles are ⅛ of the whole

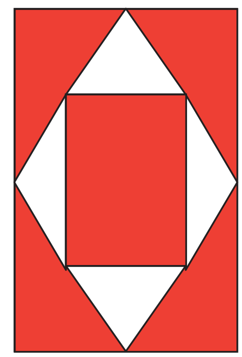


Fig. 6.8: Using puzzles to teach fractions—red triangles cover ½ of the whole

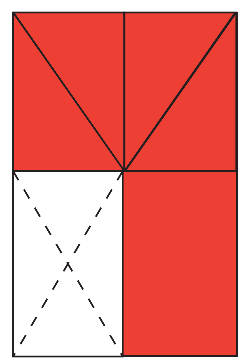


Fig. 6.9: Using puzzles to teach fractions—each white triangle is 1⁄16 of the whole

Extension 2: Finding the Area of a Different Rectangle

1. Distribute 1-inch graph paper to students and ask them to draw the outline of a rectangle, 8 inches long by 3 inches wide.

2. Ask students to count the number of square inches inside the outline, and compare this with the number of square inches in a 6 x 4 inch rectangle.

Math Note

Since an 8 inch x 3 inch rectangle has the same area as a 6 inch x 4 inch rectangle, 24 square inches, student should get the same area they did for the original rectangle. The purpose of this activity is to help students generalize their understanding of units. That is, they will begin to understand that a unit of area can be used to measure any shape, not just the shape the unit was constructed from.

Part-to-Whole Relationships: These activities can be expanded for further explorations into fractions.

3. Now ask students to nd the area of the 8 x 3 inch rectangle using their own area units. (They should get exactly the same measure that they did for their 6 x 4 inch rectangle using the same unit.)

Math Note

Just as different shapes can serve as units of area for measuring larger shapes, so different shapes can serve as the unit element (1 whole) for fractional relationships.

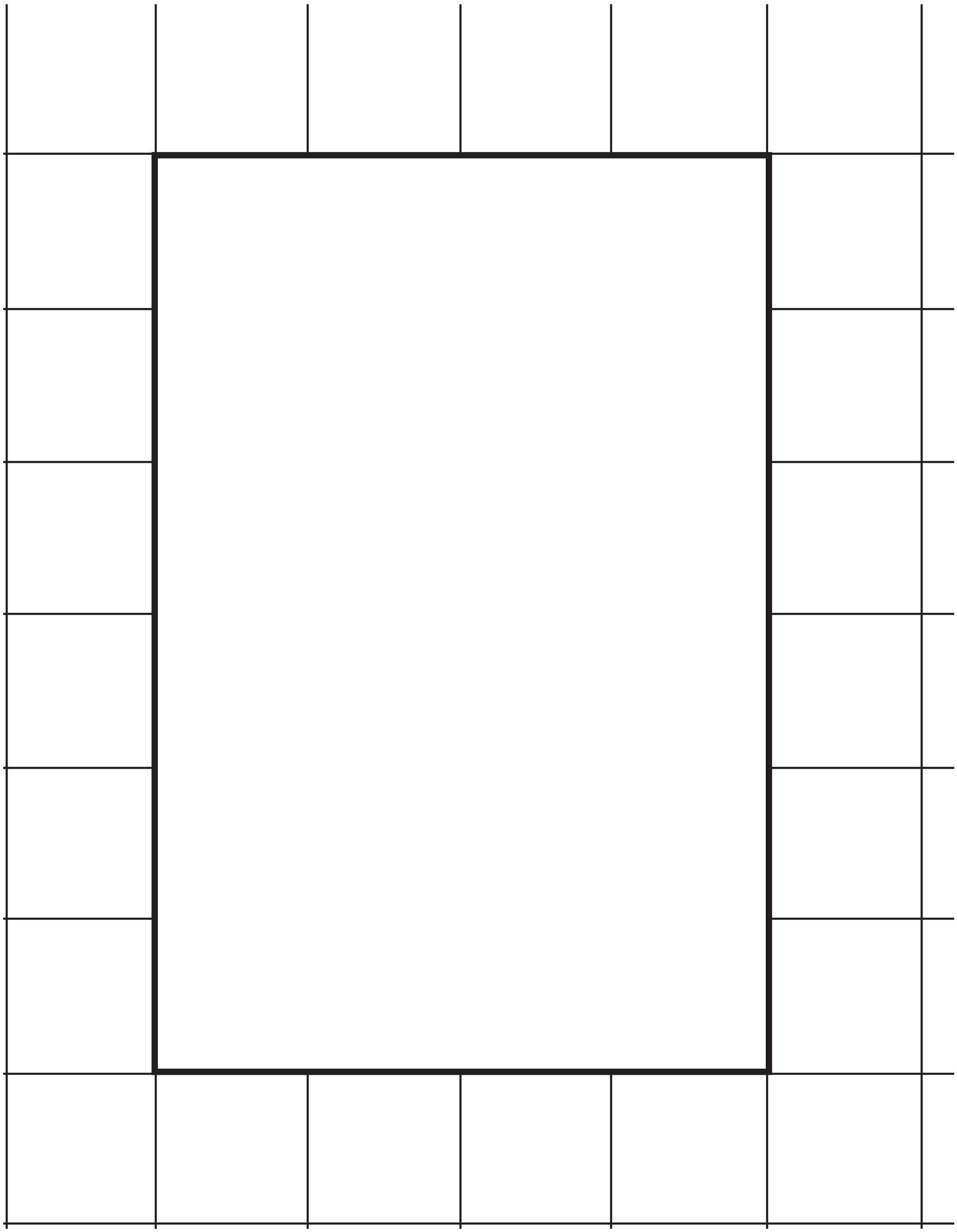
In all of the examples given, the large rectangle is considered to be the whole. However, this is arbitrary. For example, in Figure 6.8, you could choose the small red rectangle to represent 1 whole. Then the small white triangles will each represent ¼. The red right triangles will each represent ½, and the large rectangle, 4 units.

If your students are familiar with this approach to representing fractional relationships using different shapes as the “whole” or “unit,” their puzzle sets can serve as an excellent opportunity to consolidate that knowledge. If they have not investigated fractions in this way before, the extension activity can be adapted to introduce the idea.

Area Recording Chart

Unit	Shape being measured	Area of the shape

6 by 4 Inch Rectangle on 1-inch Grid



Activity 7
Calculating Areas by Counting Square Units

In this activity, students use the method of counting square units to determine the areas of their puzzle pieces. There are practical, creative, and mathematical reasons for learning this. Practically and creatively students and other designers may need to know how many pieces they will need ll the space that they are using for their design. Mathematically, the process of counting units may focus students' attention to the fact that area, the number of square units that ll a space, can be calculated more quickly through multiplying the length times the width. As students connect their design (a two-dimensional spatial array that lls space) with rst counting the number of square units to area and later to multiplying length times width as a way of determining area they will have a grounded understanding of the area formula. This activity introduces students to measuring area using standard units. They can apply this to the problem of nding the area of light and dark pieces for their headband patterns.

Students also continue to build vocabulary through word study using the Vocabulary Map.

Goals

Students will be able to:

- Determine the areas of rectangles, right triangles, isosceles triangles, and rhombi by counting square units.
- Determine the area of a right triangle or isosceles triangle by counting squares in a circumscribed rectangle and dividing in half.

Materials

- Students' puzzle pieces from Activity 5
- Transparency, Comparing Areas in Two Puzzles
- Transparency, 6 x 4 Inch Rectangle on 1-inch Grid (from Activity 6)
- Transparency, Rhombus and Four Triangles Inscribed in 6 x 4 Inch Rectangle, 1-Inch Grid
- Colored transparencies of puzzle pieces from Activity 5
- Graph paper, rulers, and scissors, if students request them
- Overhead transparency markers
- Math notebooks

Duration

One class period.

Vocabulary

Area—the amount of surface covered by a shape or region. Area is measured in square units appropriate to the size of the shape or region, such as square inches, square yards, square miles, and so forth.

Design—a pleasing shape or combination of shapes. A design may be intentionally created by someone or may be a consequence of natural forces.

Square unit—the square unit usually means the one with coordinates (0, 0), (1, 0), (1, 1), (0, 1) in the real plane. A unit of measure in area.

Preparation

Read the Math Note on page 83 and practice the suggested ways of determining area. Depending upon your students' knowledge of math vocabulary, if necessary use the Vocabulary Map for specific terms that students need to define.

