Statement of Recent and Current Research

My current research interests are focused on the development of methods and algorithms for solving control and identification problems for distributed parameter systems. These problems have important applications in science and engineering. My approach is based on deep and heretofore incompletely exploited connections between nonharmonic Fourier series, control theory for partial differential equations, inverse problems of mathematical physics, and signal processing. This approach is now recognized by specialists, and I collaborate with many mathematicians, scientists and engineers all over the world in developing my methods, in particular, the efficient Boundary Control method in inverse theory.

Using this approach we have recently solved several outstanding problems in the aforementioned areas of applied mathematics. Our results and their interdisciplinary importance can be briefly summarized as follows:

Control and inverse problems for partial differential equations on graphs. These problems belong to a rapidly developing interdisciplinary area of applied mathematics — analysis on (quantum) graphs. Differential equations on graphs are used to describe many physical processes such as mechanical vibrations of multi-linked flexible structures (usually composed of flexible beams or strings), propagation of electro-magnetic waves in networks of optical fibers, heat flow in multi-link networks, and also electron flow in quantum mechanical circuits. Recently mesoscopic quasi-one-dimensional structures (graphs), like quantum, atomic, and molecular wires, have become the subject of extensive experimental and theoretical studies. In physical terms, a quantum wire is a graph-like structure on the surface of a semiconductor, which confines an electron to potential grooves of a width of a few nanometers. A solid theory of inverse problems for the Schrödinger equation on graphs would be an important step towards designing quantum devices. Unfortunately, to date there are only a few results concerning inverse problems on graphs.

We developed a new, effective, and robust approach for solving inverse problems for partial differential equations on graphs using boundary and internal observations. Our approach is based on the Boundary Control method and uses the results on controllability of multi-link flexible structures. Since the number of edges of graphs arising in applications is typically very big, we suggested a recursive procedure for solving the inverse problem and developed an effective numerical algorithm. This procedure allows efficiently recalculating the inverse data from the original large graph to the smaller subgraphs, 'pruning' leaves of the graph step-by-step.

Our results are a breakthrough in inverse theory of differential equations on graphs. We also obtained the first numerical results solving inverse problems on graphs. Our methods allow us to recover not only the coefficients of the equations, but also the unknown topology and geometry (the angles between the adjacent edges). These results are based on new discoveries in inverse spectral theory. Our results can find numerous applications in engineering, nanotechnology, and communication theory including social networks.

Another important area of application of our results is neuroscience, in particular, dendritic trees of the central nervous system. Inverse problems here include identification of physical parameters and tree morphology from boundary observations. I continue collaboration in this field with biomathematician Jonathan Bell (UMBC).

Control and identification problems for systems with internal damping and systems with memory. Recently, we have obtained breakthrough results in control and identification problems for damped, hybrid and complex systems including controllability results for viscoelastic systems and heat equations with memory. This will have important applications in many areas, such as material science, nondestructive testing, geophysics, acoustic imaging, and remote sensing.

Our main mathematical achievement in this area is the extension of the classical Fourier method to partial differential equations with time dependent coefficients including wave and heat equations with memory. For that we proved the Riesz basis property of solutions to certain differential equations with time delays. These results are of significant interest in functional analysis in their own right.

Control theory methods in signal processing. We developed a new prospective direction in applied mathematics, which may be called "Applications of control theory to signal processing". Specifically, we demonstrated efficiency of this approach to sampling and interpolation problems and spectral estimation. We describe briefly our approach and results on these topics.

Construction of sampling and interpolating sets for signals with multi-band spectra. This problem is important for signal processing and has numerous applications in physics, engineering, and defense. Its solution yields both stable and non–redundant sampling of multi-band signals, and it gives a generalization of the Whittaker–Kotel'nikov–Shannon sampling formula, which has fundamental significance for accurate and robust transmission of information.

We have proposed a new approach to the problem based on connections between controllability of dynamical systems described by linear partial differential equations and the Riesz basis property of corresponding exponential families. We have related the problem of constructing the sampling set for spectrally constrained signals to the solution of certain kinds Wiener-Hopf equations, and we have described new techniques for solving these equations.

We have also solved a *sampling and interpolation problem for nonseparated sampling sets* and for *vector valued signals.*

Developing of an effective method in the spectral estimation problem, i.e. recovery of unknown N, a_n, and λ_n in a signal $f(t) = \sum_{n=1}^{N} a_n e^{\lambda_n t}$ by given samples $f(t_j), j = 1, \ldots, 2N$. There is an enormous literature devoted to this important problem in signal processing. We propose a new method based on our approach to dynamical inverse problems and their connections with control-theoretic ideas. This approach yields simple, fully linear recovery algorithms for the unknown parameters. The new method allows us to study various generalization of the spectral estimation problem, including cases of multiple spectra, irregular sampling, signals in the presence of noise, vector valued signals, and the case $N = \infty$.

Inverse problems in glaciology. The basal boundaries of glaciers and ice sheets are generally not observationally accessible. This poses a major problem for glacier modeling studies, because the basal boundary condition is an essential part of a well-posed problem. The surface, however, is accessible to ground based, airborne, and satellite measurements. This sets up a classic inverse problem, with too many boundary conditions at the top and not enough at the bottom for a system described by nonlinear partial differential equations.

We have applied new inverse methods to derive basal velocity fields over large areas. This is a major improvement in the understanding of the subglacial environment, and it represents a major step towards predictive glacier models. Such models are important in the study of glacier-climate interaction, dating of ice cores, and assessment of natural hazards.

We proposed several approaches to this very complicated inverse problem, including special iterative procedures. We obtained new stability estimates for this inverse problem and presented extensive numerical calculations which demonstrated good agreement with experimental glaciological data.