OLLI: History of Numbers
Day 4: The History of Irrational Numbers and Transfinite Numbers

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Recap (color coded): zero, fractions, negative numbers

- 2000 BC: Sophisticated arithmetic with fractions
- 700 BC: Zero as placeholder
- 100 BC: Arithmetic with fractions essentially equivalent to our modern approach (positive integers only)
- 600 AD: possessions, debts, and algebraic rules
- 1585: decimal representation of fractions in Europe
- 1600's: Zero is allowed to be a solution to an equation
- 1770: $(-a)(-b)$ is the opposite of $(-a)(b)$
- 1800's: Zero as type of object; division as a human-defined operation

Recap (color coded): zero, fractions, negative numbers, irrational numbers

- 2000 BC: Sophisticated arithmetic with fractions
- 700 BC: Zero as placeholder
- 430 BC: $\sqrt{2}$ is not rational
- 100 BC: Arithmetic with fractions essentially equivalent to our modern approach (positive integers only)
- 600 AD: possessions, debts, and algebraic rules
- 1585: decimal representation of fractions in Europe
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Irrational Numbers:

Terminology:

integers: $\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, 3, \cdots\}$

rational numbers: $\mathbb{Q} = \{\frac{a}{b} \mid a, b \text{ are integers, and } b \neq 0\}$

real numbers: $\mathbb{R}$, the set of values of a continuous quantity that can represent a distance along a line.

or

points on an infinitely long line

irrational numbers: $\mathbb{R} - \mathbb{Q}$

or

real numbers that cannot be written as a quotient of integers

or

numbers whose decimal representation is nonterminating and nonrepeating

Irrational Numbers: A Modern Proof that $\sqrt{2}$ is irrational

Irrational Numbers:

- (430 BC, Pythagoreans, Island of Samos) The side and diagonal of a square are incommensurable.
- (1761, J. Lambert) π is irrational, e^k is irrational if k is rational (and $k \neq 0$)
- (mid 1800's, Europe) Mathematicians started asking "What *are* irrational numbers?"
- (1858, R. Dedekind (1831-1916) Uses “cuts" of rational numbers to define irrational numbers
- (2019...???) $\pi + e$ is irrational...??

Irrational Numbers: The Idea of Dedekind Cuts

Irrational Numbers: Curious Properties

- Find a rational number between: 21.35562... and 21.35578...

- Find an irrational number between 14.22222... and 14.2212333...

Between every two rational numbers, there exists an irrational number AND between every two irrational numbers there exists a rational number.

Irrational Numbers: Curious Properties

So, the number of RATIONALs and the number of IRRATIONALs is about the same, right?

OMG, how would you ever know?

Transfinite Numbers

Georg Cantor (1845-1918) returns to one-to-one correspondence. Around 1878, he starts to count the infinite!

Transfinite Numbers

consider the two sets below:

S: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...

T: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...

Transfinite Numbers

Using the method of one-to-one correspondence it is possible to show:

the number of integers is equal to the number of rationals

Transfinite Numbers

Transfinite Numbers

Cantor was able to show that the number of irrationals was larger than the number of rationals.

Transfinite Numbers

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- 600 AD: possessions, debts, and algebraic rules
- 1585: decimal representation of fractions in Europe
- 1600's: Zero is allowed to be a solution to an equation
- 1770: $(-a)(-b)$ is the opposite of $(-a)(b)$
- 1800's: Zero as type of object; division as a human-defined operation
- 1860: Dedekind Cuts
- 1878: Cantor counts the infinite with one-to-one correspondance

Thank You for your Patience and your Questions!