Instructions

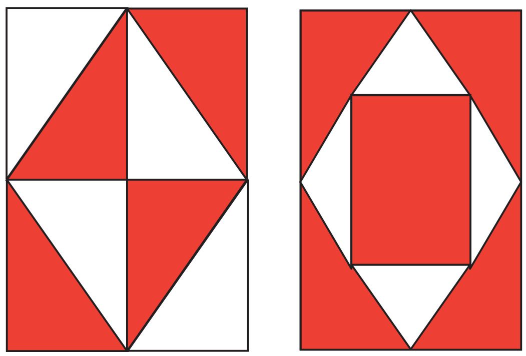
1. Remind students that in the last activity they used small puzzle pieces as units to measure the area of the entire rectangular puzzle. In this activity, they will learn how to use one unit, square inches, to measure and compare the areas of all the pieces.

2. **Review.** Display transparency, Comparing Areas in Two Puzzles, showing two different puzzles made from a 6 x 4 inch rectangle. Ask them to identify one of the smaller shapes in each puzzle; ask if they could find the area of the rectangle using that shape as a unit. Ask: Can you think of a way to measure the area of a rectangle and of other shapes using the same unit for all shapes? If students have difficulty with these you may want to pass out 6 x 4 inch rectangles and ask students to work in groups and construct one of the puzzles and work out the area relationships. You can assign different puzzles to different groups of students who can then share their work with the class.

3. Display the transparency, 6 x 4 Inch Rectangle on 1-inch Grid. Ask students if they can tell you how many square inches cover the entire rectangle (24). Today they will find the areas of all their puzzle pieces in square inch units. Square inches are an example of standard area units. Ask if students know of any other standard area units and what they are used to measure. (Square feet, yards, or meters are used to

Math Note

This offers a chance to review and consolidate students' learning about nonstandard units before going on to standard units. The total area of Puzzle 1 can be measured using either small rhombi (8 small rhombus units) or right triangles (8 right triangle units). The total area of Puzzle 2 can be measured using either the small rectangle (4 small rectangle units) or the small isosceles triangles (16 small isosceles triangle units).



Puzzle 1 Puzzle 2

Fig. 7.1: Measuring the areas of two puzzles

measure cloth, rugs, rooms in a building, and so forth.) Explain that the area of any two-dimensional shape can be measured by counting the number of square units contained within the shape.

4. **Explore.** Display the transparency, Rhombus and Four Right Triangles. Ask students to work with their own graph paper and carefully cut out a 6 x 4 inch rectangle, then cut out a rhombus and four right triangles using Winifred Beans' method. Ask them if they can determine the area of the rhombus and the four right triangles by counting squares. See Math Note on next page for a more detailed explanation of ways of nding area.

5. **Sharing.** After students have had time to work on this problem, ask them to share their methods. Display the transparency, Rhombus and Four Right Triangles again and ask students to use the transparency to show other students how they calculated the areas of the triangles and the rhombus. After one solution is offered, ask if there are any different ways to nd the area of that shape.

6. **Explore.** Now ask students to take out their puzzle pieces from Activity 5. Ask them to use graph paper to help them nd the areas of each puzzle piece in square inches. Remind them that they now know several methods to nd the areas of rectangles, triangles, and rhombi. They can use any of these methods as they nd the areas of their puzzle pieces.

Teacher Note

It's important to take time with this, both now and in reviewing students' work finding the areas of their own puzzle pieces. The concept of finding the area of a triangular region by drawing a rectangle around it and finding that the area of a triangle is exactly half the area of the rectangle is one of the most useful mathematical ideas in this module, and it is an essential step in moving to a more abstract understanding of area as a product of two lengths.

7. **Share and record.** Ask students how they determined the area of their puzzle pieces. Since several students may have constructed puzzles using the same sized pieces, this will allow them to compare results and verify that students whose pieces are the same got the same values for their areas. Ask students to record their results—and their methods of calculation—in their math notebooks.

Math Note

Students may find several different ways to work out these areas. There are two common ways to work out the area of a 2 x 3 inch triangle (a). One is to cut a 1 x 1½ inch triangle off the top (b) and move it next to the sloping line of the remaining trapezoid (c). This produces a 2 x 1½ inch rectangle with an area of 3 square inches. Another way is to draw a 2 x 3 inch rectangle (d) on graph paper around the triangle. Then cut that rectangle along its diagonal (e) and rotate the top triangle. (f) Place the two triangles on top of each other (g) to prove they are congruent. The area of the rectangle is 6 square inches, so the area of each triangle is 3 square inches. Make sure that both these methods are demonstrated by your students.

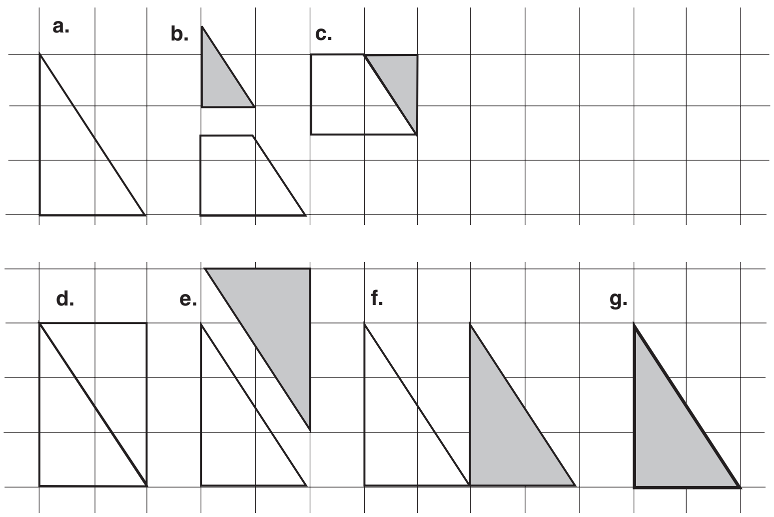
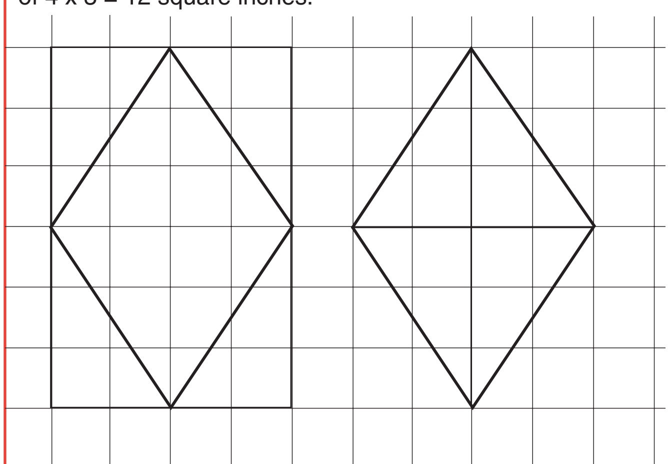


Fig. 7.2: Two ways to find the area of a 2 x 3 inch right triangle

Once the area of the right triangle is known, it's relatively easy to find the area of the rhombus. Either determine the area of the rectangle (24 square inches) and subtract four right triangles, each 3 square inches; to get an area of 24 – 12 = 12 square inches, or divide the rhombus into four congruent triangles, each with an area of 3 square inches, to get an area of 4 x 3 = 12 square inches.



