

OLLI: History of Numbers

Day 2: The History of Zero and Fractions

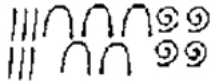
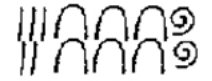
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September 20, 2018

Recap




























































- ▶ (date: too early to know) larger - smaller quantities
- ▶ (date: too early to know) counting via one-to-one correspondence, especially on body parts
- ▶ (date: 37,000 BC) tally marks on Ishango bone from centra Africa
- ▶ (date: 5000BC) writing symbols that represent counting numbers in Ancient Near East
- ▶ (date: 3000BC) Ancient Egyptians have a base 10 symbolic system
- ▶ (date: 3000BC) Mesopotamians have a *base 60 positional system*

Pics from Ancient Egyptians

	(= 456)
	(= 265)

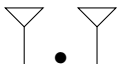
						
1	10	100	1000	10000	100000	10 ⁶
Egyptian numeral hieroglyphs						

Pics from Mesopotamians

1		11		21		31		41		51	
2		12		22		32		42		52	
3		13		23		33		43		53	
4		14		24		34		44		54	
5		15		25		35		45		55	
6		16		26		36		46		56	
7		17		27		37		47		57	
8		18		28		38		48		58	
9		19		29		39		49		59	
10		20		30		40		50			

ZERO

- ▶ (date: 700BC) Mesopotamians were using the symbol for “end of sentence” as a place-holder.



to indicate $(1 \times 60^2) + (0 \times 60) + 1$

ZERO

- ▶ (date: 700BC) Mesopotamians were using the symbol for “end of sentence” as a place-holder.
- ▶ (date: 350 AD) Earliest use of zero as placeholder in mathematical notation of the Maya
- ▶ (date: 600AD) Hindu mathematicians using small circle as a place-holder. (Base 10 representation), Brahamagupta attempts to describe computation with zero.

ZERO

- ▶ (date: 700BC) Mesopotamians were using the symbol for “end of sentence” as a place-holder.
- ▶ (date: 600AD) Hindu mathematicians using small circle as a place-holder. (Base 10 representation)
- ▶ (date 850AD) Mahavira (Jain from India) wrote (in words not symbols) how to *operate* with zero

If a is a number then:

$$a \times 0 = 0$$

$$a + 0 = a$$

$$a \div 0 = a \quad \leftarrow \text{oops!}$$

ZERO

- ▶ (date: 700BC) Mesopotamians were using the symbol for “end of sentence” as a place-holder.
- ▶ (date: 600AD) Hindu mathematicians using small circle as a place-holder. (Base 10 representation)
- ▶ (date: 850AD) Mahavira (Jain from India) wrote (in words not symbols) how to *operate* with zero
- ▶ (date: 1100AD) Baskara II (Indian) wrote

If a is a number then:

$a \div 0$ is “an infinite quantity”

ZERO

- ▶ (date: 700BC) Mesopotamians were using the symbol for "end of sentence" as a place-holder.
- ▶ (date: 600AD) Hindu mathematicians using small circle as a place-holder. (Base 10 representation)
- ▶ (date: 850AD) Mahavira (Jain from India) wrote (in words not symbols) how to *operate* with zero
- ▶ (date: 1100AD) Baskara II (Indian) still working on division by zero
- ▶ (date: 1100's) Hindu-Arabic numerical system makes it to Europe but zero is still treated as a different symbol than the other 9 symbols
- ▶ (date: 1500's) Zero is not considered a solution to an equation.
- ▶ (date: early 1600's) Thomas Harriot not only allowed zero to be a solution to an equation, but he used it as a tool to solve equations.

ZERO

Solve equations??

ZERO

Aside: Leonard Euler (1700's) wrote that $n/0$ is infinite.
 How do we handle division by zero?

FRACTIONS

- ▶ (date: 2000-1600 BC) Ancient Egyptian and Babylonians used sophisticated fractions.

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FRACTIONS

Note that in "every day life" things are set up to avoid fractions.

Example: cup, pint, quart, gallon

Example: pennies, nickels, dime, quarter, dollar

Example from science: degrees, minutes, seconds

FRACTIONS

- ▶ (date: 2000-1600 BC) Ancient Egyptian and Babylonians used sophisticated fractions.
- ▶ (date: 100 BC) Chinese manuscript *Nine Chapters on the Mathematical Art* contains notation "very similar to ours."

Chinese fractions would have written $8/5$ as $1 \frac{3}{5}$

Same algorithm for addition. Much better algorithm for division.

FRACTIONS

Chinese algorithm for division:

FRACTIONS

- ▶ (date: 2000-1600 BC) Ancient Egyptian and Babylonians used sophisticated fractions.
- ▶ (date: 100 BC) Chinese manuscript *Nine Chapters on the Mathematical Art* contains notation very similar to ours.
- ▶ (date: 600 AD) Mathematicians in India were writing fractions as one number above the other (no line separating them) and developed the *invert-and-multiply* algorithm.

FRACTIONS

- ▶ (date: 2000-1600 BC) Ancient Egyptian and Babylonians used sophisticated fractions.
- ▶ (date: 100 BC) Chinese manuscript *Nine Chapters on the Mathematical Art* contains notation very similar to ours.
- ▶ (date: 600 AD) Mathematicians in India were writing fractions as one number above the other
- ▶ (date: 1100 AD) Fractions written as $\frac{a}{b}$ began to appear in Europe as a result of Arab influence
- ▶ (date: 1850 AD) "Slash" notation appears in Europe: a/b

FRACTIONS

- ▶ (date: 1400's) Fractions in the form of percents, thanks to interest rates
- ▶ (date: 1585) Simon Stevin's book *The Tenth* popularizes decimal notation for fractions.
 - ▶ unwieldy notation

- ▶ sidesteps issues of infinite decimals

FRACTIONS: the sophistication of our modern view

- ▶ The following are all equal:

$$\frac{2}{5} = \frac{4}{10} = \frac{8\pi}{20\pi} = 40\% = 0.4 = 0.400 = 0.399999999\dots$$

- ▶ We accept the lack of precision in:

$$1/3 = 0.333 \text{ or } \pi = 3.14159$$

Summary

- ▶ 2000 BC: Sophisticated arithmetic with fractions
- ▶ 700 BC: Zero as placeholder
- ▶ 100 BC: Arithmetic with fractions essentially equivalent to our modern approach (positive integers only)
- ▶ 1585: decimals in Europe
- ▶ 1600's: Zero is allowed to be a solution to an equation
- ▶ 1800's: Zero as type of object; division as a human-defined operation