OLLI: History of Numbers
Day 3: The History of Negative Numbers and Irrational Numbers

instructor: Jill Faudree

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Recap (color coded): zero and fractions

- 2000 BC: Sophisticated arithmetic with fractions
- 700 BC: Zero as placeholder
- 100 BC: Arithmetic with fractions essentially equivalent to our modern approach (positive integers only)
- 1585: decimal representation of fractions in Europe
- 1600's: Zero is allowed to be a solution to an equation
- 1800's: Zero as type of object; division as a human-defined operation

Negative Numbers

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EXAMPLE: I am seven years old and my sister is two. When will I be exactly twice as old as my sister?

Negative Numbers

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EXAMPLE: I am eighteen years old and my sister is eleven. When will I be exactly twice as old as my sister?

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  "The second solutions is not to be taken, because of its inapplicability. For people have no clear understanding in the case of a negative quantity."
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Negative Numbers: Tide starts to turn

- (600's AD, Brahamgupta, Northwest India)
- (1655, J. Wallis, Europe) Argues negative numbers are larger than infinity.
- (1707, I. Newton) Negative Quantities are less than nothing.
- (1770, L. Euler) Justifies $(-a)(-b) = ab$ by asserting it should be "opposite" of $(-a)(b) = -ab$
- (1831, A. De Morgan, Europe) The expression $0 - a$ is as inconceivable as $\sqrt{-a}$.

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Negative Numbers: Resolution?

Negative numbers and their operations (like division and zero) are man-made.

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Two numbers *a* and *b* are *commensurable* if there exists a common measure for *a* and *b*.

Examples:

- 3 and 8 both have 1 as a common measure
- $\frac{2}{3}$ and $\frac{4}{7}$ both have ________________ as a common measure

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Irrational Numbers

Two numbers a and b are *commensurable* if there exists a common measure for a and b.

Otherwise, a and b are called *incommensurable.*

commensurable = rational   incommensurable = irrational

Irrational Numbers

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Irrational Numbers: How did they know?

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- (430 BC, Pythagoreans, Island of Samos) The side and diagonal of a square are incommensurable.
- (1761, J. Lambert) π is irrational, e^k is irrational if k is rational (and $k \neq 0$)
- (mid 1800's, Europe) Mathematicians started asking "What *are* irrational numbers?"
- (1858, R. Dedekind) Uses “cuts" of rational numbers to define irrational numbers