Area of Rhombus =

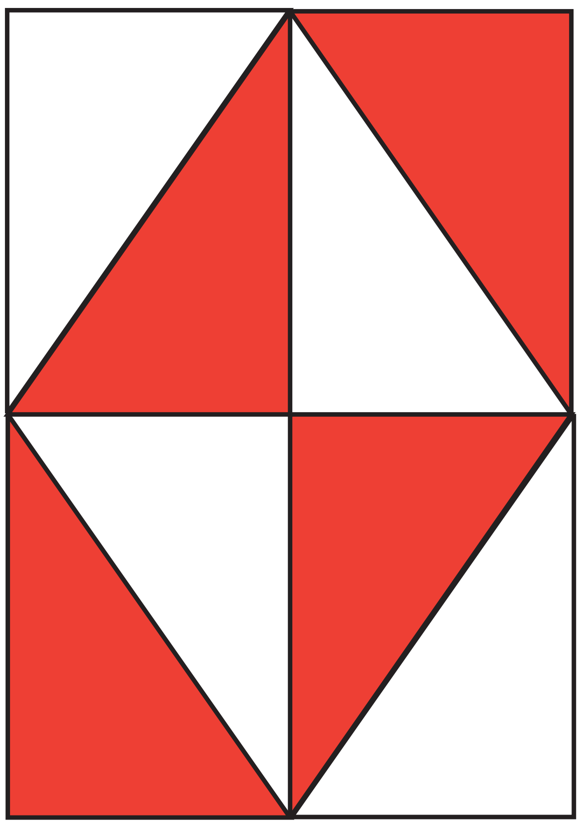
24 – 4 x 3 = 12 square inches

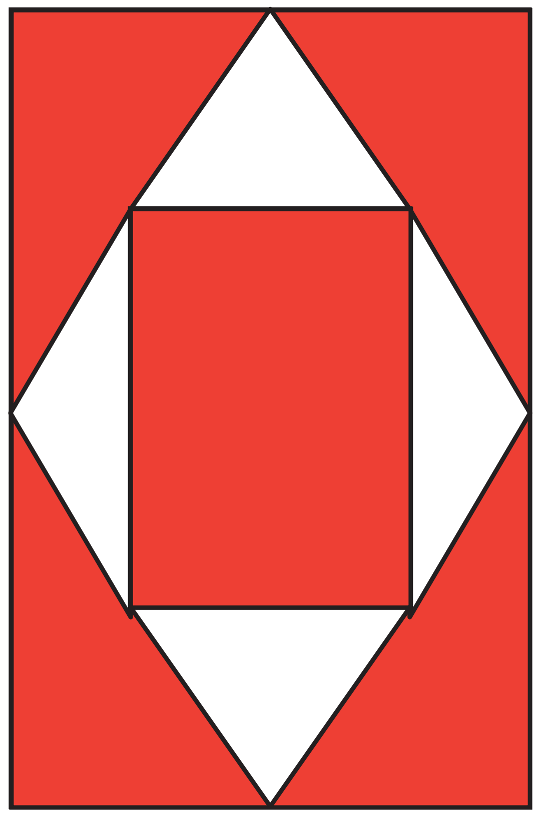
Area of Rhombus =

4 x 3 = 12 square inches

Fig. 7.3: Two ways to determine the area of a rhombus

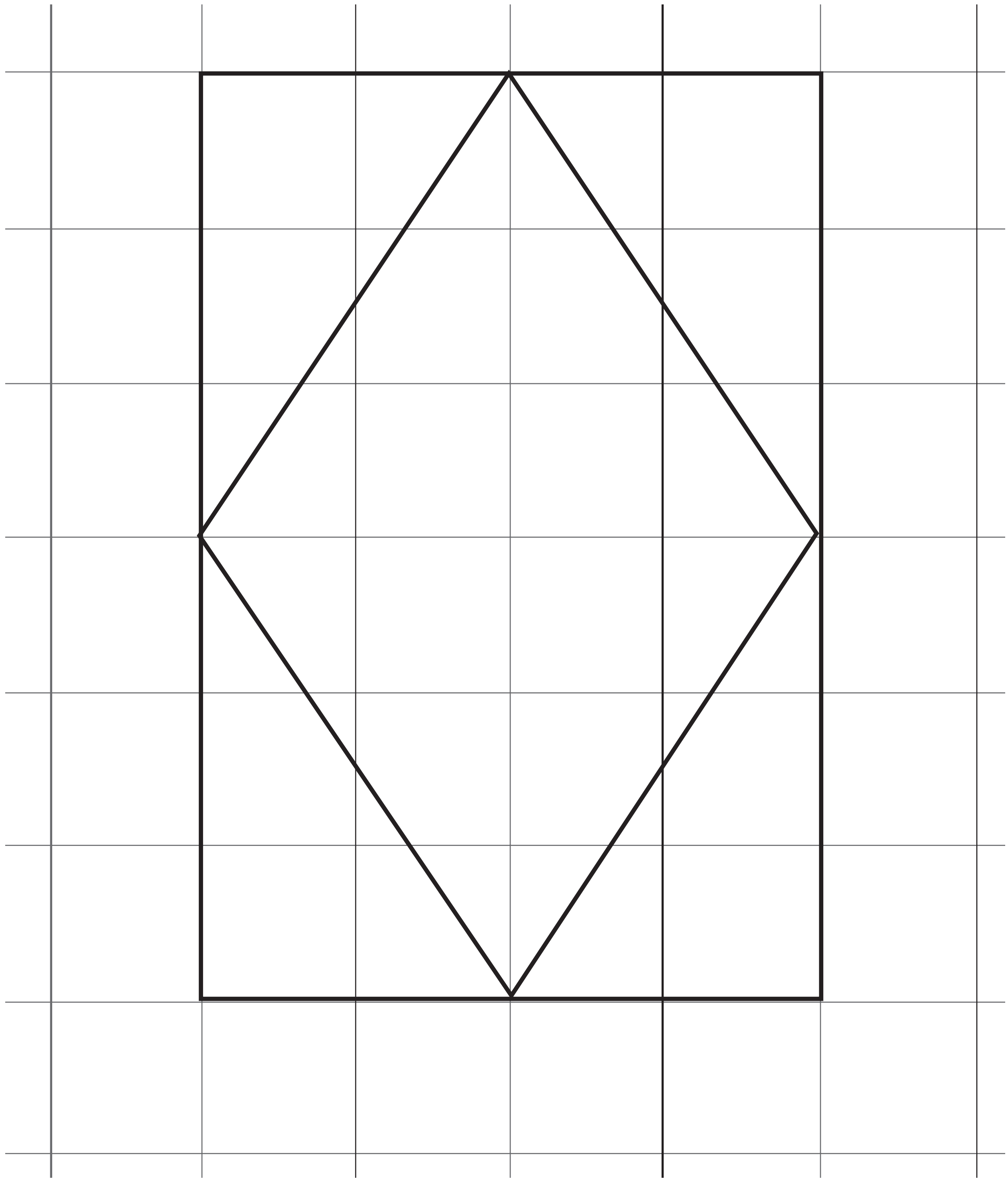
Comparing Areas in Two Puzzles





Rhombus and Four Right Triangles

Inscribed in 6 x 4 Inch Rectangle, 1-Inch Grid



Activity 8
Area Shortcuts

In this activity students learn shortcuts for determining the areas of rectangles, triangles, and rhombi. They learn the conventional formulas for computing area using the base and height of a rectangle or triangle and the diagonals of a rhombus. This is an optional activity that uses what students have already learned about areas to show them how area formulas are derived and used.

Goals

Students will be able to:

- Determine the area of a rectangle by multiplying the base times height.
- Determine the area of a right triangle, or an isosceles triangle by multiplying the base and height and dividing by two.
- Determine the area of a rhombus by multiplying the lengths of two diagonals and dividing by two.

Materials

- Transparency, Shortcut for the Area of a Rectangle
- Handout, Worksheet: Shortcut for the Area of a Rectangle
- Transparency, Shortcut for the Area of a Triangle
- Handout, Worksheet: Shortcut for the Area of a Triangle
- Transparency, Shortcut for the Area of a Rhombus
- Handout, Worksheet: Shortcut for the Area of a Rhombus
- Handout and transparency, Shortcut for the Area of a Rhombus (2)
- Math notebooks

Duration

Two or three class periods.

Vocabulary

Base—a side of a polygon by which the gure is measured

Height—a measure of a polygon, taken as a perpendicular from the base of the gure

Vertex—the point or points at which sides of a polygon, or lines of an angle, meet. Vertices of polygons are sometimes called corners.

Preparation

If necessary, review ways of calculating the area of a rectangle, triangle, and rhombus.

Instructions

1. Remind students that they have learned two methods for finding and comparing areas. Ask them to tell you what they are. They should answer:
   - folding and cutting to compare areas using nonstandard units, and
   - counting squares to measure areas using standard units.

   Explain that today they are going to learn a shortcut for finding areas of rectangles, triangles, and rhombi. The shortcut will give them a quick way to find certain areas without folding and cutting and without counting squares. The shortcuts only work for certain shapes—so if they ever come across a new shape, they can always go back to the tried-and-true methods of counting squares or folding and cutting, or they can divide the shape into smaller shapes whose areas they know how to find.

2. **Area of a Rectangle.** Distribute copies of the handout, Worksheet: Shortcut for the Area of a Rectangle and display the transparency. Call students' attention to rectangle A and ask them to find its area. Now ask them to write down the length of the base, B, and the height, H, in the space provided on the page. Then ask them to compute the area of rectangle A and write it on the page. Ask if they can see a relationship between the area and the two lengths. Then ask them to complete the other exercises on the page and answer the questions at the bottom. Give students time to complete the worksheet.

3. Display the transparency once again and ask for volunteers to share their results for rectangles B, C, and D. As you go over the worksheet, write the table below on the board or butcher paper so that you can help the students discuss and summarize their findings. Remind them that they are looking for a shortcut so that they can find the area of a rectangle without counting every square. Ask them if they can find the area of a rectangle if you give them lengths of the base and height. They should observe and discover that the area is always the product of the two lengths.

Rectangle	Base	Height	Area
A	4	8	32
B	7	3	21
C	5	5	25
D	5	2	10

Fig. 8.1: Areas of rectangles

Write on the board:
Area of a Rectangle = base × height
Tell them that this is the shortcut they are looking for.

Math Note

The convention for the formula of a rectangle is length times width. The naming of the dimensions is arbitrary. Each side is a length. We use base and height for the dimensions of a rectangle. If you and your students are more comfortable with length and width, then discuss with them how the terms are mathematically interchangeable.

