There are six problems each worth 10 points.

1. Suppose $r \geq 2$ is even. Show that no r-regular graph contains a bridge.

2. Given integers m and k where $2 \leq m \leq k$, show that there exists a graph G where G has a clique number of m and a chromatic number of k.

3. Suppose G is a graph. Show that G or the complement of G is connected.

4. Let M be a matching in a graph G, and let u be an M-unsaturated vertex of G. Prove that if G has no M-augmenting path that starts at u, then u is unsaturated in some maximum matching in G.

5. Suppose G is a planar graph and every vertex has even degree. Show that the regions can be assigned two colors so that if two regions are bounded by the same edge, then they are given different colors.

6. A graph G is said to be k-**degenerate** if every subgraph of G contains at least one vertex of degree at most k. Show that if G is k-degenerate then the chromatic number of G is at most $k+1$.