If students are not sure about this, ask them to focus on the top row of squares in rectangle A. Ask them to count the number of square units in that row (4). Then ask them to count the number of rows (8). Count with them by fours as you point to each row of squares: 4, 8, 12, 16, 20, 24, 28, 32. If students need more practice, they can draw some rectangles of their own on graph paper and work out the base, height, and area for each one.

Math Note

In this module, students only work with examples of very specialized triangles: right triangles and isosceles triangles. However, the shortcut A = ½ (*b* × *h*) is true for all triangles, where *b* is any side of the triangle, and *h* is the height of a line from the opposite vertex, perpendicular to the base, *b*.

4. **Area of a Triangle.** Repeat the process with the transparency, Shortcut for the Area of a Triangle and the handout, Worksheet: Shortcut for the Area of a Triangle. If students are not yet comfortable with the idea of drawing a rectangle around each triangle and visualizing dividing it in half along a diagonal, you may want to model the process with them again. Provide them with extra copies of the handout and suggest they draw a rectangle touching the top and base of each triangle. Then cut out the rectangle, and cut off the part or parts needed to work out the area of the triangle.

5. Give students time to complete the worksheet. Then display the transparency once again and ask for volunteers to share their results for triangles B, C, and D.

   As you go over the worksheet, write the table below on the board or butcher paper so that you can help the students discuss and summarize their ndings. Remind them that they are looking for a short cut so that they can nd the area of a triangle without counting every square. Ask them if they can nd the area of a triangle if you tell them the lengths of the base and height. They should observe and discover that the area is always ½ of the product of the two lengths. Write on the board:
   Area of a Triangle = base × height ÷ 2
   Tell them that this is the shortcut they are looking for.

Triangle	Base	Height	Area
A	5	6	15
B	7	2	7
C	6	1	3
D	4	7	14

Fig. 8.2: Areas of triangles

Again, if students are not fully con dent of this, give them some graph paper and ask them to construct triangles like the ones on the worksheet.

6. **Exploration: Area of a Rhombus.** Repeat the process a third time with the transparency Shortcut for the Area of a Rhombus and the handout, Worksheet: Shortcut for the Area of a Rhombus. Display the transparency and distribute the handout. Call students' attention to rhombus A at the top. (Cover the two lower figures.) Explain that they are now going to use what they know about rectangles and triangles to find a shortcut for the area of a rhombus. First ask a volunteer to draw a rectangle around the rhombus.

   Ask for volunteers to find the area of the rectangle. If this presents no problem then ask for volunteers to show how they can find the area of the rhombus. Then show them the second figure on the page. (Answer: area of the rectangle is 32 square units and the area of the rhombus is 16 square units.)

   Challenge them to find the area of the rhombus using any of the triangles displayed on the transparency. Display the third figure on the transparency. If the students need help, then ask them how many small triangles they see. Ask them to figure out the area of one of the small triangles (½ its base × its height = 4 sq. units). Do all four small triangles have the same area? (They do.) So the total area of the rhombus is? (16 sq. units.)

   Ask students to look at the lines dividing the rhombus into four equal triangles. These lines are called diagonals and are labeled d1 and d2. Diagonals are lines that join any two nonadjacent vertices of a polygon. Diagonal d1 goes all the way across the rhombus between corners 1 and 3. Diagonal d2 goes from top to bottom of the rhombus, between corners 2 and 4. Ask students if they can figure out the lengths of d1 and d2 (8 and 4, respectively). Explain that they are now going to practice finding the area of three more rhombi. They should use the table to record the areas along with the lengths of d1 and d2 for each rhombus, then see if they can find a shortcut for the area of a rhombus.

7. **Explore.** Distribute the handout, Shortcut for the Area of a Rhombus (2). Give students time to complete the worksheet. Then display the transparency, Shortcut for the Area of a Rhombus (2) and ask for volunteers to share their results for triangles B, C, and D. Remind them to fill in the table at the bottom of the worksheet.

   As you go over the handout, use the transparency, Shortcut for the Area of a Rhombus (2). Write the table below on the board or butcher paper so that you can help the students discuss and summarize their findings. Remind them that they are looking for a shortcut so that they can find the area of a rhombus without counting every square. Ask them if they can find the area of a rhombus if you tell them the lengths of both

Math Note

A rhombus is a quadrilateral; so is a rectangle or a square. Any quadrilateral has two diagonals. A triangle has no diagonals because each vertex is adjacent to another vertex.

diagonals. They should observe and discover that the area is always one half the product of the two lengths. Write on the board:

Area of a Rhombus = ½ $d_1 \times d_2$
Tell them that this is the shortcut they are looking for.

Rhombus	**Diagonal-1 (d_1)**	**Diagonal-2 (d_2)**	**Area**
A	8	4	16
B	4	6	12
C	10	4	20
D	2	8	8

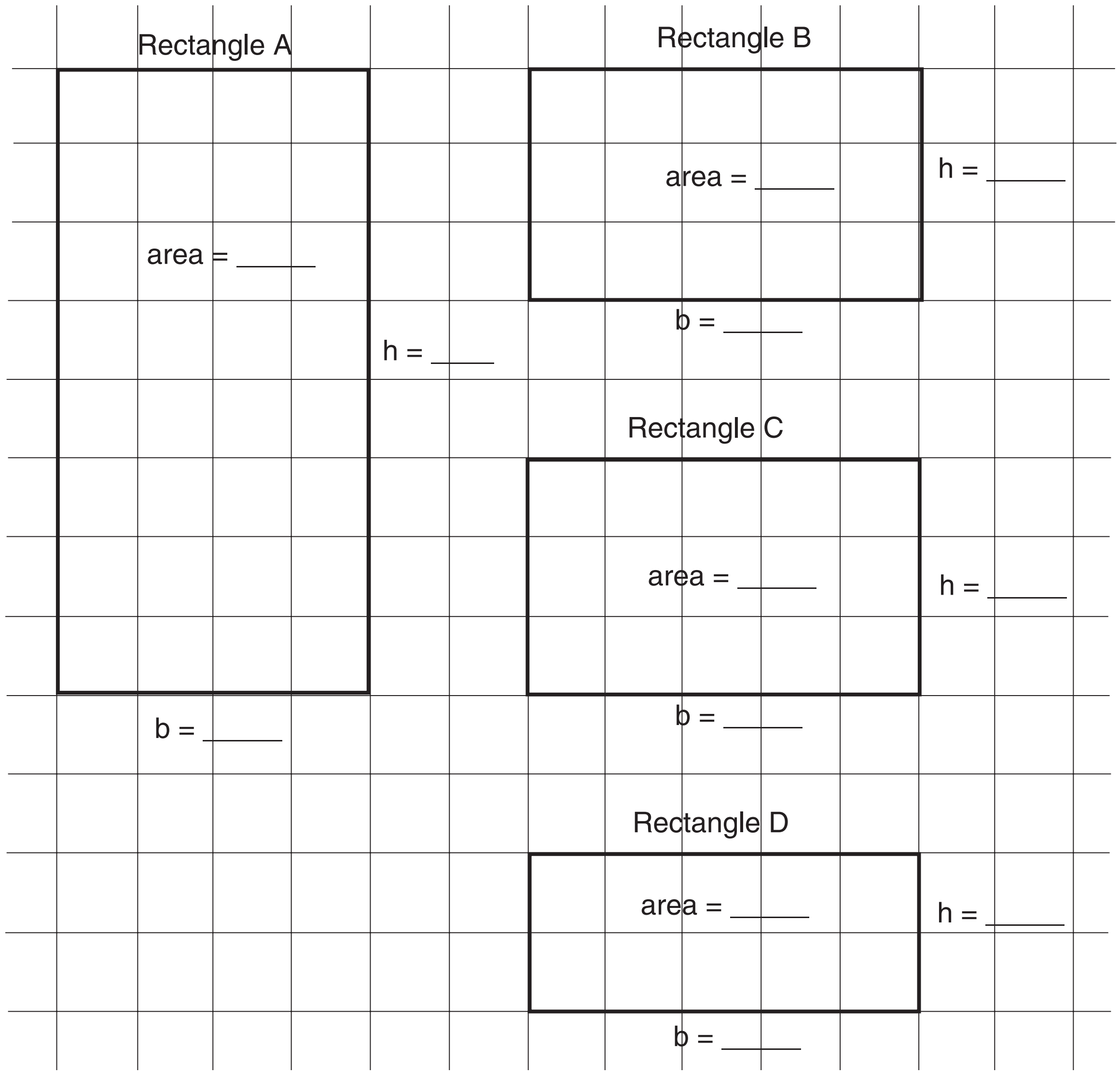
Fig. 8.3: Areas of rhombi

8. Remind students that they now know three shortcuts (or formulas) for these areas. Write these on a chart so students can see them and refer to them. Have students write the three shortcuts in their math notebooks.

Shape	**Shortcut (Long Form)**	**Area Shortcut (Short Form)**
Rectangle	base × height	b × h
Triangle	half of the base times height	½ (b × h)
Rhombus	half of the product of the diagonals	½ ($d_1 \times d_2$)

Fig. 8.4: Formulas for calculating area

Worksheet: Shortcut for the Area of a Rectangle

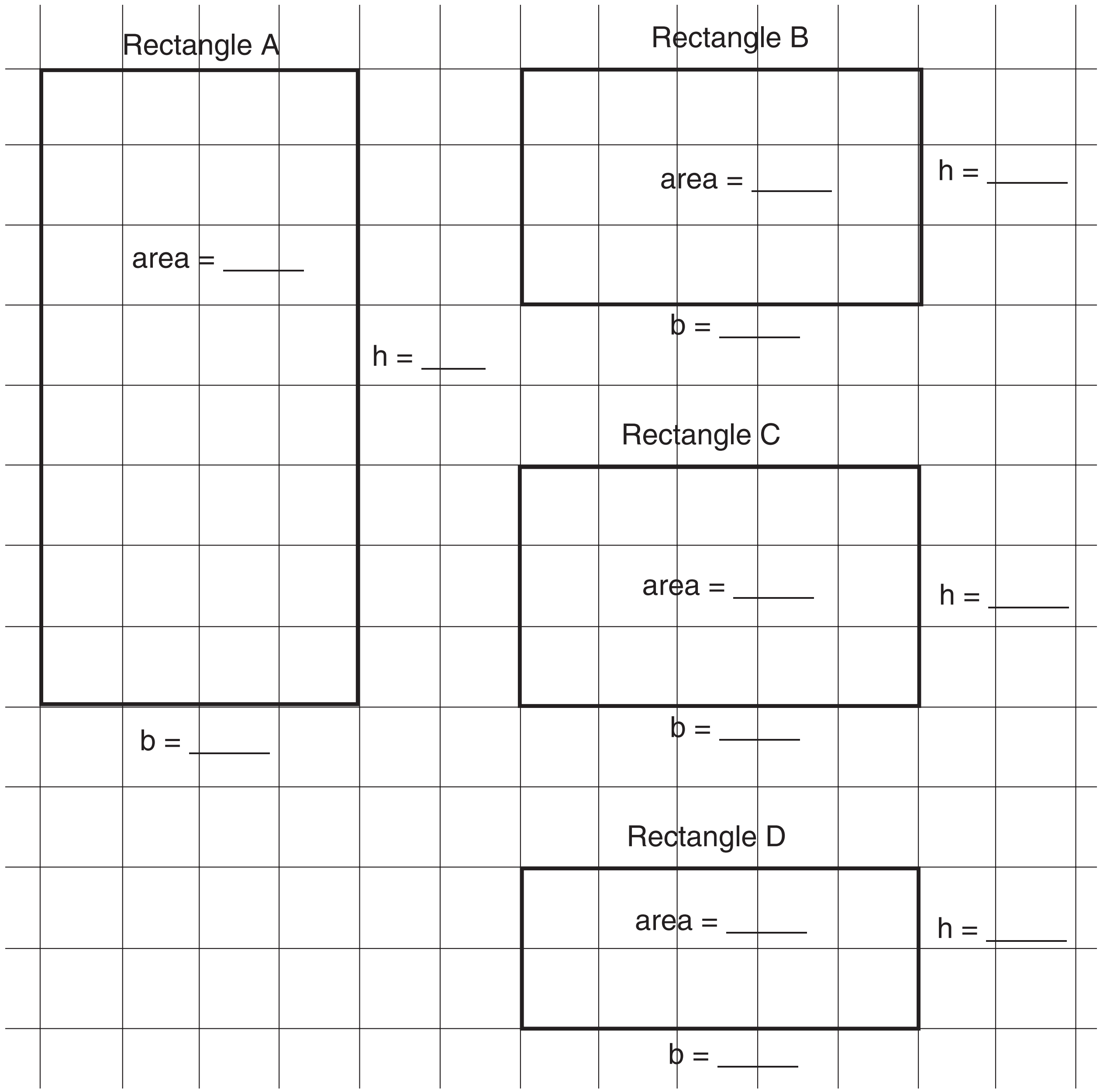


Complete the table:

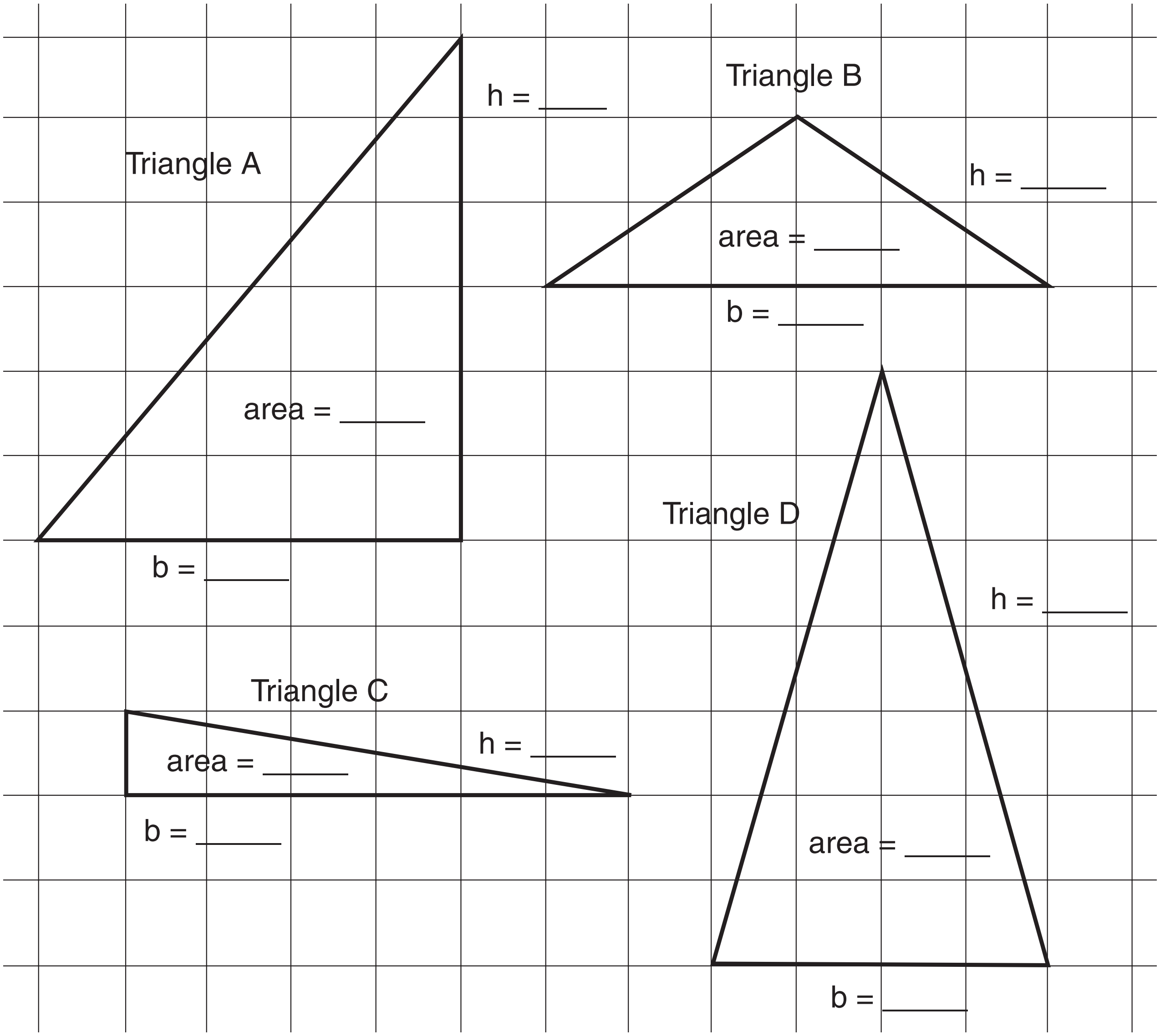
Rectangle	Base (b)	Height (h)	Area
A			
B			
C			
D			

If you know the lengths of the base and height of a rectangle, how can you find its area without a drawing?

Shortcut for the Area of a Rectangle



Worksheet: Shortcut for the Area of a Triangle

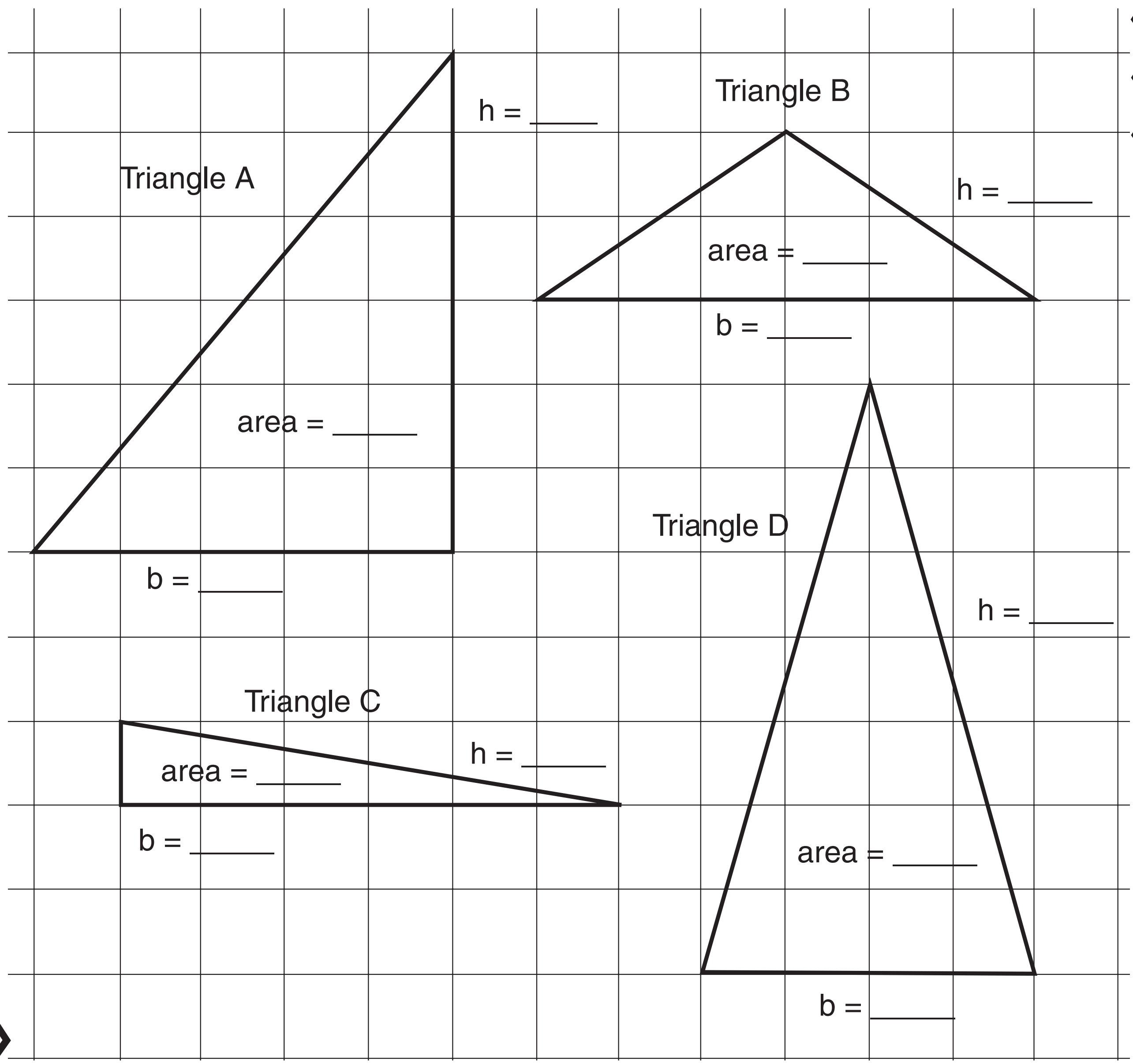


Complete the table

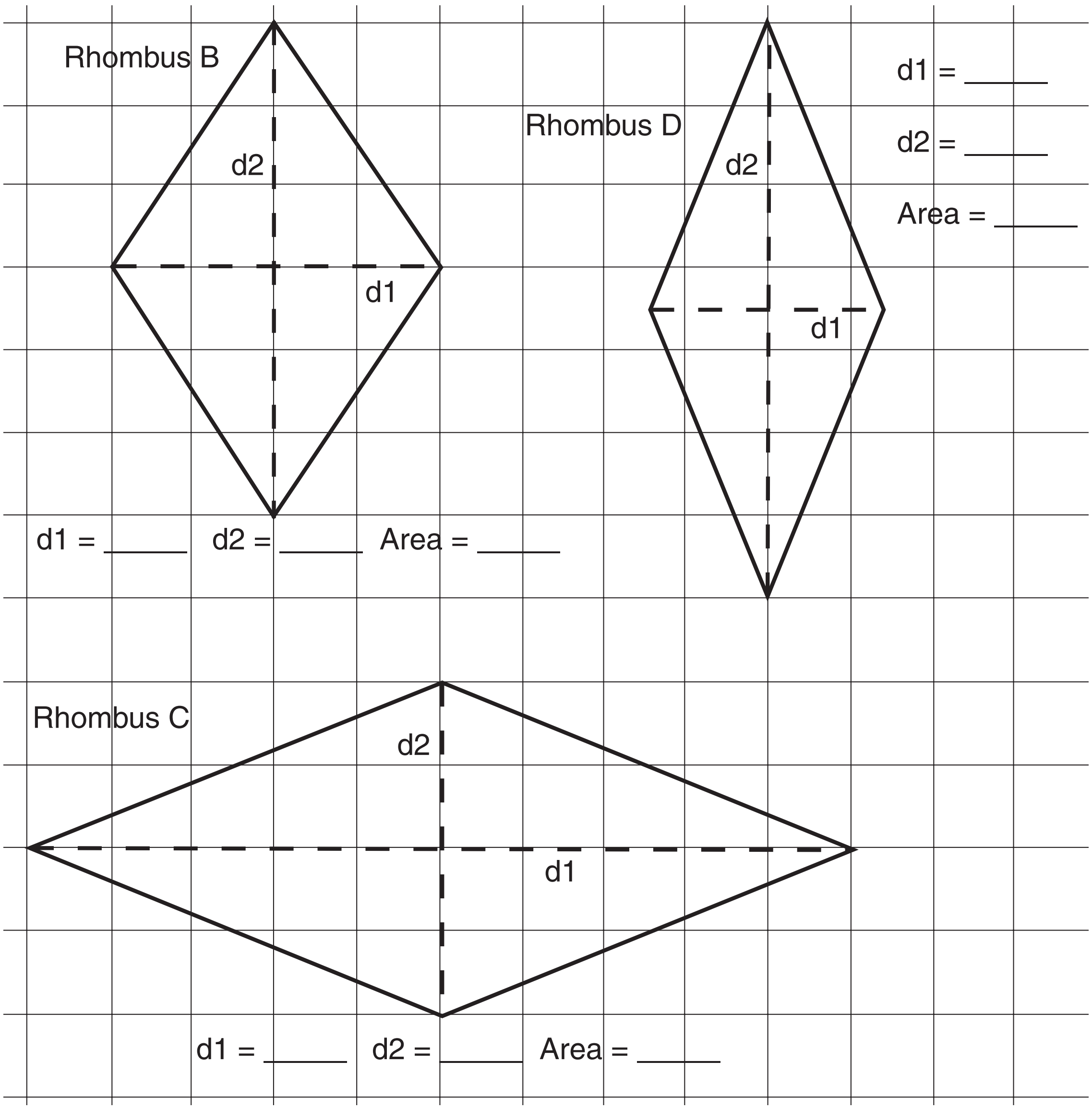
Triangle	Base (b)	Height (h)	Area
A			
B			
C			
D			

If you know the lengths of the base and height of a triangle, how can you find its area without a drawing?

Shortcut for the Area of a Triangle



Worksheet: Shortcut for the Area of a Rhombus



d1 = ______ d2 = ______ Area = ______

d1 = ______ d2 = ______ Area = ______

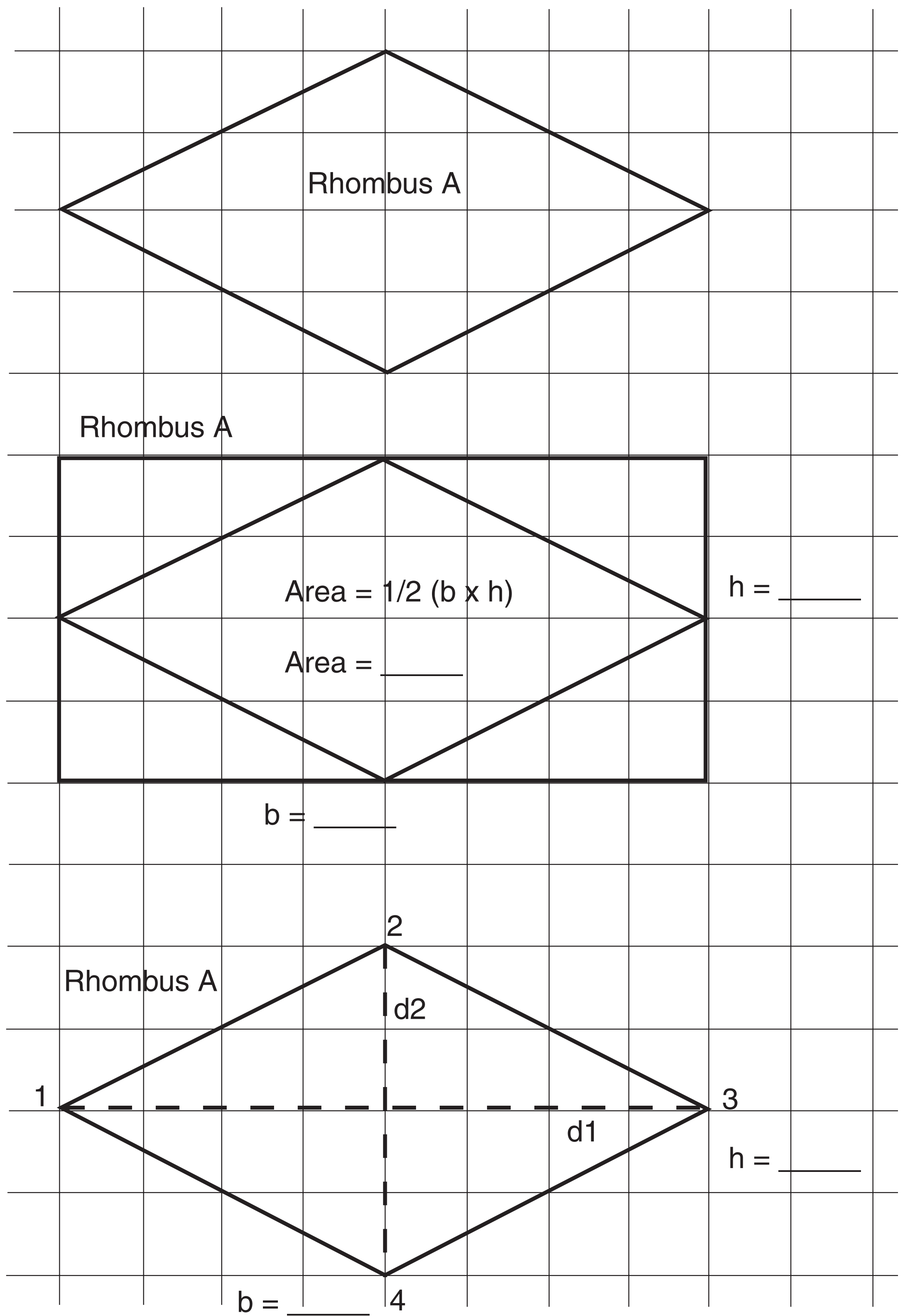
d1 = ______ d2 = ______ Area = ______

Complete the table

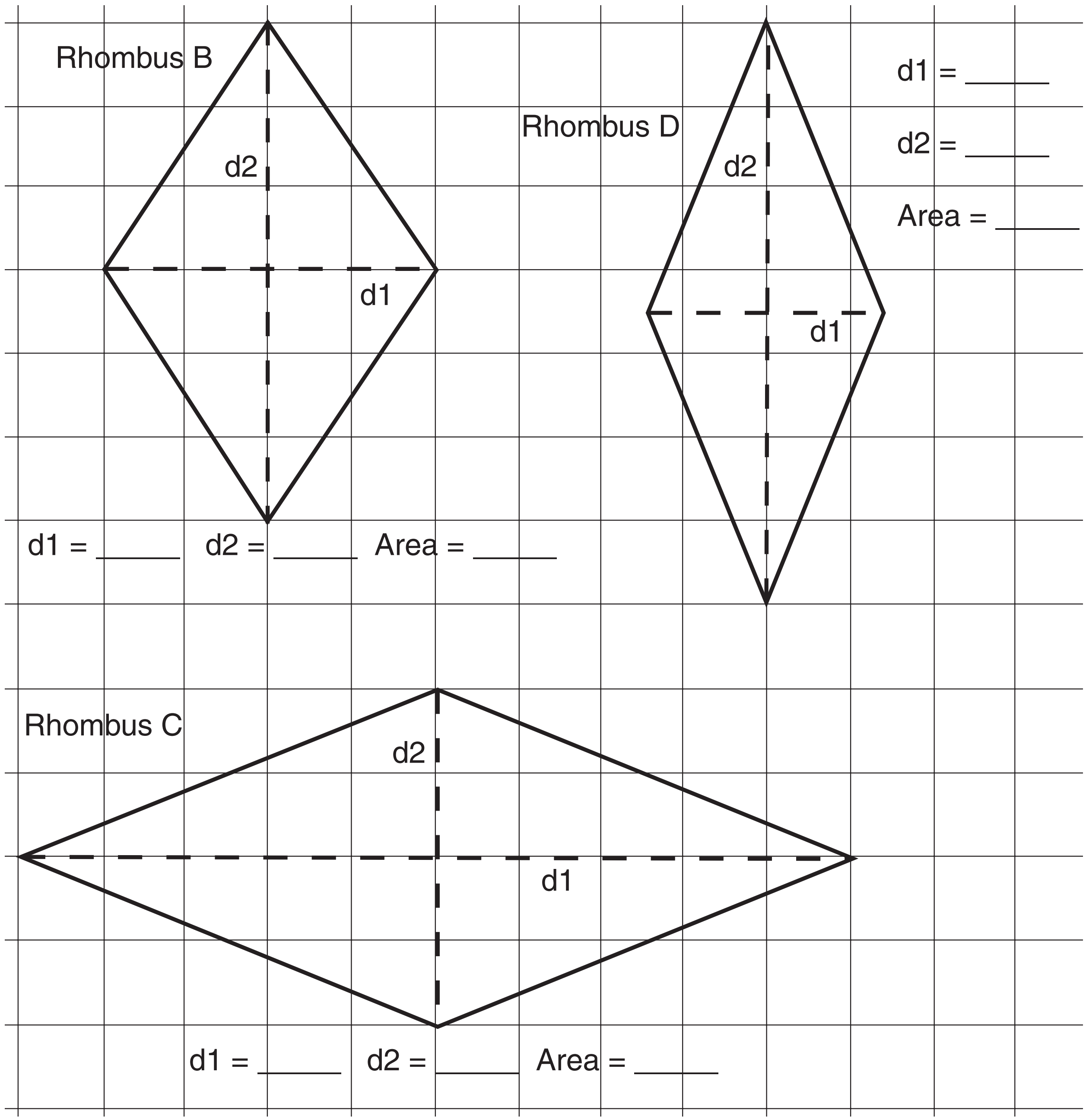
Rhombus	Diagonal 1 (d1)	Diagonal 2 (d2)	Area
A			
B			
C			
D			

If you know the lengths of both diagonals of a rhombus, how can you find its area without a drawing?

Shortcut for the Area of a Rhombus



Shortcut for the Area of a Rhombus (2)



Activity 9
Designing and Constructing a Headband Pattern

This is the culminating activity for the module, and it will take several class periods to complete all the steps. In this activity, students create a repeating element and use it to design a symmetrical headband pattern. Students examine sample patterns to investigate Yup'ik design principles for headbands, which are based on symmetry. As students create the core pattern for their headband as a template, they determine the amount (number of pieces required to make their headband as well as the amount of space that it will take up). This presents the nal challenge to the culminating activity—further explorations into area and formalizing how to derive area. After determining the total area of dark and light material needed for the complete pattern, students make templates out of stiff material like cardboard for the pattern pieces. They determine the total number of each type of piece needed, cut the pieces out of dark and light colored felt, and assemble their headbands.

This activity synthesizes the concepts that students have been working on throughout the module—design principles (symmetry and congruence) and area relationships. One natural problem that arises from making headbands is not wasting material and knowing how much material is required. An extension activity challenges students to make 15 more identical headbands for a whole dance group and determine the area of material needed. This challenge provides the context and purpose for delving into formalizing area. Moving from nonstandard ways of deriving area into a method using standard units and formulas provides students with an ef cient algorithm that saves them time and ensures that they will not buy too much or too little material.

Goals

Students will be able to:

- Use their puzzle pieces—or cut out new pieces—to make their *tumaqcat* pieces.
- Use Yup'ik design principles to create a two-color headband pattern with a repeating element.
- Determine the area of the dark and light components of the repeating pattern.
- Determine the total area of dark and light material needed for their patterns.
- Make a set of shape templates to reproduce their patterns in a more permanent material.

Materials

- Transparency and handout, Sample Headband Patterns (from Activity 1)
- Handout, Headband Area Table
- Students' puzzle pieces saved from Activity 5
- 1-inch graph paper
- Construction paper in two colors, one dark and one light
- Glue
- Paper strips, 2 x 22 inches, to use as a base for making paper patterns
- Felt strips, 2 x 22 inches, to use as a base for making headbands (one per student)
- Supplies of felt in contrasting colors
- Stiff material to form shape templates (cereal or cracker boxes or manila le folders are appropriate for this)
- 4-inch long elastic strips (one per student)
- Scissors

Teacher Note

You have the option of starting this design activity after students complete Activity 5, Constructing a Rhombus Pattern Puzzle. Before the students can finish their patterns, however, they will need to determine the total area of dark and light pieces needed. Therefore, this activity will need to run in parallel with Activities 6 and 7, in which students learn how to measure the areas of simple geometric shapes. You may want to start this activity and allow students to continue to work on cutting out their paper pieces and making their templates as a craft activity, while working through Activities 6 and 7 about measuring area. Once students are able to determine the area needed for their dark and light pieces, they can use their templates to cut out their pieces and begin gluing their headbands.

For this module we recommend that you obtain 2 x 22 inch strips of felt, and 4-inch-long elastic strips for students to use in making their headbands. We suggest that students begin by gluing their felt pattern pieces onto a felt strip. Depending on their skills and the time available, they may wish to sew their glued pieces onto the strip to make a more permanent headband.

The culminating project has three major parts:

1. Design a headband pattern according to Yup'ik design principles, using dark and light paper pieces on a paper strip.
2. Create a template for the *tumaqcat* pieces and calculate the area of dark and light felt needed to complete the project.
3. Cut out the dark and light felt pieces needed and copy the pattern by attaching pieces onto a felt strip. Join the ends of the felt strip using an elastic band.

Cultural Note

A few words about materials: Yup'ik headbands are part of ceremonial headdresses, which are traditionally constructed from skins and furs. Today black and white calf skins are widely used for making the *tumaqcat* pieces because they contain roughly equal amounts of dark and light colored fur. The small shapes are carefully sewn together to form a pattern. Then fur pieces are sewn above and below the headband to complete the headdress. The headband is sewn to a sturdy cloth strip that forms the inside lining of the headband. Then a strip of elastic material is attached to link the two ends of the headband and keep it comfortably fixed on the wearer's head. In earlier times, thin strips of animal hide were used to tie a headband in place.

Duration

Three to four class periods.

Vocabulary

Basic repeating element—a shape or collection of shapes that, when repeatedly copied and moved, make up a pattern. Basic repeating element refers to the smallest set of shapes that can be used to generate an entire pattern.

Linear pattern—a one-dimensional (potentially) infinite pattern that repeats a basic unit (geometric shape or design) over and over again, such as a band, border pattern, or strip pattern. These are sometimes called "frieze patterns" because of their use in architectural settings.

Preparation

Gather some examples of headband patterns for the students to use as models (optional: you can use the transparency). Make up 2 x 22 inch strips of paper by taping 2 inch x 11 inch strips together. Gather materials to be used for the finished headbands.

Instructions

1. Tell the class that they are about to begin designing and making their headbands. Their first task will be to decide which shapes they want to use. Brainstorm with the class on choosing their patterns. Display the transparency, Sample Headband Patterns. Ask students if they can identify some of the shapes they see in the patterns. They should easily identify rhombi, squares, triangles (right triangles, isosceles triangles, right isosceles triangles) and they may be able to identify parallelograms.

2. **Explore.** Tell students that Yup'ik pattern designers use certain basic principles in designing their patterns. Distribute copies of the Sample Headband Patterns. Ask them what is the same about all the sample patterns. Invite students to cut out the pattern strips and fold them to identify whether they have lines of symmetry. Ask them to identify the shape combinations that are repeated in each pattern. Discuss the sample patterns with the students to make sure they understand the Yup'ik design principles. The characteristics of these designs are similar to those for the 6 x 4 pattern puzzles that students made during Activity 5.

Part 1: Constructing a Paper Strip Pattern

3. Model the process of constructing a paper pattern on a strip. Use the pieces you have already selected from your pattern puzzles. As you work, review the Yup'ik design principles (see Cultural Note), and point out that you are starting from the center of your headband and working outward on both sides.

   Explain to students that they will need to start by selecting pieces (*tumaqcat*) and planning their own patterns. They will begin by making their patterns on paper strips, cutting out the construction paper shapes they will need, and gluing them in place on the strips. Remind them that when they made their pattern puzzles they learned how to make shapes that t together exactly by folding shapes along their lines of symmetry, diagonals, or lines joining midpoints.

4. Have students work in small groups. Hand out the supplies (strips of construction paper in contrasting colors, graph paper, scissors, etc.). Remind students to take out the puzzle pieces they made during Activity 5. They can make their headband patterns using the same shapes if they wish. Ask students to identify the shapes they plan to repeat to make their patterns.

5. **Joint Activity.** Begin making your pieces for your project; use the ideas about dividing larger pieces into smaller ones that were modeled in Activity 5. Some students may choose to work with you as well. Here are the steps in designing a pattern:

   1. Choose some shapes that you want to repeat.
   2. One way to estimate how many shapes you will need is to divide the 2 by 22 inch paper strip in half and then divide each half into eight equal parts.
   3. Cut out your dark and light shapes and lay out the repeating pattern, starting from the center of the headband and working outwards on both sides, placing light pieces next to dark pieces as you go.

   Allow students enough time to create their patterns. Emphasize creativity in designs, following the Yup'ik design principles.

Cultural Note

Yup'ik Design Principles for Headband Patterns

- Patterns alternate light and dark shapes to achieve a balance of color.
- Shapes line up so that sides of light shapes exactly match sides of dark shapes.
- Patterns have a symmetrical shape at their center and designs are constructed starting from the center.
- Patterns repeat the same shape sequences, with a basic repeating element and a linear pattern on both sides of their center lines.

Cultural Note

Theresa Mike, a Yup'ik pattern designer from St. Marys, divides her strips by folding each part in half, then opening the folds and folding each segment in half. She is careful to fold each segment only once, never folding double, because that would make the division less accurate.

Teacher Note

Students will need to complete Activities 6 and 7 in order to calculate the areas of dark and light material needed.

Part 2: Making a Set of Templates and Determining How Much Material is Needed

6. Once students complete their paper patterns, ask them to determine how many different shapes they used and how many of each. Tell the students that before they cut pieces for their final project, they will first have to figure out how much dark and light material they will need. This will enable everyone in the class to complete their headbands without wasting material. Students may use the methods they learned in Activities 6 and 7 to determine the areas needed. Ask different groups to share how they figured out how much material they would need to complete the project.

   Hand out the Headband Area Table and suggest that students use it to record the information they'll need to calculate the total area. Remind students that the total area of the shapes must equal the area of the 2 by 22 inch strip (44 square inches). This activity can be supplemented with the extension at the end of this activity.

Teacher Note

This part of the project offers a good opportunity to consolidate learning about area that has been developed through Activities 6–8. If necessary, take some time to review Activity 7 about calculating shape areas by counting squares. This would be a good time to review Activity 8, Area Shortcuts, as well.

7. Ask students to make a template for each shape using stiff material cut from a cereal or cracker box or a manila folder. They will copy one of their paper shapes to make one template for each separate shape. They will use the same template, carefully tracing around it, to copy each type of shape onto the felt.

Part 3: Making the Headband Out of Felt

8. **Joint Activity.** Make your headband pieces at the same time the students make theirs. Once you have determined how much dark and light material you will need, carefully trace and cut all the pieces. One objective is to cut the pieces so that there is as little waste material as possible. If you have small pieces of material left over, put them back in the general supply so that other people can use them to make their shapes.

9. **Joint activity.** Complete your project while the students are completing theirs and display it for students to see. Once all the pieces have been cut, begin gluing them onto a felt strip, starting at the center and working outward. Pay attention to any difficulties that you might have such as the end piece not fitting and how you solve that problem. If possible, observe difficulties that students may have. Share problems and solutions.

10. When the headband pattern is complete, join both ends with a short piece of elastic band.

11. If your students are skilled at sewing, they may want to make their headbands more permanent by sewing the glued pieces onto their felt strips.

12. When the headbands are nished, invite guests—parents or students from other classes—to a celebration to mark the successful completion of a major project. The celebration provides an opportunity for students to share what they have learned about shape-making and pattern design.

Fig. 9.1: Theresa Mike

Extension Activity: Calculating Areas for a Class Project

Ask students to suppose that a class of 15 students all want to make identical headbands. The student's job is to choose a pattern—one of the ones created by someone in the class—and determine how much dark and light material would be needed to make 15 headbands exactly like that. The class will need to know how much material to purchase before they begin the project. The Headband Area Table handout can help them calculate the area needed for one headband.

Headband Area Table

Shape of Piece (sketch shape below)	Area of Piece	Number of Pieces Used	Total Area Needed for Pieces
Light Colored Pieces			
Total area of light pieces needed:			
Dark Colored Pieces			
Total area of dark pieces needed:			